INTRODUCTION OF THREE FUNCTIONAL MODELS MATCHED TO THE STOCHASTIC RESPONSE EVALUATION OF ACOUSTIC ENVIRONMENTAL SYSTEM AND ITS APPLICATION TO A SOUND INSULATION SYSTEM

Mitsuo OHTA† and Yoshifumi FUJITA††

†Dept. of Electronic Engineering and Computer Science,
Faculty of Engineering, Kinki University
Umenobe 1, Takaya, Higashi-Hiroshima, 729-17 Japan
††Dept. of Management and Information Science,
Onomichi Junior College,
Hisayamada-chou 1600, Onomichi, 722 Japan

ABSTRACT For evaluating the response fluctuation of the actual environmental acoustic system excited by arbitrary random inputs, it is important to predict a whole probability distribution form closely connected with evaluation indexes L_x, L_{eq} and so on. In this paper, a new type evaluation method is proposed by introducing three functional models matched to the prediction of the response probability distribution from a problem-oriented viewpoint. Because of the positive variable of the sound intensity, the response probability density function can be reasonably expressed theoretically by a statistical Laguerre expansion series form. The relationship between input and output is described by the regression relationship between the distribution parameters (containing expansion coefficients of this expression) and the stochastic input. These regression functions are expressed in terms of the orthogonal series expansion and their parameters are determined based on the least-squares error criterion and the measure of statistical independency.

1. INTRODUCTION

For the evaluation of the actual environmental acoustic system, there are many kinds of evaluation indexes such as L_x, L_{eq} and so on. As is well known, these are closely related to the whole probability distribution form. We have already proposed some methods to predict the probability distribution form of the response fluctuation of the acoustic system with arbitrary random inputs. In this paper, a new type evaluation method is proposed by introducing three functional models matched to the prediction of the response probability distribution from a problem-oriented viewpoint. Since originally the sound intensity fluctuates only within a positive amplitude region, the probability distribution of the output sound intensity can be reasonably expressed in a form of statistical Laguerre series expansion. Here, as the factor reflecting functionally the stochastic relationship between input and output for the acoustic system, the regression relationship between the distribution parameters including the coefficients of the expansion form can be taken. However, there remains the problem on how to identify the regressions coefficients by use of the measurements of the observation as the problem to be solved. These regression functions are expressed generally in terms of the orthogonal series expansions for the probability distribution of only input or the joint probability distribution of input and output. The distribution parameters of the above functional model are determined based on the least-squares error criterion and the measure of statistical independency.

2. THEORETICAL CONSIDERATION

2.1 Three functional models of acoustic system contaminated by background noise

For the acoustic environment system, let us consider concretely the problem evaluating the probability distribution of the output sound intensity responding to a stochastic input sound intensity fluctuating in the non-Gaussian probability distribution form, after defining x as the sound intensity of the input, y as the sound intensity of the output, z as the sound intensity of the observation and v as the sound intensity of the background noise. In consideration of this problem, it is naturally assumed that the background noise v and the output y are independent each other(of course, x and v are originally independent each other). The probability density function (abbreviation, p. d. f.) p(y) of output y, fluctuating only in the positive values region can be reasonably expressed in the following statistical type Laguerre series expansion form:

$$P(y)=P_\Gamma(y;m_y,s_y)\sum_{n=0}^{\infty} A_n L_n^{(m_y-1)}\left(\frac{y}{s_y}\right) \tag{1}$$

with

$$m_y=\frac{\langle y\rangle^2}{\langle(y-\langle y\rangle)^2\rangle},\; s_y=\frac{\langle(y-\langle y\rangle)^2\rangle}{\langle y\rangle},\; A_n=\frac{\Gamma(m_y)n!}{\Gamma(m_y+n)}\left\langle L_n^{(m_y-1)}\left(\frac{y}{s_y}\right)\right\rangle, \tag{2}$$

where $\Gamma(m)$ denotes a gamma function and < > denotes the expectation of y. In addition, $p_\Gamma(\zeta;m,s)$ and $L_n^{(\alpha)}(x)$ denote respectively a gamma p.d.f. and an associated Laguerre polynomial, defined as follows:

$$P_\Gamma(\zeta;m,s)=\frac{1}{\Gamma(m)s}\left(\frac{\zeta}{s}\right)^{m-1}e^{-\frac{\zeta}{s}},\quad L_n^{(\alpha)}(x)=\sum_{j=0}^{n}\binom{n+\alpha}{n-j}\frac{(-x)^j}{j!}. \tag{3}$$

