WIGNER-VILLE INTERPRETATION OF MUSICAL SOUND AND TRANSIENT VIBRATION SIGNALS

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ABSTRACT Very often, one would like to have visual image of mechanical or acoustical events such as musical sound and transient vibrations. Conventional methods to visualize the signal, such as power spectrum, do not normally allow to cultivate the signal of interests due to their inherent limitation on transient signals. Other than the conventional method, one could use an instantaneous frequency which can reveal the variation of frequency in terms of time. Nevertheless it is quite sensitive to noise and can not resolve the frequency components of signals; normally produces additional components other than those of the signals. In this paper, we introduce the Wigner-Ville spectrum to see the transient characteristics of signal, especially musical sound and transient mechanical vibration signatures. For musical sound, several popular western classic music have been selected for the analysis. For the transient mechanical signature, the signals obtained from the car door experiment and the beam experiment are interpreted in terms of Wigner-Ville spectrum. Results demonstrate the visual expressions of transient signals; musical sound and vibrations.

1. MOTIVES

Mechanical signals such as acceleration signals depicted by electrometers or sound pressure signals obtained by microphones are often acknowledged to be a rich and compact source of information which contains the condition or operating mechanism of a machine. This rather general postulate can be extended to whatever produces sound or vibration as long as the signals can be measured. It is noteworthy that, not surprisingly though, human being has used various signals depicted by his/her sensors; ear is one of the typical examples, to understand or analyze what he/she has interests.

A typical implementation of aforementioned realization which basically assumes that there must be a relation between signals and what is vibrating; structure or air for instance, in other words signal is a representative of effects of any cause, is 'machine condition monitoring' which nowadays has been very often tried to many sophisticated machines such as large turbo-machinery and space aircraft.

It is often in and practice that the response signals are only measurable or available ones; any signals which express the cause of the motion sometimes are not usually measurable. Therefore, one must have certain model to inter-relate between cause and effect; in fact, one must have numerous hypothetical model which can interpret the measured signals so that he/she can understand what in fact causes the responses. This situation is in strict sense, mathematically ill-posed problem since one would only have the response signal and nothing about the signals for the cause and those about the system between cause and effect. This ill-posed problem is in fact very popular in medical practice; medical doctors do not normally have unique solution; their hypothesis would be confirmed by surgery in usual practice. A doctor who got the name in his/her practice normally has various ways to examine the signals which are based not only on modern medical instruments but also on his/her past experiences on the signals. In fact, best doctor has an ability to analyze the signal in numerous ways; in other words, multi-dimensional, colorful view on the signals empowers his/her medical practice.

One of conventional ways to see the measured signals is to look at the signal in 'amplitude-time domain', in which one can see the change of amplitude-time signal. In fact, this representation of time signal contains every useful information of signal but sometimes it is rather difficult to interpret when it has many frequency components. This actually motivated other representation of signal, that is frequency domain expression of signal by using fast Fourier transform. For stationary signals, frequency domain interpretation leads many useful and powerful methods for the analysis of signals; i.e., transfer function estimation, spectral analysis, and cross-spectrum analysis are well known typical examples. For transient signals; it is noteworthy that in fact any signal can be regarded to be transient depending on the record length with respect to period of signal of interest, either amplitude-time interpretation or amplitude-frequency representation is not enough to fully visualize the signals. For example, the change of frequency in time can not be well expressed by those two expressions. Next introduces the methods which can see the signals of interests with respect to amplitude-frequency-time, so that one can extend the dimension of signal interpretations, getting more colorful view on the signals, as distinguished medical doctors do.

2. INSTANTANEOUS FREQUENCY AND WIGNER -VILLE SPECTRUM

Instantaneous frequency is nothing but an expression of signal in terms of frequency and time so that one could reveal the transient characteristics of the signal. Basic method for obtaining instantaneous frequency comes from the realization of the fact that frequency is the change of phase with respect to time; any signal can be expressed in terms of amplitude and phase. This interpretation of the signal in terms of complex variable is known to be 'analytic signal'. The instantaneous frequecny(f(t)) then can be obtained as

$$f(t) = \frac{\dot{\tilde{x}}(t)\, x(t) - \dot{x}(t)\, \tilde{x}(t)}{2\pi \left\{ x^2(t) + \tilde{x}^2(t) \right\}} \tag{1}$$

where x(t) denotes any signal which is measured in time domain, $\tilde{x}(t)$ expresses the Hilbert transform of x(t), and $\dot{}$ expresses differentiation with respect to time; for the details, see Kim and Lim [1].

