A NEW STOCHASTIC EVALUATION THEORY OF ARBITRARY ACOUSTIC SYSTEM RESPONSE AND ITS APPLICATION TO VARIOUS TYPE SOUND INSULATION SYSTEMS
——EQUIVALENCE TRANSFORMATION TOWARD THE STANDARD HERMITE AND/OR LAGUERRE EXPANSION TYPE PROBABILITY EXPRESSIONS

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ABSTRACT In the actual sound environmental systems, it seems to be essentially difficult to exactly evaluate a whole probability distribution form of its response fluctuation, owing to various types of natural, social and human factors. Up to now, we very often reported two kinds of unified probability density expressions in the standard expansion form of Hermite and Laguerre type orthonormal series to generally evaluate non-Gaussian, non-linear correlation and/or non-stationary properties of the fluctuation phenomenon. However, in the real sound environment, there still remain many actual problems on the necessity of improving the above two standard type probability expressions for practical use. In this paper, first, a central point is focused on how to find a new probabilistic theory of practically evaluating the variety and complexity of the actual random fluctuations, especially through introducing some equivalence transformation toward two standard probability density expressions mentioned above in the expansion form of Hermite and Laguerre type orthonormal series. Then, the effectiveness of the proposed theory has been confirmed experimentally too by applying it to the actual problems on the response probability evaluation of various sound insulation systems in an acoustic room.

1. INTRODUCTION

Generally, it is usual that the actual stochastic phenomena is analyzed theoretically as if it fluctuates within an infinite or a half-infinite amplitude ranges. For example, we very often employed the well-known Gaussian distribution or Gamma distribution of standard types as the conventional methods for evaluating the probability density function of the actual random phenomena. However, the actual latent environmental constraint like a finite dynamic range of the observation mechanism should be considered in the actual sound measurements. In reality, such an amplitude constraint owing to natural, social and human factors is sometimes ignored. Furthermore, to be exact, owing to the non-linear, non-stationary characteristics of the system and the contamination by a background noise, the non-Gaussian or non-Gamma properties of the observed fluctuation data should be taken into consideration, too. Accordingly, the new evaluation method based on the equivalence transformation to the standard case for practical use under the above specific conditions must have been required.

If taking the above amplitude limitation into consideration, we already proposed the evaluation method by use of the unified probability density expression in the standard expansion expression form of Jacobi type orthonormal series (ref. 1), by taking Beta function type probability density function (abbr. p.d.f.) as the first expansion term. Although surely this Beta function type p.d.f. or Jacobi expansion type p.d.f. describes flexibly the actually observed p.d.f. under the amplitude limitation, it is not widely well known. In addition to this, the statistical benefit of employing the well-known Gaussian distribution or Gamma distribution seems to be very important especially for the practical utilization, even if they are approximately employed.

More concretely, the following two equivalence transformation methods have been newly introduced :

[1] First, an equivalence transformation from the Beta distribution to the well-known Gaussian distribution has been found in order to utilize positively much well-known knowledge connected with Gaussian p.d.f. (fluctuating within an infinite amplitude range). Then, the unified expression type p.d.f. in the orthonormal expansion series form of Hermite type taking a Gaussian p.d.f. as the first expansion term (reflecting approximately the dominant part of the actually observed p.d.f.) has been adapted after applying this equivalence transformation to the actual data, in order to compensate some kind of deviation from a dominant Gaussian p.d.f. (ref. 2,3,4).

[2] Next, an equivalence transformation from the Beta distribution to the well-known Gamma distribution has been found in order to utilize positively much well-known knowledge connected with Gamma p.d.f. (fluctuating within a half-infinite amplitude range). Then, the unified expression type p.d.f. in the orthonormal expansion series form of Laguerre type taking a Gamma p.d.f. as the first expansion term (reflecting approximately the dominant part of the actually observed p.d.f.) has been adapted after applying this equivalence transformation to the actual data, in order to compensate some kind of deviation from a dominant Gamma p.d.f. (ref. 2,3,4).

Finally, the effectiveness of the proposed theory has been experimentally confirmed too by applying it to the actual problems on the response probability evaluation of various type sound insulation systems in an acoustic room.

