A METHOD TO FORMULATE ALGORITHM OF ADAPTIVE CONTROL OF ACTIVE NOISE CONTROL

Takahiko Ono

ONOSOKKI Co., Ltd.
Hakusan 1-16-1
Midori-ku
Yokohama-shi 226
Japan

Ken'iti Kido

Department of Computer Science
CHIBA INSTITUTE OF TECHNOLOGY
Tudamuma 2-17-1
Narashino-shi 275
Japan

ABSTRACT A new simple method to formulate the adaptive algorithm to control the coefficients of FIR filter is introduced. The filter is used in the active noise control system. The introduced algorithm includes the LMS algorithm as a special case. The validity of the theoretical result is confirmed by the computer simulation.

1. INTRODUCTION

In the active control of noise, the noise is canceled out by superposing sound wave of the same wave form and the inverse polarity to the original noise. The noise source and the noise transmission system usually vary with the passage of time and the wave form of the noise changes. Therefore the wave form of the cancellation sound should be always renewed and the technology of adaptive control is indispensable in the active noise control system. This paper explains a new simple consideration method of FIR filter.

2. ACTIVE NOISE CONTROL BY USE OF ADAPTIVE FILTER

When the primary noise source passes through a transfer system and comes to an observing point, the relation between the primary noise x(t), the observed noise y(t) and the impulse response of the transfer system h(t) is expressed by the following equation.

$$y(t) = x(t)\cdot h(t) = \int_0^{\infty} x(t-\tau)h(\tau)d\tau \qquad (1)$$

The noise is canceled if the sound of the same waveform of opposite polarity -y(t) is superimposed on the noise. When the source signal x(t) can be observed, it is possible to synthesize -y(t) from -x(t) and the impulse response of the system $h(\tau)$. But $h(\tau)$ varies with the air flow, the atmospheric temperature and the other boundary conditions. Therefore, the use of the adaptive filter to synthesize the cancellation wave is indispensable.

3. FORMULATION OF ADAPTIVE ALGORITHM

Figure 1 shows the schematic diagram of the noise transmission system with the adaptive control system. The notations are shown in the same figure.

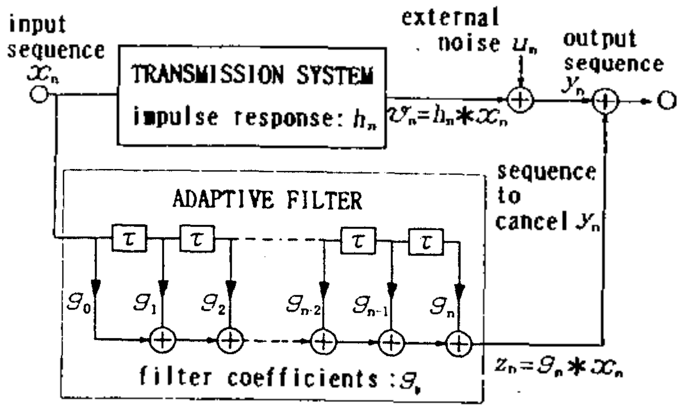

Fig.1 Schematic explanation of the system

The output of the noise transmission system y is expressed by the following equation:

$$y_n = \sum_{i=0}^{n} h_i x_{n-i} + u_n \tag{2}$$

Extracting the relation between y_n and h_p, Eq.(2) can be rewritten as the following.

$$y_n = h_p x_{n-p} + \Delta H_{np} + u_n \tag{3}$$

where,

$$\Delta H_{np} = \sum_{j(\neq p)} h_j x_{n-j}$$

The summation with j excludes the term of j=p.

The output of sequences z of the system g is expressed as the following.

$$z_n = \sum_{i=0}^{n} g_i x_{n-i} \tag{4}$$

In the same way as Eq.(3), the relation between z_n and g_p can be written as the following

$$z_n = g_p x_{n-p} + \Delta G_{np} \tag{5}$$

where,

$$\Delta G_{np} = \sum_{j(\neq p)} g_i x_{n-j}$$

Therefore, the n-th sample of canceled signal at O that is the difference between y_n and z_n is expressed as the following.

$$E_n = y_n - z_n = (h_p - g_p) x_{n-p} + D_{n,p} + u_n \tag{6}$$

where,

$$D_{n,p} = \Delta H_{np} - \Delta G_{np} = \sum_{j(\neq i)} x_{n-j}(h_j - g_j) \qquad (7)$$

Dividing Eq.(6) by x_{n-p}, the following equation is established.

$$(y_n - z_n) / x_{n-p} = h_p - g_p + D_{n,p} / x_{n-p} + u_n / x_{n-p} \qquad (8)$$

Equation (8) shows that the quantity $(y_n-z_n) / x_{n-p}$ is the sum of the difference $h_p - g_p$ and the remaining terms. And $h_p - g_p$ can be obtained as the many times average of $(y_n-z_n)/x_{n-p}$ if the averaged value of remaining terms is null.