Two parameters m_y, s_y and every expansion coefficient A_n depend on the fluctuation of input x and the stochastic relationship between input x and output y. From the problem-oriented viewpoint to predict the p.d. f. of the output y, it is desirable that the relationship should be described functionally in terms of the regressions styles: $\langle y|x\rangle$, $\langle (y-\langle y\rangle)^2|x\rangle$ and $\langle L_k^{(m_y-1)}(y/s_y)|x\rangle$ rather than in terms of only the physical rule of correspondence between x and y. Here, $\langle\cdot|x\rangle$ denotes the conditional expectation conditioned by x directly connected with regression analysis. These regressions functions are defined as follows:

$$\left.\begin{aligned}&\langle y|x\rangle=\int_0^\infty y\,p(y|x)\,dy,\quad \langle(y-\langle y\rangle)^2|x\rangle=\int_0^\infty (y-\langle y\rangle)^2\,p(y|x)\,dy,\\&\left\langle L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)\middle|x\right\rangle=\int_0^\infty L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)p(y|x)\,dy,\end{aligned}\right\} \tag{4}$$

where p(y|x) is the conditional p.d. f. of y conditioned by x. Since these regressions are obviously nonlinear, for the purpose of getting one of general representations, let us use the method of orthogonal series expansion, by letting p(x) and p(x,y) denote the p. d. f. of x and the joint p.d. f. of x and y. After expanding p(x) and p(x,y) into the orthonormal expansion series forms with weighting functions $p_\Gamma(x;m_x,s_x)$ and $p_\Gamma(x;m_x,s_x)p_\Gamma(y;m_y,s_y)$ as basic p.d.f., respectively and using the definition of the conditional probability p(y|x)=p(x,y)/p(x), p(y|x) can be expressed as follows:

$$p(y|x)=\frac{p_\Gamma(y;m_y,s_y)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}\varphi_m^{(1)}(x)\varphi_n^{(2)}(y)}{\sum_{m=0}^{\infty}A_{m0}\varphi_m^{(1)}(x)}, \tag{5}$$

where

$$\varphi_m^{(1)}(x)=\sqrt{\frac{\Gamma(m_x)m!}{\Gamma(m_x+m)}}\,L_m^{(m_x-1)}\left(\frac{x}{s_x}\right),\quad \varphi_n^{(2)}(y)=\sqrt{\frac{\Gamma(m_y)n!}{\Gamma(m_y+n)}}\,L_n^{(m_y-1)}\left(\frac{y}{s_y}\right), \tag{6}$$

$$A_{m0}=\langle\varphi_m^{(1)}(x)\rangle \quad\text{and}\quad A_{mn}=\langle\varphi_m^{(1)}(x)\,\varphi_n^{(2)}(y)\rangle. \tag{7}$$

Here, $\varphi_m^{(1)}(x)$ and $\varphi_n^{(2)}(y)$ are the orthonormal polynomials that satisfy the following relationships:

$$\int_0^\infty p_\Gamma(x;m_x,s_x)\varphi_m^{(1)}(x)\,\varphi_n^{(1)}(x)\,dx=\delta_{mn}\ ,\quad \int_0^\infty p_\Gamma(y;m_y,s_y)\varphi_m^{(2)}(y)\,\varphi_n^{(2)}(y)\,dy=\delta_{mn}\ , \tag{8}$$

where δ_{mn} denotes Kronecker's delta. Upon expanding y and $(y - <y>)^2$ into the orthogonal series expansion forms as follows:

$$y=\sum_{k=0}^{1} C_k^{'}\,\varphi_k^{(2)}(y)\ ,\quad (y-<y>)^2=\sum_{k=0}^{2} C_k^{''}\,\varphi_k^{(2)}(y)\ , \tag{9}$$

the above regression functions can be respectively expressed as follows:

$$\langle y|x\rangle=\frac{\sum_{m=0}^{\infty} B'_m\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}\ ,\quad \langle(y-\langle y\rangle)^2|x\rangle=\frac{\sum_{m=0}^{\infty} B''_m\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}\ ,\quad \left\langle L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)\Big|x\right\rangle=\frac{\sum_{m=0}^{\infty} D_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}\ , \tag{10}$$

where

$$B'_m=\sum_{n=0}^{1} C'_n\,A_{mn}\ ,\quad B''_m=\sum_{n=0}^{2} C''_n\,A_{mn}\ ,\quad D_{mk}=\sqrt{\Gamma(m_y+k)/\Gamma(m_y)\,k!}\ A_{mk}\ , \tag{11}$$