2. THEORETICAL CONSIDERATIONS

2.1 Derivation of Equivalence Transformation from Beta P.D.F. to Gaussian P.D.F.

For finding an equivalence transformation from Beta p.d.f. to Gaussian p.d.f., first, let us pay our attention to only Beta p.d.f. defined by :

$$P_x(x) = \frac{1}{B(\gamma,\alpha-\gamma+1)} \cdot \frac{1}{b-a} \cdot \left(\frac{x-a}{b-a}\right)^{\gamma-1} \cdot \left(1-\frac{x-a}{b-a}\right)^{\alpha-\gamma}, \tag{1}$$

where x denotes the observation datum fluctuating under the amplitude limitations within the lower level a and the upper level b (Here, after the non-dimensional transformation : $u = \frac{x-a}{b-a}$, $\gamma \to \alpha'$ and $\alpha - \gamma + 1 \to \beta$, Eq. (1) can be rewritten to the expression form of standard type Beta p.d.f. : $p_x(u) = u^{\alpha'-1}(1-u)^{\beta-1} / B(\alpha',\beta)$.).

Hereupon, a mean value μ_x, variance σ_x^2 and the n-th order moments around μ_x (n=3, 4) can be easily derived from Eq. (1) respectively, as follows :

$$\mu_x = a + (b-a)\cdot\frac{\gamma}{\alpha+1}, \tag{2}$$

$$\sigma_x^2 = (b-a)^2\cdot\frac{\gamma(\alpha-\gamma+1)}{(\alpha+1)^2(\alpha+2)}, \tag{3}$$

$$\left\langle (x-\mu_x)^3 \right\rangle = (b-a)^3\cdot\frac{2\gamma(\alpha-\gamma+1)(\alpha-2\gamma+1)}{(\alpha+1)^3(\alpha+2)(\alpha+3)}, \tag{4}$$

$$\left\langle (x-\mu_x)^4 \right\rangle = (b-a)^4\cdot\frac{3\gamma(\alpha-\gamma+1)\left\{\gamma(\alpha-\gamma+1)(\alpha-5)+2(\alpha+1)^2\right\}}{(\alpha+1)^4(\alpha+2)(\alpha+3)(\alpha+4)}. \tag{5}$$

Now, for purpose of approximately finding the equivalence transformation $f(x)$ from Beta p.d.f. to Gaussian p.d.f., let us notice the fundamental property that the 3rd order moment around a mean value : $\left\langle \left(f(x)-\langle f(x)\rangle\right)^3 \right\rangle$ after this transformation must be disappeared. That is, by letting this transformation into the Taylor expansion form as follows :

$$f(x) = f(\mu_x) + f'(\mu_x)(x-\mu_x) + \frac{1}{2}f''(\mu_x)(x-\mu_x)^2 + \cdots \tag{6}$$

and employing some roughly approximated relationship $\langle f(x)\rangle \cong f(\mu_x)$ ($\langle f(x)\rangle$: a mean value of $f(x)$), the following equation must be approximately satisfied :

$$\begin{aligned}\left\langle \left(f(x)-f(\mu_x)\right)^3 \right\rangle &\cong \{f'(\mu_x)\}^3\left\langle (x-\mu_x)^3 \right\rangle + \frac{3}{2}\{f'(\mu_x)\}^2 f''(\mu_x)\left\langle (x-\mu_x)^4 \right\rangle \\ &\cong 0.\end{aligned} \tag{7}$$

After substituting Eqs. (4) and (5) into Eq. (7), the following equation is found :

$$\begin{aligned}&(\alpha+1)(\alpha+4)(\alpha-2\gamma+1)f'(\mu_x) \\ &\quad + \frac{9}{4}(b-a)\left\{\gamma(\alpha-\gamma+1)(\alpha-5)+2(\alpha+1)^2\right\}f''(\mu_x) \cong 0.\end{aligned} \tag{8}$$

Moreover, by substituting two parameters α and γ directly derived from Eqs. (2) and (3) into this equation and using an approximate relationship $\mu_x \cong (a+b)/2$, a fairly simplified equation :

$$(a+b-2\mu_x)f'(\mu_x) + (\mu_x-a)(b-\mu_x)f''(\mu_x) = 0 \tag{9}$$

can be obtained. Accordingly, this equivalence transformation $f(x)$ can be derived by solving the above differential equation Eq. (9), as follows :

$$f(x) = C_1 \log \frac{x-a}{b-x} + C_2, \tag{10}$$

where C_1 and C_2 denote the integral constants. In this paper, these constants are especially selected as $C_1 = 1$, $C_2 = \log b$. After all, the objective equivalence transformation and the standard expansion expression form of Hermite type orthonormal series after this transformation can be obtained as follows :

