EFFECTS OF PARTICLE RESONANCE ON DISPERSION OF ELASTIC WAVES IN PARTICULATE COMPOSITES

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ABSTRACT Elastic wave propagation in discrete random medium is studied to evaluate the effects of particle resonance on dispersion and attenuation of composite materials containing spherical inclusions. The frequency-dependent wave speed and attenuation coefficient can be obtained from proposed self-consistent method. It can be observed that the abrupt increase of effective wave speed and the concurrent peak of attenuation at low frequency is due to the lowest resonance of particles, whereas those in high frequency region are due to higher ones. The lowest resonance is mainly caused by the density mismatch and higher resonances by the stiffness mismatch between matrix and particles. The dispersion and attenuation of elastic waves in particulate composites are affected by the lowest resonance much more than by higher ones.

1. INTRODUCTION

In this paper, three conditions that must be satisfied by the dynamic effective density and Lame parameters of composite medium are derived without limit of frequency in a self-consistent way. The method for predicting the dynamic effective properties of composites proposed in this study is closely related with the coherent potential approximation in disordered alloy physics.[1-3] Frequency dependent effective density and Lame parameters can be directly obtained by solving the aforementioned self-consistent conditions. This also differentiates the present method with others in which the parameters are usually obtained from the wave speeds by assuming the dynamic effective density as the volume-weighted average density. Comparing the dispersion and attenuation curves with those of effective properties, one can clearly realize that an important physical phenomenon related with the dynamic properties of discrete random medium is the resonant scattering of particles in effective medium. Moreover, one can easily observe the influence of resonance modes on the dispersion and attenuation. In this manner, more insights can be given into the underlying physics of elastic wave propagation in discrete random media. Resultant wave speeds and attenuation coefficients calculated from dynamic properties are compared with the experimental results of Kinra *et al.*[3,4] as well as with the theoretical results of Waterman and Truell[10] where the same order of approximation with the present theory was employed.

2. THEORY

The self-consistent way to predict the effective properties can be summarized as follows: firstly, the effective medium is defined that the field in that medium is the mean field. Therefore, the sum of fluctuations due to the variations of material property from that of effective medium should be vanished on the average. To obtain the self-consistency, the true matrix and inclusions are embedded in the effective medium having undetermined elastic properties. The property of effective medium is then determined by setting the ensemble average of the total scattering operator, $\mathbf{T}$, to be zero. The total scattering operator contains the whole multiple scattering processes due to the embedded true matrix and inclusions in effective medium. However, since the computation of total scattering operator is not feasible in general, the self-consistency condition is usually approximated by using the single scattering operator, $\mathbf{t}$.[2,3] That is, the properties of effective medium can be determined from the following condition:

$$\langle \mathbf{t} \rangle = 0, \tag{1}$$

where $\langle\ \rangle$ is an operator for ensemble averaging over composition, orientation, and shape of scatterers. For the sake of simplicity, the scatterer will be modeled as an equivalent sphere that is identical to all constituents as shown in Fig.1, and thus the averaging is performed only for the compositions as follow:

$$\langle \mathbf{t} \rangle = \sum_j v_j \mathbf{t}^j = 0, \tag{2}$$

where v_j and $\mathbf{t}^j$ denote the volume fraction and scattering operator of the j-th composition, respectively. When the mean field is assumed to be the plane longitudinal wave field propagating in the effective medium as shown in Fig.1, then the displacement of the mean field can be expressed as

$$\bar{\mathbf{u}} = \hat{\mathbf{a}} \exp\left[i(k_l^e \hat{\mathbf{a}} \cdot \mathbf{r} - \omega t)\right], \tag{3}$$

where $\hat{\mathbf{a}}$ means the unit vector of motion having the same direction with the propagating direction. The superscript 'e' means a quantity of the effective medium. The effective longitudinal wavenumber, $k_l^e (= \omega / c_l^e)$, remains unknown yet. The self-consistency condition of Eq.(1) can be equivalently rewritten as

$$\left\langle \hat{\mathbf{a}} \cdot \mathbf{t} \cdot \hat{\mathbf{a}} \right\rangle = 0. \tag{4}$$

where the dyadic notation is used. In Eq.(4), the self-consistency condition is reexpressed by the average scattered wave which is made to be vanished in forward direction. In order to obtain three self-consistency conditions, the far field scattering displacement is to be evaluated. The scattered field is given by

$$\tilde{\mathbf{u}} \sim \mathbf{f}(\mathbf{k}_l^e) \frac{\exp(ik_l^e r)}{r} + \mathbf{g}(\mathbf{k}_s^e) \frac{\exp(ik_s^e r)}{r}, \tag{5}$$