It is noteworthy that the instantaneous frequency (Equation (1)) is in fact non-linear with respect to the measured and its Hilbert pair. This implies that if there is more than one frequency component at any instant of time, then the estimated instantaneous frequency based on Equation (1) would produces additional components in general. Only for some special cases of which the amplitude ratio between the signals which have different frequency components at an instant is much larger than unity, or when the difference of frequencies are very large, then the instantaneous frequency has been proved to show the main frequency component of interests[1].

Wigner-Ville spectrum which was introduced in 1932, considers a transformation of a wave function into probability function of the simultaneous values of n independent variables for the coordinates and n momenta. If one considers only instant of time and frequency then Wigner-Ville spectrum W(t,ω) can be written as

$$W(t,\omega) = \int_{-\infty}^{\infty} s(t+\frac{\tau}{2})\, s^*(t-\frac{\tau}{2})\, e^{-j\omega\tau}\, d\tau \qquad (2)$$

which in fact represents instantaneous energy of signal along frequency axis if one see Equation (2) at fixed or arbitrary time, or amplitude variation of signal along time axis for an arbitrary frequency. As one could see from the expression of Equation (2), the estimation of Wigner-Ville spectrum involves the use of window function due to finite record length. This effect on the bias error on the estimation is in fact proportional to 2nd derivatives of Wigner-Ville and the variance of window with respect to time and frequency [3].

Fig. 1 illustrates various realization of the vibration signal measured by an accelerometer which was mounted on the surface of a passenger car. The vibration is due to the slamming of car door; for detail explanation, see [4]. As one can easily anticipates the power spectrum of the signal just exhibits very broad distribution of vibration energy over a wide frequency range; 0-1kHz, which does not show any information concerning the action of door slamming. On the other hand, the instantaneous frequency essentially demonstrates that the door excited 400 Hz in the beginning then 200 Hz component and lastly 100 Hz. These frequency components might not well represents any specific mechanical components of car door since as pointed out, the instantaneous frequency produces additional components and exhibits a frequency which may have a relation with frequencies which were excited, when there are many frequencies at the time of interest. This turns out to be true, since Wigner-Ville representation of the signal (Fig. 1) says that in the beginning of slamming, there are mainly two components of frequency; about 200 Hz and 550 Hz. The time of which the 2nd mechanism of the door was operated can also be seen by the amplitude-time graph, instantaneous frequency graph, and more colorfully by the Wigner-Ville spectrum. In fact, Wigner-Ville spectrum shows that the 2nd action generates rather broad band vibration; the band width is about 550 Hz. From this typical application of Wigner-Wille spectrum along with the plots of power spectrum, instantaneous frequency, and amplitude-time, one now has better visual image of the signal, which of course leads colorful and rational imaginations concerning the causes and dynamic systems, therefore can do whatever he/she wants without mechanical surgery. Fig. 2 and Fig. 3 are other examples which shows a kind of 'finger print' of signal, especially for whom wants to have visual image of music, Fig. 3 would be what he/she want to spend some time to have various imaginations.

REFERENCES

1. Yang-Hann Kim and Byoung-Duk Lim, "Instantaneous frequency of a transient mechanical signature and its estimation by a moving window; applicability and physical interpretation," J. of Mechanical Systems and Signal Processing, 1994, (in press).

2. E. Wigner, "On the quantum correction for thermodynamic equilibrium,' Physical Review, Vol. 40, pp 749-759, June 1, 1932.

3. Yon-Kyu Park and Yang-Hann Kim, "The effects of window on the bias error of Wigner-Ville spectrum estimation," in preparation.

4. Y.-K. Park, and Y.-H. Kim, "Comparison of estimation methods of instantaneous frequency and examples of its application to beam, engine block, and car door vibration, " J. of the Korean Society of Noise and Vibration Engineering, Vol. 3, No. 4, pp 341-359, 1993.