$$z = \log b \frac{x-a}{b-x}, \quad P(z) = P_0(z)\left\{1 + \sum_{n=1}^{\infty} A_n H_n\left(\frac{z-\mu_z}{\sigma_z}\right)\right\}, \tag{11,12}$$

where $P_0(z)$, $A_n\left(= \frac{1}{n!}\left\langle H_n\left(\frac{z-\mu_z}{\sigma_z}\right)\right\rangle\right)$ and $H_n(\cdot)$ denote Gaussian p.d.f., the n-th order expansion coefficient and the Hermite polynomials, respectively.

Hereupon, especially the following property must be noticed. That is, in the limitation of $a \to 0$ and $b \to \infty$, Eq. (11) becomes $z = \log x$ and $P_0(z)$ in Eq. (12) corresponds exactly to the well-known log-Normal p.d.f. of x.

2.2 Derivation of Equivalence Transformation from Beta P.D.F. to Gamma P.D.F.

Now, for purpose of approximately finding the equivalence transformation $g(x)$ from Beta p.d.f. to Gamma p.d.f., let us notice the fundamental property that after this transformation the 3rd order moment around a mean value must be disappeared for Gamma p.d.f. That is, by letting this transformation into the Taylor expansion form given by :

$$g(x) = g(\mu_x) + g'(\mu_x)(x-\mu_x) + \frac{1}{2} g''(\mu_x)(x-\mu_x)^2 + \cdots, \tag{13}$$

and by employing some roughly approximated relationship $\langle g(x)\rangle \cong g(\mu_x)$ ($\langle g(x)\rangle$: a mean value of $g(x)$), the following equation must be approximately satisfied :

$$\begin{aligned}\left\langle \{g(x)-g(\mu_x)\}^3 \right\rangle - 2\frac{\left\langle \{g(x)-g(\mu_x)\}^2 \right\rangle^2}{g(\mu_x)} &\cong \{g'(\mu_x)\}^3 \left\langle (x-\mu_x)^3 \right\rangle \\ &+ \frac{3}{2}\{g'(\mu_x)\}^2 g''(\mu_x) \left\langle (x-\mu_x)^4 \right\rangle \\ &- 2\frac{\{g'(\mu_x)\}^4 \sigma_x^4}{g(\mu_x)} \\ &\cong 0.\end{aligned} \tag{14}$$

After substituting Eqs. (4) and (5) into Eq. (14), the following equation is found :

$$\begin{aligned}&(\alpha+1)(\alpha+4)(\alpha-2\gamma+1)h(\mu_x)-\frac{9}{4}(b-a)\left\{\gamma(\alpha-\gamma+1)(\alpha-5)+2(\alpha+1)^2\right\}h'(\mu_x)\\&+\frac{9}{4}(b-a)\left\{\gamma(\alpha-\gamma+1)(\alpha-5)+2(\alpha+1)^2\right\}-\sigma_x^4\frac{(\alpha+1)^4(\alpha+2)(\alpha+3)(\alpha+4)}{(b-a)^3\gamma(\alpha-\gamma+1)}\cong 0,\end{aligned} \tag{15}$$

$$h(\mu_x)=\frac{1}{\left(\log|g(\mu_x)|\right)'} \tag{16}$$

In the same manner as in section 2.1, by substituting two parameters α and γ directly derived from Eqs. (2) and (3) into this equation and using an approximate relationship $\mu_x \cong (a+b)/2$, fairly simplified equation :

$$(a+b-2\mu_x)h(\mu_x)-(\mu_x-a)(b-\mu_x)h'(\mu_x)=0 \tag{17}$$

can be obtained. Accordingly, this equivalence transformation $g(x)$ can be derived by solving the above differential equation Eq. (17), as follows :

$$g(x)=C_1\cdot\left(\frac{x-a}{b-x}\right)^{C_2}, \tag{18}$$

where C_1 and C_2 denote the integral constants. In this paper, these constants are especially selected as $C_1=\sqrt{b}$, $C_2=1/2$. After all, the objective equivalence transformation and the standard expansion expression form of Laguerre type orthonormal series after this transformation can be obtained as follows :