where $\mathbf{f}(\mathbf{k}_l^e)$ and $\mathbf{g}(\mathbf{k}_s^e)$ denote the scattering amplitudes of the longitudinal and shear waves at far field in the direction of $\mathbf{r}$, respectively. The longitudinal scattering amplitude can be expressed as

$$\mathbf{f}(\mathbf{k}_l^e) = \frac{ik_l^e}{4\pi\rho^e\omega^2}\hat{\mathbf{r}}\int_\Omega \left\{\delta\rho\omega^2\mathbf{u}\cdot\nabla + \delta\lambda k_l^{e^2}\Delta - 2\delta\mu\mathbf{E}:\nabla\nabla\right\}\exp(-i\mathbf{k}_l^e\cdot\mathbf{r})dr \tag{6}$$

where $\Delta(=\varepsilon_{ii})$ and $\mathbf{E}$ are dilatation and strain tensor inside the scatterer, respectively, and the colon implies the scalar product of second order tensors.

By using the relationship between scattering operator and forward scattering field,[5] one obtains

$$\hat{\mathbf{a}}\cdot\mathbf{t}\cdot\hat{\mathbf{a}} = \frac{4\pi\rho^e\omega^2}{k_l^{e^2}}\hat{\mathbf{a}}\cdot\mathbf{f}, \tag{7}$$

and Eq.(2) becomes

$$\sum_i v_i\hat{\mathbf{a}}\cdot\mathbf{f}_i = 0. \tag{8}$$

From Eq.(8), one can observe that, in the context of scattering problem, the properties of effective medium can be obtained by vanishing the average of the forward scattering amplitudes by local variations from effective medium. Moreover, since the forward scattering amplitude is proportional to the total cross section as stated by the forward scattering theorem for elastic waves,[6] aforementioned condition says that the total power abstracted from the mean field must be vanished in the effective medium. From Eqs.(6)-(8), three independent conditions that should be satisfied by the elastic properties of effective medium can be derived as follows:

$$\sum_i v_i\delta\rho^i\omega^2\int_\Omega \tilde{\mathbf{u}}^i\cdot\nabla\exp(-i\mathbf{k}_l^e\cdot\mathbf{r})d\Omega = 0, \tag{9}$$

$$\sum_i v_i\delta\lambda^i\int_\Omega \Delta^i\exp(-i\mathbf{k}_l^e\cdot\mathbf{r})d\Omega = 0, \tag{10}$$

$$\sum_i v_i\delta\mu^i\int_\Omega \mathbf{E}^i:\nabla\nabla\exp(-i\mathbf{k}_l^e\cdot\mathbf{r})d\Omega = 0. \tag{11}$$

3. RESULTS AND DISCUSSION

Numerical calculations are performed for a random particulate composite of lead particles in epoxy (EPON 828-Z) matrix. The material properties of particles and matrix are presented in Table I. Experimental study for this material was carried out by Kinra *et al.*[7-9] The longitudinal wave speed obtained from the coherent potential approximation expanded to finite frequency in this paper is compared with the theoretical results by Waterman and Truell[10] as well as with the experimental results by Kinra *et al.* as in Fig. 2, and the corresponding attenuation coefficients obtained from both theories are shown in Fig. 3. In these figures, it is noted that a lot of resonance modes of particle affect on the dispersion and attenuation for volume fractions considered. Because the multiple scattering effect can not be considered in the Waterman and