after substituting Eqs. (5) and (9) into Eq. (4) and using Eq. (8). Since the observed quantity is not y but z, it is necessary to find how to identify the above regression parameters in Eq. (10) by use of observed z values. On the basis of the additivity of sound intensity quantity and the addition theorem for the associated Laguerre polynomial, the following relationships can be found :

$$z=y+v\ , \tag{12}$$

$$(z-<z>)^2=(y-<y>)^2+(y-<y>)\,(v-<v>)+(v-<v>)^2\ , \tag{13}$$

$$L_n^{(m_y)}\left(\frac{z}{s_y}\right)=L_n^{(m_y)}\left(\frac{y+v}{s_y}\right)=\sum_{k=0}^{n} L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)L_{n-k}^{(0)}\left(\frac{v}{s_y}\right)\ . \tag{14}$$

By replacing y, (y- <y>), (y - <y>)^2 and $L_k^{(m_y-1)}(y/s_y)$ by <y|x>, $\langle y-\langle y\rangle|x\rangle$, $\langle(y-\langle y\rangle)^2|x\rangle$ and $\langle L_k^{(m_y-1)}(y/s_y)|x\rangle$, the following three functional models for the resultant observation z are introduced corresponding to Eqs. (12), (13) and (14):

$$z=\frac{\sum_{m=0}^{\infty} B'_m\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}+v\ , \tag{15}$$

$$(z-\langle z\rangle)^2=\frac{\sum_{m=0}^{\infty} B''_m\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}+(v-\langle v\rangle)^2+2\,\langle y-\langle y\rangle|x\rangle\,(v-\langle v\rangle)\ , \tag{16}$$

$$L_n^{(m_y)}\left(\frac{z}{s_y}\right)=\sum_{k=0}^{n}\frac{\sum_{m=0}^{\infty} D_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{\infty} A_{m0}\,\varphi_m^{(1)}(x)}\,L_{n-k}^{(0)}\left(\frac{v}{s_y}\right)\ . \tag{17}$$

2.2 Identification of functional models and prediction of the response probability distribution

Since $\langle y\rangle = \langle\langle y|x\rangle\rangle$ and $\langle (y-\langle y\rangle)^2\rangle = \langle\langle (y-\langle y\rangle)^2|x\rangle\rangle$ can be evaluated once after $\langle y|x\rangle$ and $\langle (y-\langle y\rangle)^2|x\rangle$ are determined, two distribution parameters m_y and s_y can be estimated from Eq. (2). Accordingly, first let us determine $\langle y|x\rangle$ and $\langle (y-\langle y\rangle)^2|x\rangle$. By replacing infinity by finite number M in the first equation of Eq. (10), its model error ε is defined as follows:

$$\varepsilon = z - \frac{\sum_{m=0}^{M} B'_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} - v \ . \tag{18}$$

The unknown parameters B'_m's (k=0,1,...,M) are determined by the well-known least-squares error criterion . That is, these parameters are chosen to minimize the expectation of ε^2:

$$\langle \varepsilon^2\rangle = \left\langle \left(z - \frac{\sum_{m=0}^{M} B'_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} - v \right)^2 \right\rangle_{x,z,v} = \left\langle \left(z - \frac{\sum_{m=0}^{M} B'_m \varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} - \langle v\rangle \right)^2 \right\rangle_{x,z} \ , \tag{19}$$

where $\langle\ \rangle_{x,z,v}$ or $\langle\ \rangle_{x,z}$ denote expectations about x, z and v, or about x and z, respectively. Upon equating to zero the first derivatives $\partial\langle \varepsilon^2\rangle / \partial B'_k$ (k=0,1,..., M), the following simultaneous equations are obtained:

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_k^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle B'_m = \left\langle \frac{\varphi_k^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} (z - \langle v\rangle) \right\rangle_{x,z} \quad (k=0,1,\ldots,M) . \tag{20}$$

Similarly, minimizing the expectation of squares of the model error for the first equation of Eq. (10) to determine B''_m's (k=0, 1,..., M) yields

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_k^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle B''_m = \left\langle \frac{\varphi_k^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \{ (z-\langle z\rangle)^2 - \langle (v-\langle v\rangle)^2\rangle \} \right\rangle_{x,z} \tag{21}$$

(k=0, 1, 2,..., M)

