ON-LINE ESTIMATION PROCEDURES OF DIGITAL FILTER TYPE FOR REVERBERATION CHARACTERISTICS IN CLOSED ACOUSTIC SYSTEMS BASED ON NOISY OBSERVATION

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ABSTRACT

The acoustic phenomena in the actual sound systems involve a variety of compound problems. In this paper, the well-known Bayes' theorem is first employed and expanded into orthonormal and non-orthonomal series forms matched to the digital processing of lower and higher order statistical informations and the noisy observations. Proposed on-line algorithms of digital filter type are applied to the actual state estimation for a reverberation characteristics in a room under contamination of background noises.

1. INTRODUCTION

The acoustic phenomena in the actual sound environmental systems involve a variety of compound problems. Not only the physical, social but also human factors make them further complicated and diversified. Thus, the frequency and amplitude ranges of fluctuation of the environmental sound spread widely and so its fluctuation can exhibit every variety in pattern, which shows very often non-stationary and non-Gaussian properties.

On the basis of the above viewpoint, this paper discusses a new proposal of a general type of stochastic signal processing procedure which can evaluate an arbitrary type of statistics including not only the lower order moments like mean and variance supported by a large amount of sampled data, but also the higher order moments supported by a small amount of sampled data reflecting the end of the probability distribution form. More concretely, the well-known Bayes' theorem is first expanded into the orthonomal series forms matched to the digital processing of higher order statistical informations and successive noisy observations. Especially in a specific case when up to the second order moments are of main concern, it is proven that one of the proposed estimation algorithm becomes completely consistent with the well-known Kalman filter[1], which is called a wide sense digital filter. The effectiveness of the proposed methods is experimentally confirmed by applying them to the actual acoustic environments — more specifically, the estimation and prediction problems for the reverberation characteristics in a room.

2. BASIC FORMULATION

First, it is necessary to establish a model for the environmental sound systems in order to derive its dynamical state estimation algorithms, *i.e.*, wide sense digtal filters. Now, let the system dynamics be given in the following general form based on the time-series analysis[2]:

$$x_{k+1} = F_k(x_k, u_k), \tag{1}$$

where x_k is the sound intensity and called a state variable, u_k is a non-Gaussian random input sound, and $F_k(.)$ is a non-linear function. The observation mechanism can be generally modelled as:

$$y_k = G_k(x_k, v_k), \tag{2}$$

where y_k is an observation of the system, v_k is an additive background noise(called an observation noise) of arbitrary distribution type, and $G_k(.)$ is a non-linear function.

Before proceeding further analysis for the theoretical forms of Eqs.(1) and (2), the introduction of the well-known Bayes' theorem is basically important, which is given as:

$$P(x_k|Y_k) = \frac{P(x_k|Y_{k-1})P(y_k|x_k, Y_{k-1})}{P(y_k|Y_{k-1})} = \frac{P(x_k, y_k|Y_{k-1})}{P(y_k|Y_{k-1})}, \tag{3}$$

where $Y_k \equiv \{y_1, y_2, ..., y_k\}$ is a set of observation up to a k-time stage. As mentioned above, the main purpose of the introduction of Bayes's theorem is that it can generally match with a variety of non-linear and non-Gaussian properties, and furthermore a possible variety of human-side evaluation indexes. The problem here is to estimate successively the probability density function(abbr. *p.d.f.*) of a state variable, x_k, based on the noisy successive observation data, y_k, under the presence of a background noise.

3. ON-LINE ESTIMATION OF REVERBERATION TIME BASED ON NOISY OBSERVATION

Now, in order to reflect hierarchically the linear and/or non-linear correlation effects of the successive observation, y_k, on the state variable, x_k, we expand the joint *p.d.f.*, $P(x_k, y_k|Y_{k-1})$, into the orthonormal series form, because all the correlation informations are included in this expression of joint *p.d.f.* Here, the base *p.d.f.*'s refecting the human-side advance planning may be employed as the first expansion term. Let $P_0(x_k|Y_{k-1})$ and $P_0(y_k|Y_{k-1})$ be such base *p.d.f.*'s and expand $P(x_k, y_k|Y_{k-1})$, as follows:

$$P(x_k, y_k|Y_{k-1}) = P_0(x_k|Y_{k-1})P_0(y_k|Y_{k-1}) \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}(k)\varphi_m{}^{(1)}(x_k)\varphi_n{}^{(2)}(y_k), \quad (4)$$