Truell theory, the resonance frequencies in the results are those of a single particle embedded in the matrix material. It is also noted that resonance frequencies shift to higher ones from those of single particle in matrix as the particle volume fraction increases. In Fig.2, the lowest order resonance which creates an abrupt rise in wave speed shifts from the single scattering resonance at $k_l^1 a \approx 0.3$ to those in effective medium at $k_l^1 a \approx 0.4$ and $k_l^1 a \approx 0.6$ as volume fraction increase to 5% and 15%, respectively. It is observed that the dispersion obtained from the coherent potential approximation predicts the shift of resonance frequency. In contrast, the Waterman and Truell theory can not predict this frequency shift when the volume fraction exceeds 5%. As mentioned in the previous section, the multiple scattering effects can be considered in coherent potential approximation at least on the average. The shifts of higher order resonances are rather small. In Figs. 4, 5 and 6, the complex spectra of effective density and elastic moduli illustrate the aforementioned resonant behaviors. As can be seen in Eq.(7), scattering of elastic waves can be caused by the mismatching of stiffness and that of density as well. An abrupt variation of the magnitude appears in the density spectrum at low frequency that can not be observed in the stiffness spectra. Consequently, one can say that the density mismatch (*viz.* the difference in the inertia between matrix and particles or $\Delta\rho\omega^2\mathbf{u}$) for incident excitation gives rise to the lowest resonance. From these facts, the lowest resonance mode can be conjectured as the rigid-body oscillation. Therefore, the composite medium in this low frequency region can be modeled as an equivalent medium in which simple oscillators are randomly distributed. By this simple model, an unversal trend can be understood which is common to nearly all the fiber-reinforced[11] and particulate[12] composites in this frequency region: that is, the rapid increase after gradual decrease of wave speeds in accordance with the increase of frequency. In addition, the shifting of the lowest resonance frequency to higher frequency region can be explained from the fact that the particles oscillate in more and more stiffened surrounding medium along the increase of volume fraction. As a matter of course, the particle can not oscillate in phase with the matrix by the aforementioned difference in the inertial force. Therefore, in inhomogeneous materials the effective density should be complex and frequency dependent, whereas this can never be so in homogeneous materials. At higher frequencies, resonances can occur, which are mainly caused by the mismatch of dynamic stiffness, also that can be seen by comparing Figs. 2 and 3 with Figs. 4-6. In these figures, positions of peaks and troughs coincide with each other. In this frequency region, dynamic stiffness shows resonant behavior again. As the particle volume fraction increases, the resonant modes become more damped because the coherent attenuation increases and their frequencies shift slightly to higher frequencies. The effects of higher order resonances on dispersion are rather small compared with that from the zero order resonance.

4. CONCLUSION

The ordinary coherent potential approximation method that has been used in alloy physics is modified to investigate the frequency dependent behaviors of the dynamic stiffness and density. Self-consistency conditions for effective medium are derived without limit of frequency. The wave speed predicted by using the present theory agrees better with the experimental results than that by Waterman and Truell theory. The shift of resonance frequencies can be predicted by the present theory. The effect of particle resonances on dispersion can be estimated qualitatively.

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Table I. Material properties of constituents.

Materials	Density(kg/m^3)	λ(GPa)	μ(GPa)
Lead	11300	39.02	8.36
EPON 828Z	1200	4.90	1.73

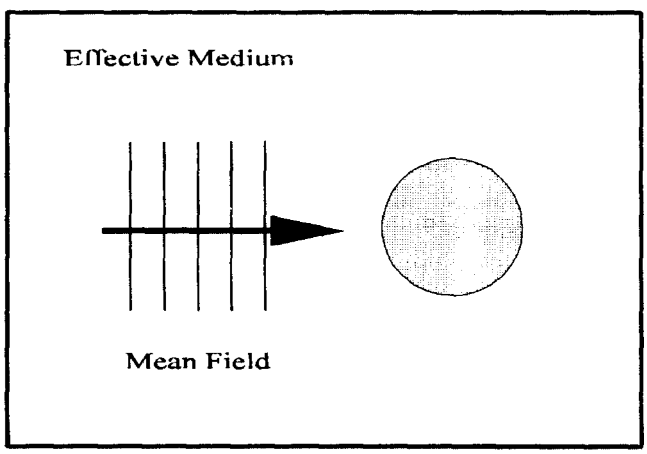


Fig. 1 Mean field propating in effective medium with an inhomogeneity.

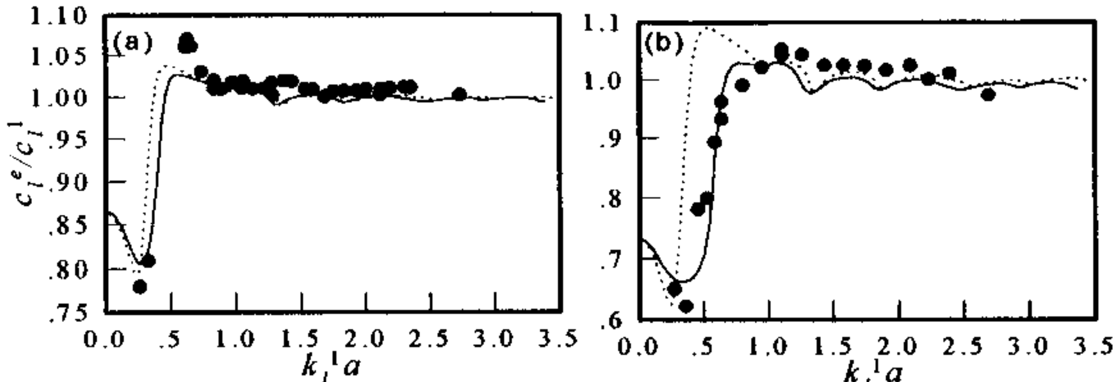


Fig. 2 Effective longitudinal wave speed. (——), Present theory; (·······), Waterman-Truell theory; (●) exp. by Kinra *et al.* (a) Volume fraction of 5%, (b) volume fraction of 15%.