The renewal algorithm of the filter coefficient g_p can be made from Eq.(8).

Under the assumption that the renewal of the filter coefficient g_p is carried out adding $\mu\,(y_n-z_n) / x_{n-p}$ to the m-th renewed value $g_p(m)$ using a small positive value of μ, the m+1-th renewed value $g_p(m+1)$ is written as the following.

$$\begin{aligned} g_p(m+1) &= g_p(m) + \mu \cdot (y_n-z_n) / x_{n-p} \\ &= g_p(m) + \mu h_p - \mu g_p(m) + \mu R \qquad (9) \end{aligned}$$

where,

$$R = D_{n,p} / x_{n-p} + u_n / x_{n-p} \qquad (10)$$

If R=0, $g_p(m+1)$ will be hp under the condition μ=1. But R can not be null. R takes random value and it's mean value is zero, as the input sequence x_n and the external noise u_n are random and their mean value is zero. And it can be seen from Eq.(7) that the value of R approaches to zero by the renewal for the value of $D_{n,p}$ decreases as g_j approaches to h_j.

Therefore, it is expected that $g_p(m)$ approaches to h_p by the renewal through $g_p(m+1)$ is occasionally more apart from h_p than $g_p(m)$ due to the fluctuation of the value of μ R.

This renewal algorithm is expressed by the following equation.

$$g_p(m+1) \leftarrow g_p(m) + \mu \cdot (y_n-z_n) / x_{n-p} \qquad (11)$$

This algorithm can not be used when x_{n-p}=0 as division by x_{n-p} is included. And renewal should be skipped when x_{n-p}=0.

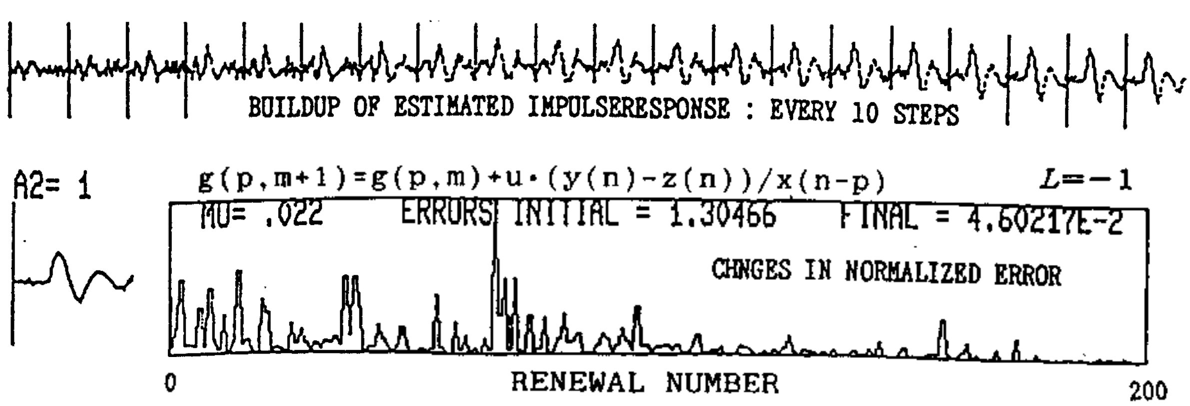

Fig.2 Computer simulation of ANC using the algorithm of Eq.(11)
(a) Filter coefficients in every 10 renewals.
(b) Changes in value of residuals with renewal number.

The change in the residual y_n-z_n by the renewal according to Eq.(11) is examined by the computer simulation. Figure 2 shows an example of the results. From those computations, it can be said that the residual converges to zero using adequate value of μ. But these is an operation which is seemed to be unreasonable in the algorithm given by Eq.(11), that is, the renewal is big when x_{n-p} is small and small when x_{n-p} is big. The improvement of the algorithm will be necessary.

4. IMPROVEMENT OF THE ALGORITHM

It is not necessary to be a constant for the value of μ in Eq.(11). Only the required condition is positive. We can change the value of μ in each renewal. We can multiply the absolute value of x_{n-p} to the 2nd term of the right hand side of Eq.(11). And the following algorithm is formulated.

$$G_p(m+1) \leftarrow g_p(m) + \mathrm{sgn}(x_{n-p}) \cdot \mu \cdot \{y_n(m) - z_n(m)\} \qquad (12)$$

The following algorithm can be formulated as the general algorithm.

$$g_p(m+1) \leftarrow g_p(m) + \mathrm{sgn}(x_n\text{-}p) \cdot |x_{n-p}|^L \mu \cdot \{y_n(m) - z_n(m)\} \qquad (13)$$

In this expression, Eq.(11) is the case of L=-1 and Eq.(12) is the case of L=0. Using bigger L, the bigger renewal is conducted when the value of x_{n-p} is big.

