CORRECTION METHOD OF ESTIMATED INSERTION-LOSS WITH FLOW

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ABSTRACT The four-terminal transmission matrix method has been widely used to estimate the insertion-loss. However, the predictions using the equations in the four-terminal transmission matrix method do not reflect a practical phenomenon accurately. In this paper, the correction method to derive the insertion-loss for a constant sound pressure source is presented. The method of correction to the four-terminal transmission matrix method was proposed by rewriting the real and imaginary parts as they depend solely on the flow velocity. Then the result was compensated for by adding the component of the temperature gradient.

1. INTRODUCTION

Insertion-loss is widely used to predict the acoustic performance of mufflers , and it is evaluated by B or D parameter of the four-terminal matrix method. However, the prediction by means of this method does not reflect a practical phenomenon accurately. Because of the assumptions such as no-flow, constant temperature distribution in the analysis, the acoustic characteristic obtained by calculation are different largely from measured ones. We have already discussed about the errors ensuing from some assumptions in the four-terminal transmission matrix method by using the Characteristic Curve Method and proposed a method to correct the insertion-loss for a constant volume velocity source (1). In this paper, the correction method to derive the insertion-loss for a constant sound pressure source is presented and we compare our results with Prasad (2).

2. METHODS

As far as constant sound pressure source is concerned, The insertion-loss IL defined by (3)

$$IL = 10\,\mathrm{Log}\frac{W_r}{W_0} = 20\,\mathrm{Log}|B| + 20\,\mathrm{Log}\,R_r \qquad (1)$$

where W_r and W_0 are the radiated power at one point in space with or without the straight pipe or muffler inserted between that point and the source. R_r is the radiation impedance.
As the attenuation constant α is small compared with wave number k , for $\alpha L \ll 1$ in general, the B and D parameters can be described by the following approximation equation

$$B = Z\cdot\sinh(\alpha+jk)L \qquad (2)$$

$$20 \, \mathrm{Log}|B| = 10 \, \mathrm{Log}|\sin^2 kL + (\alpha L)^2 \cos^2 kL| + 10 \, \mathrm{Log}|Z| \tag{3}$$

where Z is the characteristic impedance defined by $Z=\rho c / S$, ρ is the mass density, c is the wave velocity, S is the cross-sectional area and L is the pipe length.

The B constant is computed under the assumption in which the system would be linear, isentropic, zero mean flow, zero temperature gradient, and so on. However, these assumptions are not realized in the actual exhaust system. A strict result can be obtained only when the pressure P, velocity V and mass-density ρ at any point in the exhaust pipe satisfy the following three basic equations [4]

Continuity $$\frac{\partial \rho}{\partial t} + \rho \frac{\partial V}{\partial x} + V \frac{\partial \rho}{\partial x} = 0 \tag{4}$$

Motion $$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + F = 0 \tag{5}$$

Energy $$\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x} - c^2 \left(\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} \right) - E = 0 \tag{6}$$

where $E=\rho (h-1)(q+V \cdot F)$, h is the ratio of the specific heat, F is the wall-friction, c is the wave velocity and q is the heat added per unit mass per unit time.

The Characteristic Curve Method [5] converts the partial differential equation of Eq. (4) to Eq. (6) into six total differential equations as follows

$$C^+ \begin{cases} \dfrac{dP}{dt} + c\rho \dfrac{dV}{dt} - E + c\rho F = 0 & (7) \\ \dfrac{dx}{dt} = V + c & (8) \end{cases}$$

$$C^- \begin{cases} \dfrac{dP}{dt} - c\rho \dfrac{dV}{dt} - E - c\rho F = 0 & (9) \\ \dfrac{dx}{dt} = V - c & (10) \end{cases}$$

$$C^d \begin{cases} \dfrac{dP}{dt} - c^2 \dfrac{d\rho}{dt} - E = 0 & (11) \\ \dfrac{dx}{dt} = V & (12) \end{cases}$$

Equations (7), (9) and (11) are the compatibility equations which are only valid along the respective characteristic lines, Eq. (8),(10) and (12).

In conventional method, boundary conditions has often used [1], i.e.

[1] At the inlet of pipe(x=0), pressure-time pattern P_I is applied.

[2] At the effective length of pipe ($L_{eff} = L+0.6 \cdot d$), $P = P_0$. Where d is the diameter of pipe and P_0 is the ambience pressure.

The numerical calculation is carried out through Eq.(7) to Eq.(12) to find the flow velocity V_1 and V_2 at the inlet and outlet pipe, respectively. The B parameter can be evaluated by $B= P_{SI} / U_2$ in the frequency domain where P_{SI} is the sound-pressure transformed from P_I and $U_2 = V_2 \cdot S$. The wall-friction F and the heat added per unit mass per unit time q are defined by [4]

$$F = \zeta \frac{L}{d} \frac{\overline{V}^2}{2g} \tag{13}$$

$$q = \frac{4\alpha_p}{d \cdot \rho} (T_0 - T_G) \tag{14}$$

where ζ is the friction coefficient, $\bar{V}$ is the average velocity, d is the pipe diameter, T_G is temperature of gas and T_0 is external temperature.