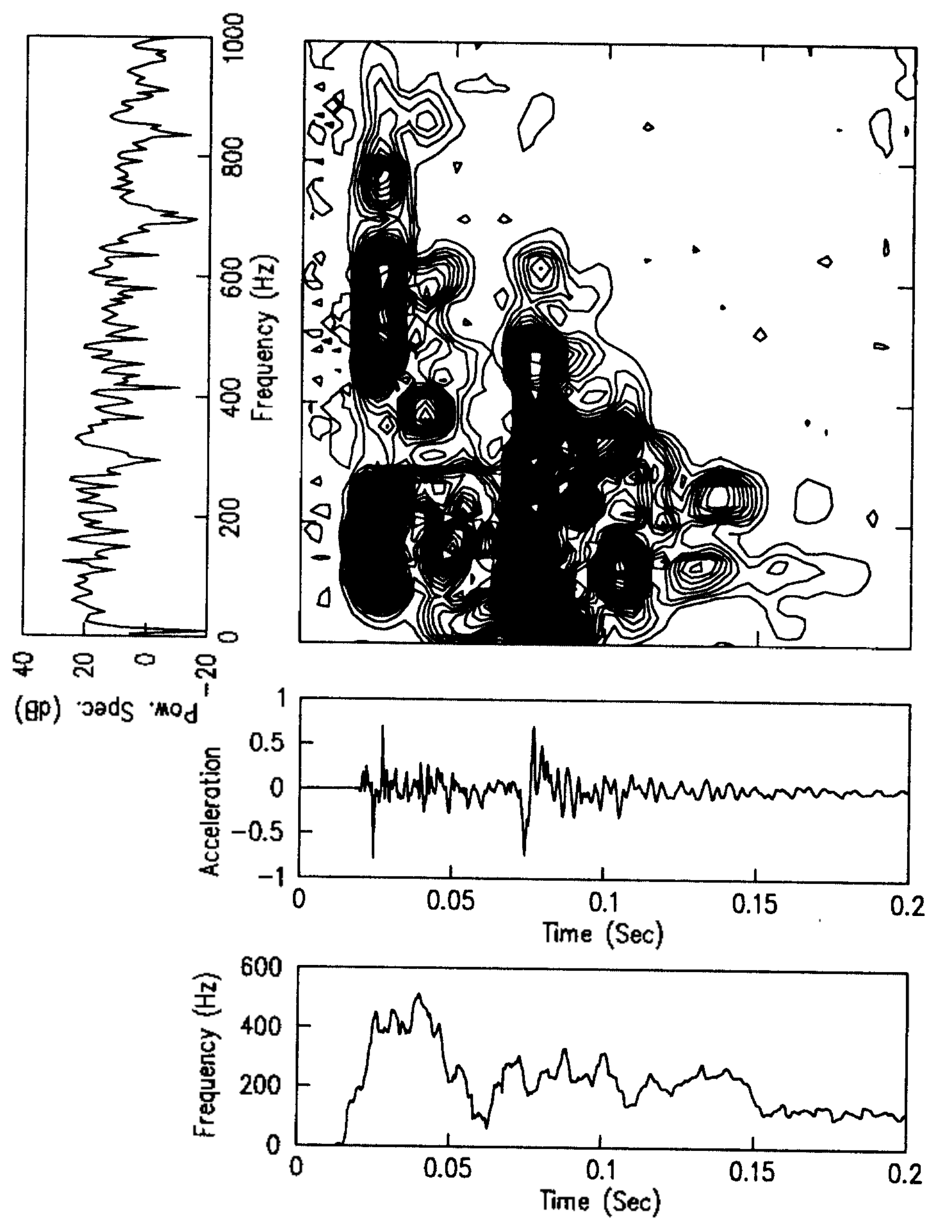

Fig. 1 Wigner-Ville distribution of the acceleration signal caused by closing the car door (Smoothing window = Gaussian window : $\sigma_t = 20\Delta t$, $\sigma_\omega = 20\Delta\omega$, Number of data = 1024, Sampling frequency = 5.12 kHz)