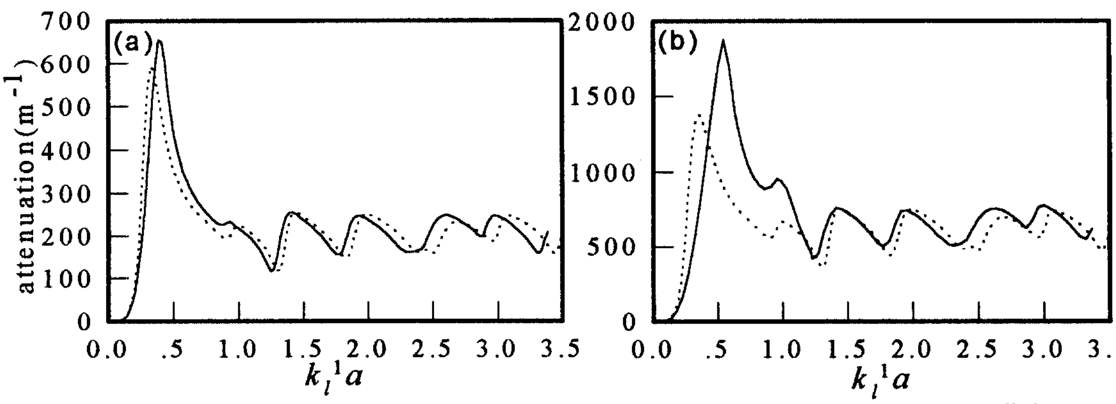
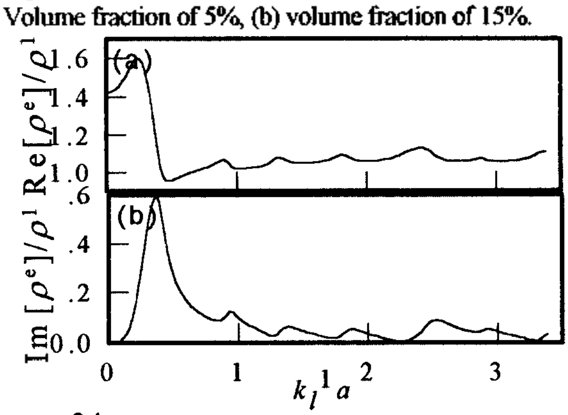
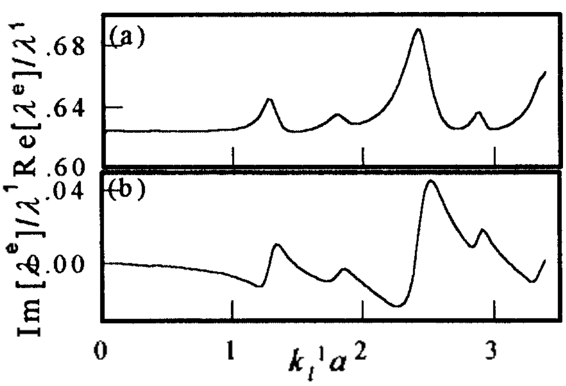


Fig. 3 Coherent attenuation of longitudinal wave. (——), Present theory; (·······), Waterman-Truell theory. (a) Volume fraction of 5%, (b) volume fraction of 15%.





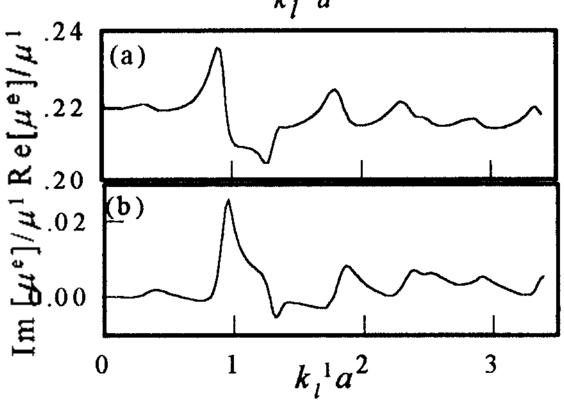


Fig.4 Effective density.(upper left)

Fig.5 Effective 1st Lame parameter.

Fig.6 Effective 2nd Lame parameter.