When L=+1, Eq.(13) is the same as the following algorithm which is well known as the LMS algorithm.

$$g_p(m+1) \leftarrow g_p(m) + x_{n-p} \cdot \mu \cdot \{y_n(m) - z_n(m)\} \qquad (14)$$

We can set L=1, 2, 3 or more. The adaptation will be faster when the bigger value of L is used. The sign of
x_{n-p} is necessary when L is even number for the value of $x_n\text{-}p^L$ becomes always positive. And the computer simulations were carried out for L=0, 1 and 3. The results are shown in Figs. 3, 4 and 5.

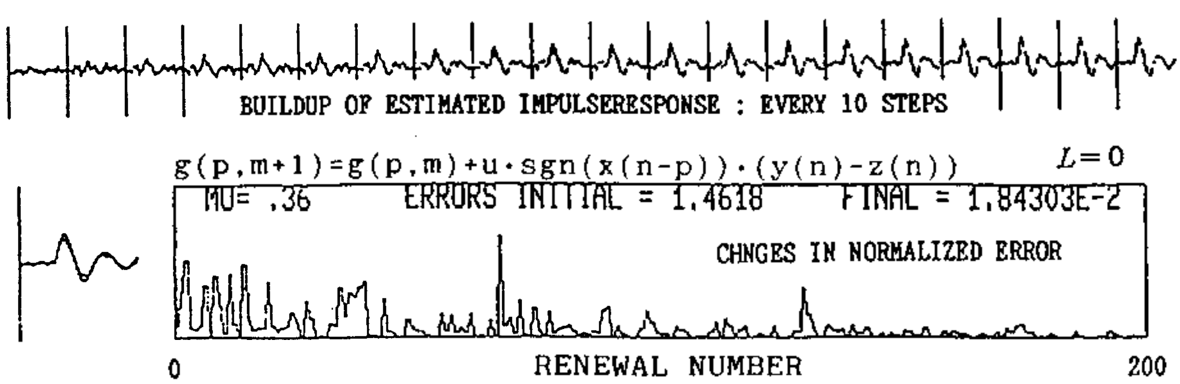

Fig.3 Computer simulation of ANC using the algorithm of L=0
(a) Filter coefficients in every 10 renewals.
(b) Changes in value of residuals with renewal number.

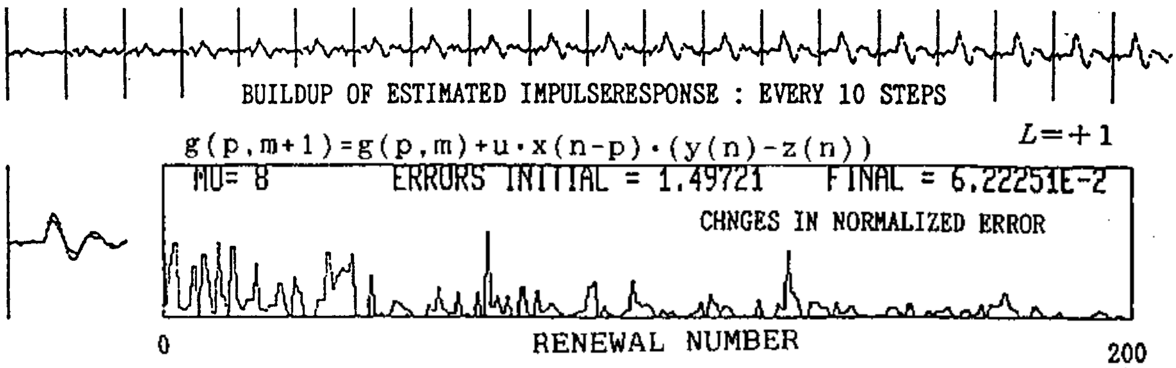

Fig.4 Computer simulation of ANC using the algorithm of L=1

(a) Filter coefficients in every 10 renewals.

(b) Changes in value of residuals with renewal number.