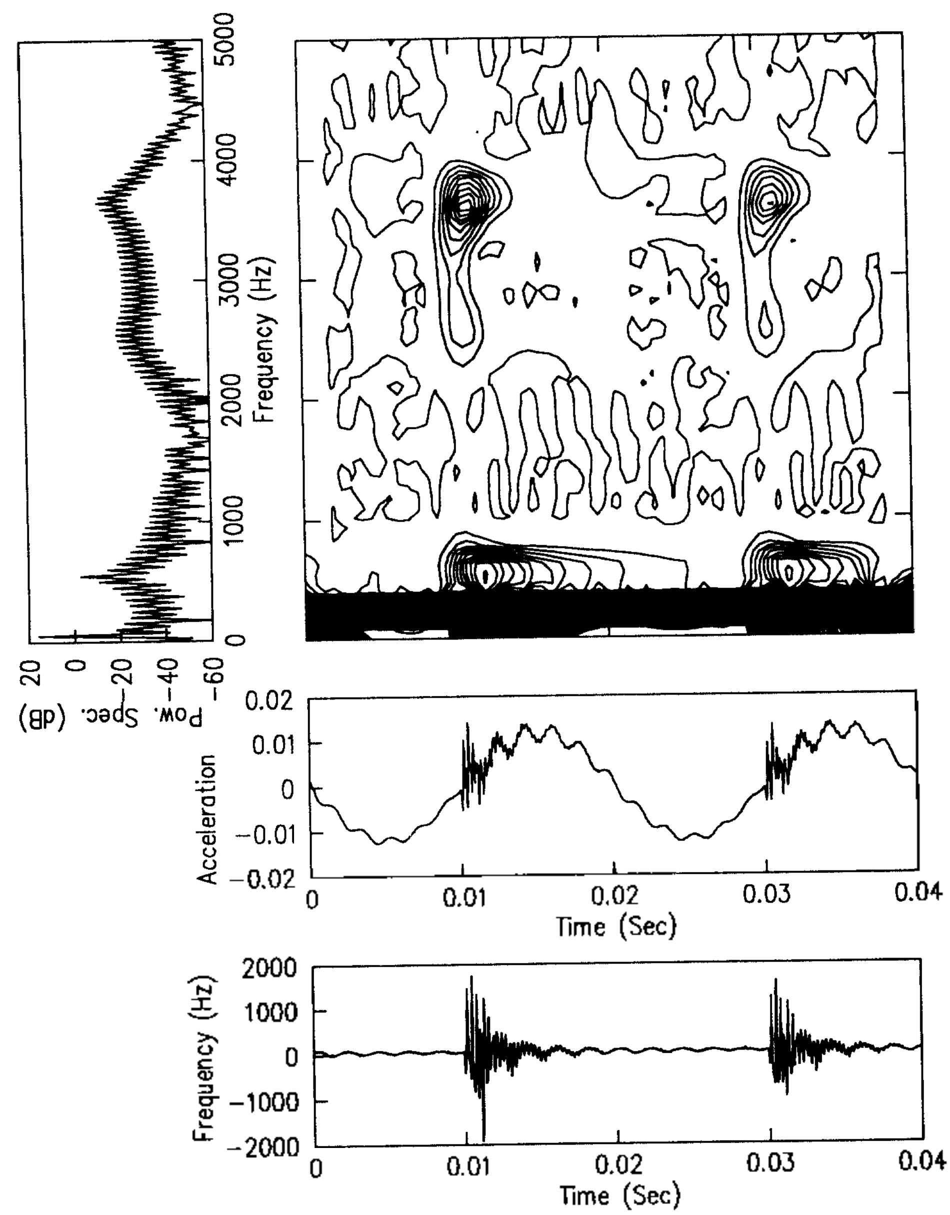

Fig. 2 Wigner-Ville distribution of the acceleration signal caused by the impact on a beam which is under sinusoidal motion

(Smoothing window = Gaussian window : $\sigma_t = 20\Delta t$, $\sigma_\omega = 20\Delta\omega$, Number of data = 1024, Sampling frequency = 25.6 kHz)