The friction coefficient ζ and the heat transfer coefficient α_p are given by Colebrook and Kays as follows [6]

$$\frac{1}{\sqrt{\zeta}} = 2\mathrm{Log} \left| \frac{\varepsilon}{3.71d} + \frac{2.51}{Re\sqrt{\zeta}} \right| \tag{15}$$

$$\alpha_p = 0.023\, Re^{0.8}\, Pr^{0.5}\, \lambda / d \tag{16}$$

in which Re is Reynolds number defined by $Re = \bar{V} d / \mu$, μ is the coefficient of viscosity, ε is the standard roughness, Pr is Prandtl number and λ is the friction factor.

The measurements data such as the pressure-time pattern P_1 and the gas temperature characteristic are required to calculate and examine the correctness of the results obtained by the Characteristic Curve Method. The experiment was carried out with a straight pipe which was connected to an engine system (four-stroke engine, 144cc, 3600rpm). The pressure sensor P_{S1} was attached at the inlet of pipe to measure the pressure waveform P_1 which is then used in calculations under boundary conditions [1]. The one P_{S2} was located at 20cm from P_{S1} to measure the pressure waveform P_2 which is used to compare with the calculation result to assert the correctness of the method. The measurements of gas temperature and the average flow velocity were also done in several points in the pipe by using a thermoelectric couple and anemometer after the engine had reached a stable state. The results of the measurements are shown in Fig. 1. Like the case of a car climbing uphill, if a fixed RPM is to be maintained, it is necessary to increase the acceleration which will cause the flow velocity and temperature at the exhaust valve to rise accordingly as the gas is forced through the pipe. The increase of temperature along the pipe as well as the increase of temperature gradient can affect the acoustic propagation considerably.

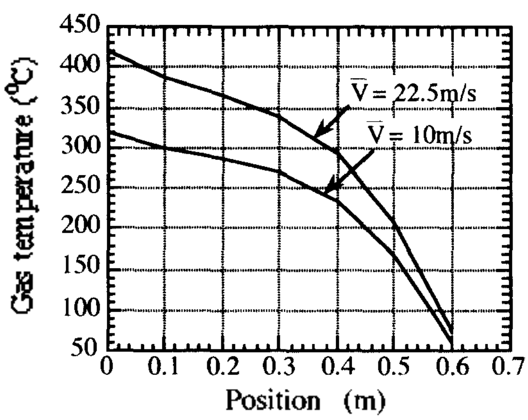

Fig. 1 Measurement results of gas temperature (L=0.6m, d=0.042m)

3. RESULTS AND DISCUSSION

Figure 2 shows the B parameter calculated by the four-terminal transmission matrix method and Characteristic Curve Method of a straight pipe of 60cm long and 4.2cm in diameter . The upper and lower figures show the results when mean flow velocities are 10m/s and 22.5m/s, respectively. The average of gas temperatures are 230 ^{0}C for 10m/s case and 306 ^{0}C for 22.5m/s case. In the computation based on the four-terminal transmission matrix method , sound velocity c is defined by $c=331.5+0.61\bar{T}$ in which $\bar{T}$ is given by the mean value of the gas temperature measured in the pipe. Similarly, the mass-density ρ is also given by the mean value. The follows are observed; (a) In the low frequency range, the results obtained from the four-terminal transmission matrix method and those from the Characteristic Curve Method are very similar. However, the difference of two results become large along with an increase of frequency. (b) The frequencies at which the resonances occur in the two methods are slightly different. Moreover, the values at resonance and antiresonance are not uniform.

The correction of B parameter is performed by making the real and imaginary parts of Eq. (2) as they depend solely on the flow velocity. Then we compensate for the result by adding the contribution part of the temperature gradient. This correction method is the extension of the method proposed by Nishimura, *et al.* [1].

To meet the condition (a) requiring the resonance frequencies to be shifted to the lower frequency range we suggest to replace k with $\beta k (\beta>1)$. Equation (2) becomes

$$B = Z \cdot \sinh(\alpha + j\,\beta \cdot k)L \tag{17}$$

To meet the condition (b), we propose to add a second term to the original by replacing $\sinh(\alpha+jk)L$ with

$$B = Z \cdot \{ \sinh(\alpha_M + j\,\beta k)L + j\,\gamma \sin^2(\beta kL) \} \tag{18}$$

in which α_M, a factor dependent on mean flow V should be chosen so that $\alpha_M > \alpha$. The second term is to create different values at antiresonances at odd and even modes. γ constant is used to adjust their amplitude.