$$z=\sqrt{b\frac{x-a}{b-x}},\quad P(z)=P_0(z)\left\{1+\sum_{n=1}^{\infty}B_nL_n^{(m_z-1)}\left(\frac{z}{s_z}\right)\right\}, \tag{19,20}$$

where $P_0(z)$, $B_n\left(=\frac{n!\Gamma(m_z)}{\Gamma(m_z+n)}\left\langle L_n^{(m_z-1)}\left(\frac{z}{s_z}\right)\right\rangle\right)$ and $L_n^{(\cdot)}(\cdot)$ denote Gamma p.d.f., the n-th order expansion coefficient and the Laguerre polynomials, respectively ($\Gamma(\cdot)$ denotes the Gamma function). In the above two cases (Eqs. (11) and (19)), we have to point out that some distortion of p.d.f. form based on the introduction of roughly approximation in the above calculation process can be compensated by employing two kinds of unified expansion type series expression of Hermite and Laguerre polynomials in the final procedures.

3. EXPERIMENTAL CONSIDERATIONS

The effectiveness of the proposed theory has been experimentally confirmed too by applying it to the actual problem of evaluating the response probability distribution of various type sound insulation systems. Owing to the page limitation, only the following two experimental results have been shown here.

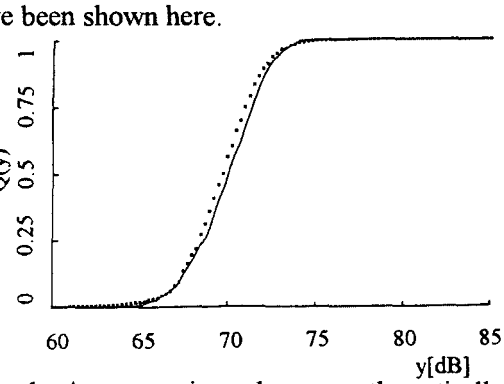

Fig. 1 A comparison between theoretically predicted curve (———) by use of the proposed equivalence transformation (Eq.(11)) and experimentally sampled points (×) on the output probability distribution for a double-wall type sound insulation system.

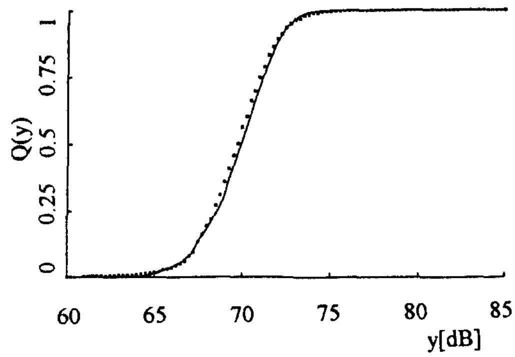

Fig. 2 A comparison between theoretically predicted curve (———) by use of the proposed equivalence transformation (Eq.(19)) and experimentally sampled points (×) on the output probability distribution for a double-wall type sound insulation system.

4. CONCLUSION

In this paper, the problem on how to find a new probabilistic theory of practically evaluating the variety and complexity of the actual random fluctuations has been discussed especially through introducing some equivalence transformation toward two standard type probability density expressions in the expansion form of Hermite and Laguerre type orthonormal series. Then, the effectiveness of the proposed theory has been confirmed experimentally too by applying it to the actual problems on the response probability evaluation of various sound insulation systems in an acoustic room.

Nevertheless, such a research seems to be at an early stage of study. So, there remain many future problems, such as : how to define the integral constant of the proposed equivalence transformation, or how to truncate the higher order expansion terms in the expansion expression of p.d.f. We would express our hearty thanks to K. Hatakeyama, A. Ikuta and S. Tsushimi for their helpful assistance.

REFERENCES

1. M. Ohta, A. Ikuta and N. Takaki, "A stochastic signal processing of incomplete observation data with amplitude limitation and state estimation under the existence of additional noise," the trans. of the IEICE vol. E-71 No. 1, 8-15 (1988)

2. M. Ohta, "On the composition of waves in the random phase problem," proceedings of the 5th Japan National Congress for Applied Mechanics, 489-492 (1955)

3. M. Ohta, "On the general energy distribution of Brownian Motion," proceedings of the 7th Japan National Congress for Applied Mechanics, 317-322 (1957)

4. M. Ohta and T. Koizumi, "General statistical treatment of the response of a non-linear rectifying device to a stationary random input," IEEE trans. on Info. Theory vol. IT-14 No. 4, 595-598 (1968)