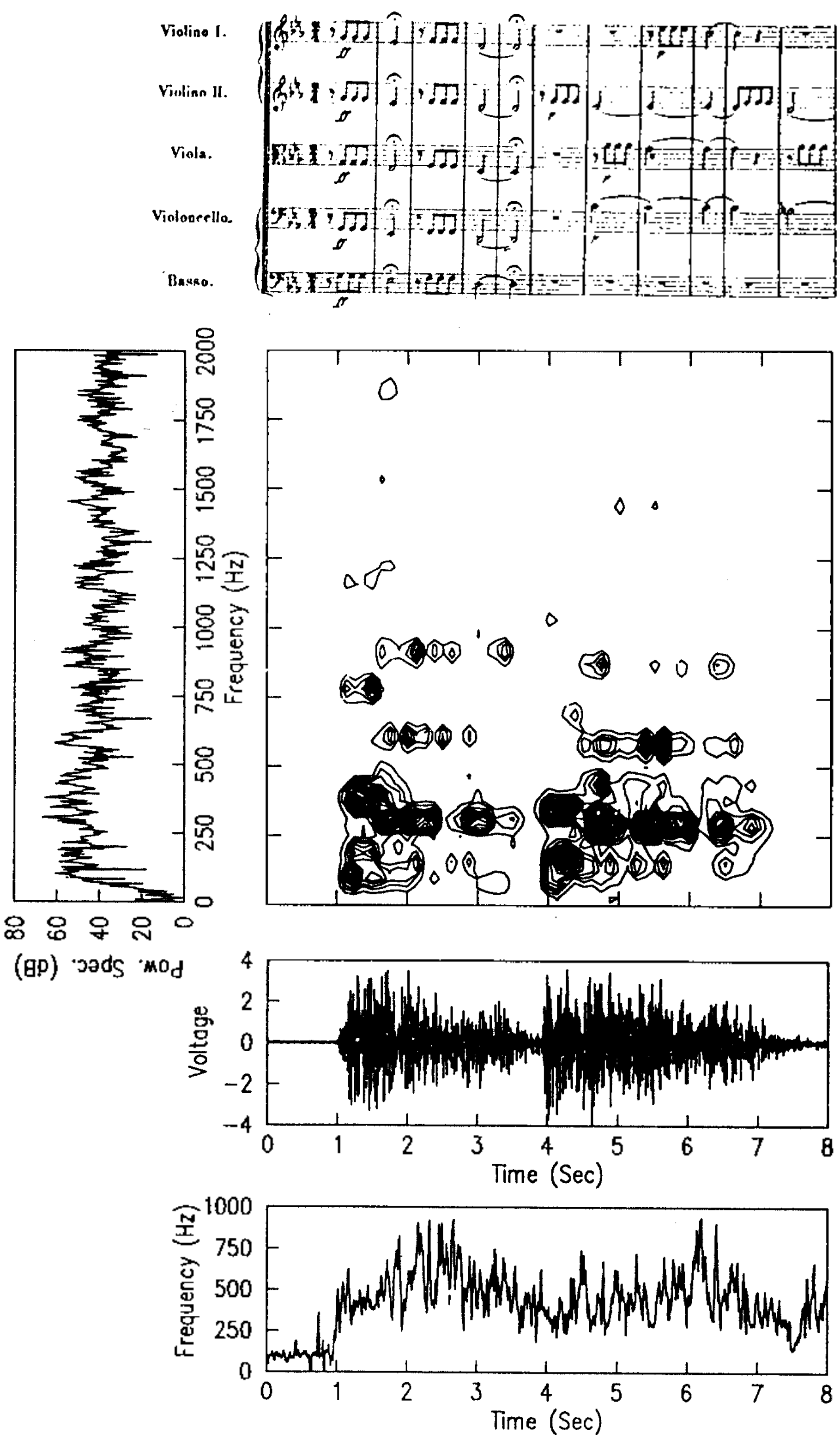

Fig. 3 Wigner-Ville distribution of a musical signal : Beethoven Symphony No. 5 (Smoothing window = Gaussian window : $\sigma_t = 110\Delta t$, $\sigma_\omega = 110\Delta\omega$, Number of data = 65536, Sampling frequency = 8.0 kHz)