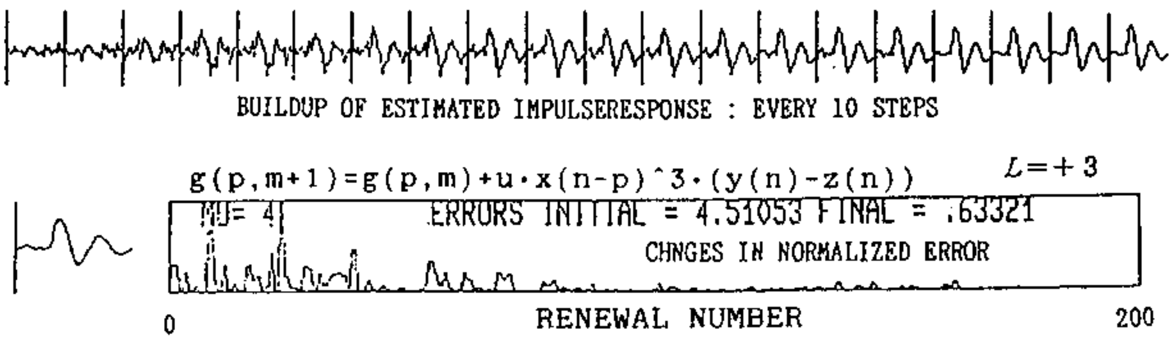

Fig.5 Computer simulation of ANC using the algorithm of L=3

(a) Filter coefficients in every 10 renewals.

(b) Changes in value of residuals with renewal number.

5. THE CASE OF SINUSOIDAL SOURCE SIGNAL

The above discussion is base on the assumption that the primary noise source is random. But the practical noise source is not always random. There are often the noise of sinusoidal wave or periodic wave and the sinusoidal noise is usually more annoying than random noise in hearing. The system should work well in the case of such the noise.

In the standpoint of system identification, the primary noise is required to be random. If the primary noise is not random, it takes longer time for identifying the system. But the identification of the transmission system is not necessary in the active noise control. Important problem is to reduce the resultant noise level.

The computer simulations were carried out to investigate what phenomena come out in the case of sinusoidal source signal. The results are shown in Fig.6. As seen in this figure, the output $y_n(m)$-$z_n(m)$ decreased very rapidly. In the case of L=1, the coefficients of filter is sinusoidal as anticipated. But, the coefficients are not sinusoidal in the cases of L=-1 and 0. It is considered for those cases that the system fallen down into a local minimum and the values at the local minima become sufficiently small. In all cases, the adaptation speed is more than 10 times comparing to the case of random input. Why the speed is so fast is not yet clear. And the same phenomenon was observed in the acoustical experiments using a duct.

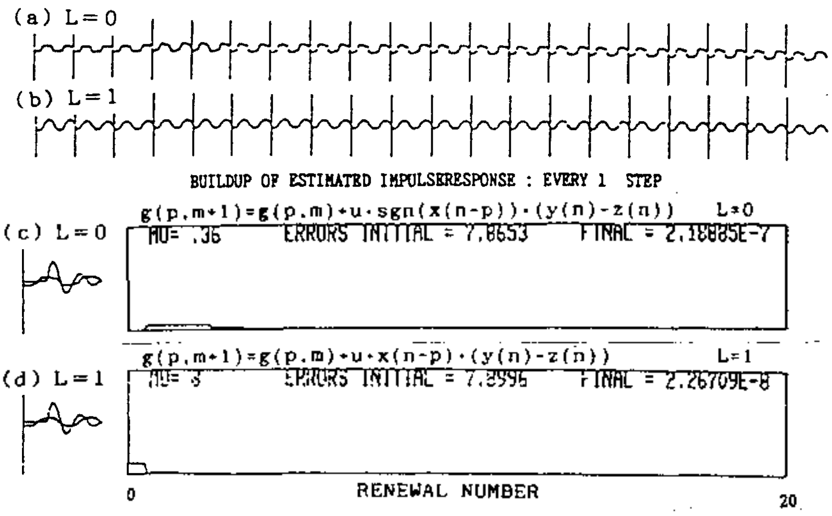

Fig.6 Computer simulation of ANC using the sinusoidal wave as the source signal for L=0, +1.
(a), (b) Filter coefficients in every 10 renewals.
(c), (d) Changes in value of residuals with renewal number.

6. SUMMARY

The principle of algorithm of the adaptive FIR filter is introduce based on a method different from the conventional method and a general adaptation algorithm is given with physical interpretation. The case of L=1 is the well known LMS algorithm. All the results are verified by the computer simulations.

The introduction of the algorithm is based on the assumption that the source signal is random sequence. But, according to the computer simulations and the acoustical experiments, the system converges more quickly in the case of sinusoidal input than in the case of random input. The reason is considered and discussed in this paper, but it is not yet fully clear.

Literature: B.Widrow, S.D.Stears : "Adaptive Signal Processing," Printice-Hall, Inc. Englewood Cliffs, N.J. (1985)