Next, let us estimate the coefficients A_n's.
Since

$$\left\langle L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)\right\rangle = \left\langle\left\langle L_k^{(m_y-1)}\left(\frac{y}{s_y}\right)\middle| x\right\rangle\right\rangle \tag{22}$$

can be evaluated once after $\langle L_k^{(m_y-1)}(y/s_y)|x\rangle$ (k=1, 2,...) are determined, the coefficient A_n (n=1, 2,...) can be estimated from Eq. (2). Hence, the problem is reduced to the estimation of the parameters D_{mn} in Eq. (10). Here, to estimate D_{mn}, the least-squares error criterion and the measure of statistical independency between the input x and the observation z are employed. First, in the similar way as stated above, the least-squares error criterion leads to

$$\sum_{m=0}^{M} \left\langle \frac{\varphi_i^{(1)}(x)\varphi_m^{(1)}(x)}{\left(\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)\right)^2} \right\rangle D_{mn}$$

$$= \left\langle \frac{\varphi_i^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \left\{ L_n^{(m_y)}\left(\frac{z}{s_y}\right) - \sum_{k=0}^{n-1} \frac{\sum_{m=0}^{M} D_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{M} A_{m0}\varphi_m^{(1)}(x)} \left\langle L_{n-k}^{(0)}\left(\frac{v}{s_y}\right)\right\rangle \right\} \right\rangle_{x,z} \quad (23)$$

(i=0, 1, 2,...., M and n=1,..., M)

Next, let us introduce the evaluation criterion based on the measure of statistical independency. The joint p.d. f. of x and v, the marginal p.d.f. of x and the marginal p.d.f. of v are denoted by p(x,v), p(x) and p(v), respectively. Then, by assuming that p(v) is expressed by the gamma distribution $p_\Gamma(v;m_v,s_v)$ and expanding p(x,v) into the orthonormal series expansion with a weighting function $p(x)p_\Gamma(v;m_v,s_v)$, p(x,v) is first expressed as follows:

$$p(x,v)=p(x)p_\Gamma(v;m_v,s_v)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} K_{ij}\theta_i^{(1)}(x)\sqrt{\frac{\Gamma(m_v)\,j!}{\Gamma(m_v+j)}}\; L_j^{(m_v-1)}\left(\frac{v}{s_v}\right), \quad (24)$$

where $\theta_i^{(1)}$'s are an orthonormal polynomial such that

$$\int_0^\infty p(x)\theta_i^{(1)}(x)\theta_j^{(1)}(x)\,dx=\delta_{ij}, \quad (25)$$

and are determined by employing Schmidt's orthogonalization technique and K_{ij}'s are defined by

$$K_{ij}=\sqrt{\frac{\Gamma(m_v)\,j!}{\Gamma(m_v+j)}}\left\langle \theta_i^{(1)}(x)L_j^{(m_v-1)}\left(\frac{v}{s_v}\right)\right\rangle . \quad (26)$$

As is well known, if x and v are independent, then p(x,v) = p(x) p(v). Hence, by comparing this with Eq.(24), the coefficients K_{ij} except for i=0 and j=0 must satisfy K_{ij}=0. That is, this leads to

$$\left\langle \theta_i^{(1)}(x)L_j^{(m_v-1)}\left(\frac{v}{s_v}\right)\right\rangle = 0 \quad (27)$$

By using Eq. (17) and the definition formula of Laguerre polynomial

$$L_j^{(m_v-1)}\left(\frac{v}{s_v}\right)=\sum_{l=0}^{j}(-1)^l L_l^{(0)}\left(\frac{v}{s_y}\right)\sum_{r=l}^{j}(-1)^r\binom{r}{l}\binom{j+m_v-1}{j-r}\left(\frac{s_y}{s_v}\right)^r, \quad (28)$$

Eq. (27) yields

$$\sum_{m=0}^{M}\left\langle \frac{\theta_i^{(1)}(x)\varphi_m^{(1)}(x)}{\sum_{m=0}^{M}A_{m0}\varphi_m^{(1)}(x)}\right\rangle D_{m1}=\left\langle \theta_i^{(1)}(x)\left\{ m_v\frac{s_v}{s_y}-1+L_1^{(m_y)}\left(\frac{z}{s_y}\right)\right\}\right\rangle_{x,z}, \quad (29)$$