Moreover, in order for the values at antiresonances to decrease exponentially with an increase of frequency as shown in Fig. 2. We propose to multiply Eq.(18) with $\exp(\alpha'_M \beta kL)$

$$B = Z \cdot \exp(\alpha'_M \beta kL) \{ \sinh(\alpha_M + j\,\beta k)L + j\,\gamma \cdot \sin^2(\beta kL) \} \tag{19}$$

where α'_M is a correction factor. The value of γ, α_M and α'_M could be determined by finding $|B|^2$ using Eq.(19) and applying the values of resonance and anti-resonance at some points in Fig. 3, namely :

$$|B|^2 = Z \cdot \exp(\alpha'_M \beta kL) \left[\left\{ \cosh(\alpha_M L) \cdot \sin(\beta kL) + \gamma \cdot \sin^2(\beta kL) \right\}^2 + \cos^2(\beta kL) \cdot \sinh^2(\alpha_M L) \right] \tag{20}$$

At resonance frequency ($\beta \cdot kL = n\pi$) Eq.(20) becomes

$$|B|^2 = Z \cdot \exp(\alpha'_M \beta kL) \sinh^2(\alpha_M L) \tag{21}$$

At antiresonance frequency ($\beta \cdot kL = n\pi/2$) Eq.(20) becomes

$$|B|^2 = Z^2 \cdot \exp(\alpha'_M \beta kL) (\gamma + (-1)^n \cosh(\alpha_M L))^2 \quad (n=1, 2, 3,.....) \tag{22}$$

To include the temperature gradient characteristic in the equation, we propose to add a term $\exp(\alpha_T)$ to Eq.(19)

$$B = Z \cdot \exp((\alpha'_M + \alpha_T)\ \beta kL) \{ \sinh(\alpha_M + j\ \beta k)L + j\ \gamma \sin^2(\beta kL) \} \qquad (23)$$

where α_T is the correction factor; function of temperature gradient Tr defined by Tr = (Tin - Tout) / (Tin + Tout).

With $\overline{V}$ varying from 8m/s to 32m/s, Tr varying from 0 to 0.75, using the proposed method, we obtain the following values for the factors :

$$\alpha_M = \alpha + 0.56\,M\ ;\ \alpha'_M = -0.43\,M\ ;\ \alpha_T = -0.043\,Tr\ ;\ \gamma = 8.26\,M \qquad (24)$$

where M is Mach number defined by $M = \overline{V} / c$.

Note that Eqs (23) will reduce to Eq.(2) when no mean flow and no temperature gradient are present. Also, the four-pole parameters could be computed for the case whether the mean flow is present or not, and also for the case when the temperature gradient is present or not.

Similar study has been reported by Prasad and Crocker [2]. They derived the four-pole parameters for a straight pipe in the presence of a uniform mean flow and a linear temperature gradient by using first-order perturbation theory and a Green's function approach. No loss was assumed in their analysis, therefore the B and D parameters which they derived differ from our equation. However, it is found that the general tendency of mean flow and temperature gradient characteristic which they derived are similar to those of our results as shown in Fig. 4. Note that the calculation in Fig. 4 was performed on the same conditions as Fig. 3 and in order to satisfy a condition of linear temperature gradient, the pipe is divided into 32 segments in our computation.The fact that Prasad did not take loss due to the friction and heat transfer in a straight pipe, leads us to believe that their equation can be used only in the limited areas of velocity and temperature gradient.

3. CONCLUDE

The method of correction to the four-terminal transmission matrix method based on the Characteristic Curve Method was proposed by rewriting the real and imaginary parts as they depend solely on the flow velocity. Then the result was compensated for by adding the component of the temperature gradient. The proposed formula is given by Eq. (23) which could be computed whether mean flow or temperature gradient is present or not.