$$\sum_{m=0}^{M}\left\langle \frac{\theta_i^{(1)}(x)\varphi_m^{(1)}(x)}{\sum_{m=0}^{M}A_{m0}\varphi_m^{(1)}(x)}\right\rangle D_{mj}=\left\langle \theta_i^{(1)}(x)\left[\sum_{l=0}^{j-1}(-1)^l\left\langle L_l\left(\frac{v}{s_y}\right)\right\rangle \sum_{r=l}^{j}(-1)^{j+r}\binom{r}{l}\binom{j+m_v-1}{j-r}\left(\frac{s_v}{s_y}\right)^{j-r}+L_j^{(m_x)}\left(\frac{z}{s_y}\right)-\sum_{k=1}^{j-1}\left(\frac{\sum_{m=0}^{M}D_{mk}\varphi_m^{(1)}(x)}{\sum_{m=0}^{M}A_{m0}\varphi_m^{(1)}(x)}\right)\left\langle L_{j-k}^{(0)}\left(\frac{v}{s_y}\right)\right\rangle\right]\right\rangle_{x,z} \quad (30)$$

(j ≥ 2)

After all, by using Eq. (1) with estimated values of m_y, s_y and A_n, the p.d.f of y can be predicted.

3. EXPERIMENTAL CONSIDERATION

By using the proposed method, the sound insulation system was identified and the p.d.f. of the response sound fluctuation without a background noise was predicted. In the experiment, when the road traffic noise was excited, the input and output were sampled every one second by using a sound level meter. After measurement, to confirm the effectiveness of the proposed method, the white noise generated by a noise generator was added as the background noise. In Fig. 1, the result for the case of M=2 is shown.

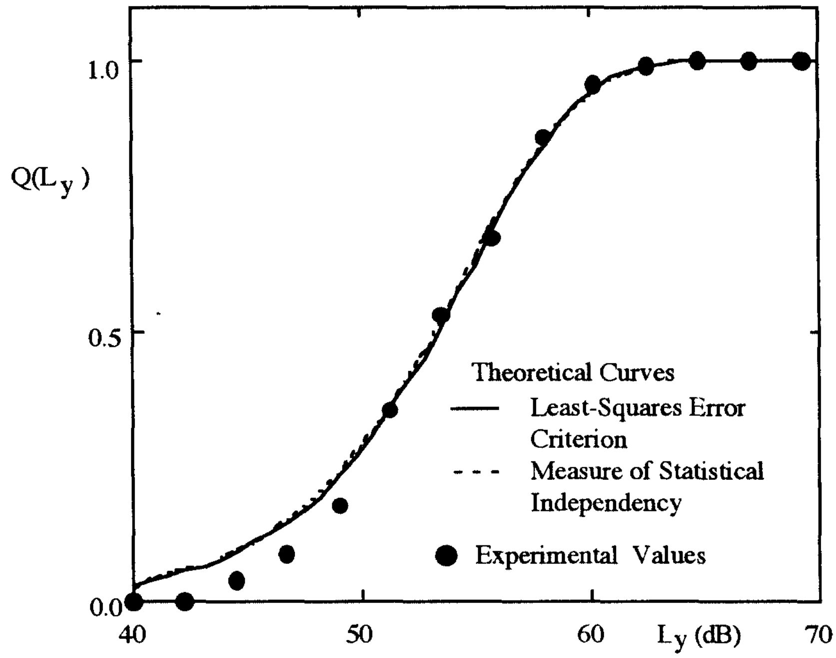

Fig.1 A comparison between the theoretically predicted cumulative probability distributions and experimentally sampled values for a sound-bridge insulation system.

4. CONCLUSION

To predict the probability distribution of the response fluctuation of the environmental acoustic system with an arbitrary sound input under the existence of the background noise, a new method has been proposed by introducing three functional models in the sound intensity scale. The response probability distribution has been expressed in a statistical Laguerre expansion. Then, the relationship of input and output has been described by the regression of the moments directly connected to this expansion form. After the regression has been related with observation through the functional models and has been expressed in terms of the orthogonal series expansion, the parameters of the regressions have been estimated based on the least-squares error criterion and the measure of statistical independency. Finally, the proposed method has been experimentally confirmed by applying it to an actual acoustic system.

ACKNOWLEGEMENT

We would like to express our cordial thanks to Mr. T. Nakano and Mr. M. Masuno for their assistance.

REFERENCES

1. M. Ohta and T. Koizumi, "General statistical moment of the response of a non-linear rectifying devices to a stationary random input," IEEE Trans. Inf.Theory IT-14,595-598(1968).
2. S. Moriguchi, K. Udagawa and S. Hitotumatu, *Formulas of Mathematics III* (Iwanami,Tokyo,1973)(in Japanese).
3. A.Papoulis, *Probability, Random Variables, and Stochastic Processes*(McGraw-Hill, New York,1965).