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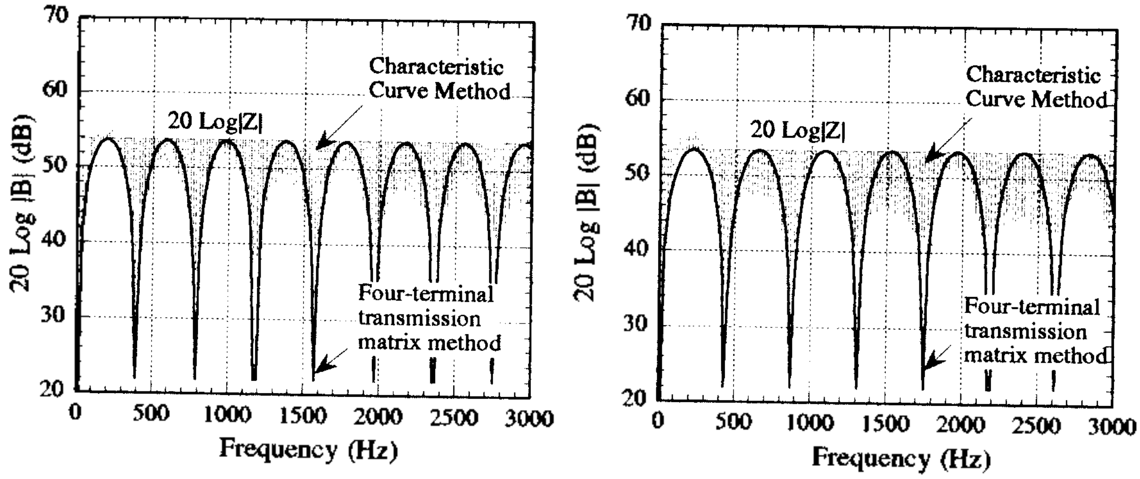
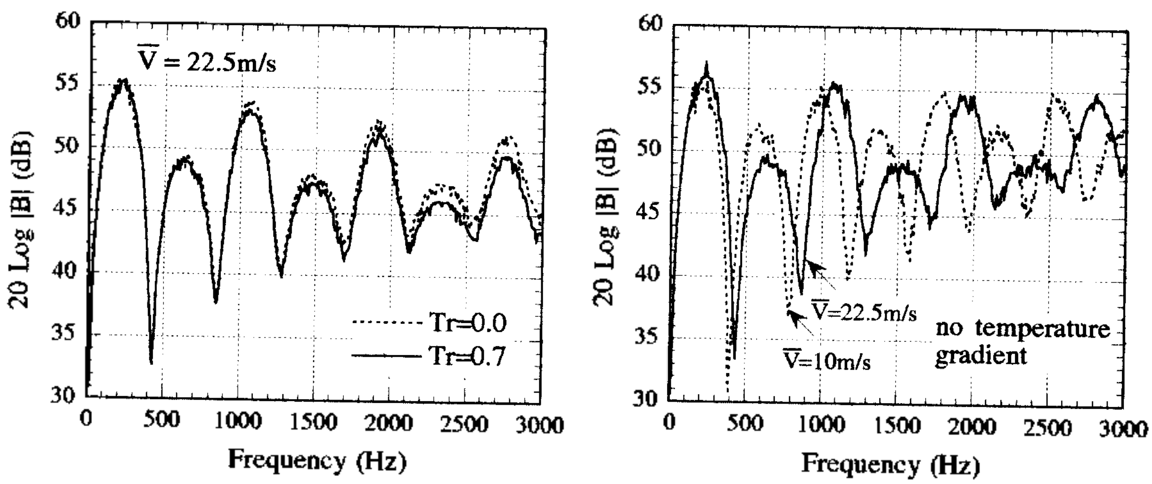


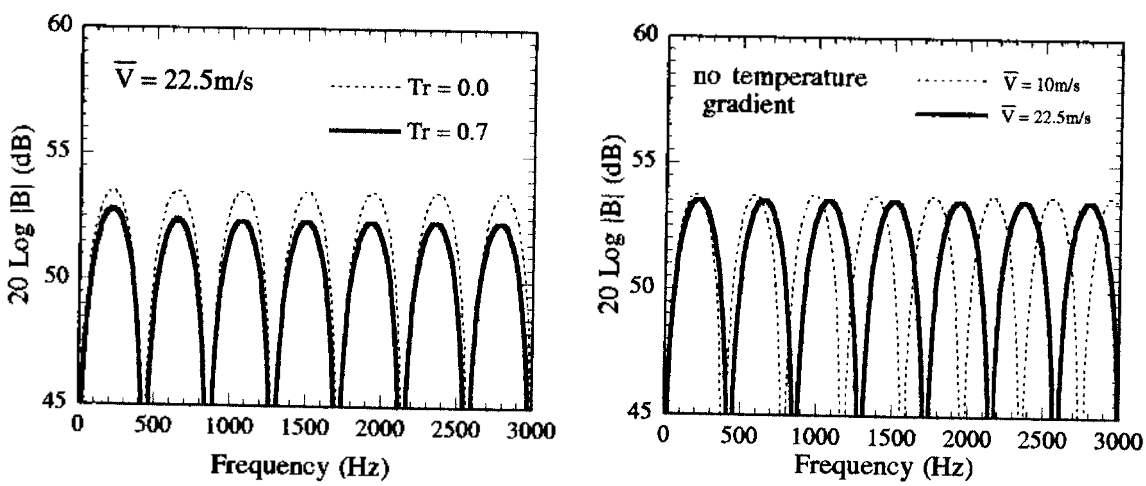
(a) (b)

Fig. 2 B parameter with mean flow of 10m/s and 22.5m/s.



(a) (b)

Fig. 3 Effects of mean flow and temperature gradient .



(a) (b)

Fig. 4 Results of Prasad.