where $\varphi_m{}^{(1)}(x_k)$ and $\varphi_n{}^{(2)}(y_k)$ are the orthonormal polynomials with the weighting functions, $P_0(x_k|Y_{k-1})$ and $P_0(y_k|Y_{k-1})$, respectively as follows:

$$\int \varphi_m{}^{(1)}(x_k)\varphi_n{}^{(1)}(x_k)P_0(x_k|Y_{k-1})dx_k = \delta_{mn}, \quad (5)$$

$$\int \varphi_m{}^{(2)}(y_k)\varphi_n{}^{(2)}(y_k)P_0(y_k|Y_{k-1})dy_k = \delta_{mn}. \quad (6)$$

For example, the follwoings can be considered as $P_0(x_k|Y_{k-1})$ and $P_0(y_k|Y_{k-1})$: (1)When each marginal *p.d.f.*, *i.e.*, $P(.|Y_{k-1})$, is correspondingly taken as $P_0(.|Y_{k-1})$, the state estimation algorithm which can bring all of only mutual correlation effects of y_k on x_k in strong relief along the essence of estimation principle is obtained. (2)When the *p.d.f.* which describes approximately the main or distinctive portion of the phenomenon of concern is skillfully extracted as such $P_0(.|Y_{k-1})$, the state estimation algorithm, in which the series expansion converges rapidly only with use of a few beginning expansion terms, can be obtained. (3)When the standard probability distribution, *e.g.*,Gaussian, gamma or Poisson distribution, or one whose statistical properties are *a priori* well-known, is taken as $P_0(.|Y_{k-1})$, the state estimation algorithm, in which plenty of intellectual data processing techniques are effectively used, can be obtained in close connection with well-known standard informations. In fact, when Gaussian, gamma or Poisson distribution is especially taken, the corresponding orthonomal polynomials are realized directly by Hermite, Laguerre or Charlier polynomials, without using the complicated Schmidt's orthogonalization method. (4)When the artificially optimum *p.d.f.'s* are used as $P_0(.|Y_{k-1})$, giving priority to the convergence of computation, artificially skillful state estimation algorithm can be obtained.

Substituting Eq.(4) into Eq.(3), we have the following estimation algorithm:

$$\hat{f}_N(x_k) = \frac{\sum_{m=0}^{N}\sum_{n=0}^{\infty} A_{mn}(k)C_{Nm}\varphi_n{}^{(2)}(y_k)}{\sum_{n=0}^{\infty} A_{0n}(k)\varphi_n{}^{(2)}(y_k)}, \quad (7)$$

where,

$$f_N(x_k) = \sum_{m=0}^{N} C_{Nm}\varphi_m{}^{(1)}(x_k). \quad (8)$$

In order to show the actual effectiveness of the present method, the application is made to the estimation problem of a reverberation time in a room from the noisy observation data. It is well-known by Sabine's reverberation theory that the sound energy density, $x(t)$, in a room is subject to the well-know differential equation,

$$\frac{dx(t)}{dt} = -F_0x(t), \quad F_0 \equiv \frac{6}{T\log_{10} e}, \quad (9)$$

where T is a reverberation time of the room. Eq.(9) is transformed into the following discrete form:

$$x_{k+1} = Fx_k, \quad F \equiv \exp(-F_0 \Delta t), \tag{10}$$

with a sampling interval, Δt. Letting $\Phi \equiv F_0 \Delta t$, and the initial sound level be X_0 ($x_0 = E_0 \cdot 10^{X_0/10}; E_0 \equiv 10^{-12}$watt/m^2) and considering the additivity property of energy on an objective sound and a background noise, the observed sound level, Y_k, is expressed by

$$Y_k = 10 \log_{10}(10^{X_0/10} e^{-\Phi \cdot k} + 10^{V_k/10}). \tag{11}$$

Considering the reverberation decay of Eq.(10), we have

$$\Phi_k = \Phi_{k-1}. \tag{12}$$

Eqs.(11) and (12) respectively correspond to observation and system equations. The estimate of the reverberation time at the k-th stage, denoted by $\hat{T}_k$, is calculated by the estimate of Φ_k as $\hat{T}_k = 6\Delta t/(\hat{\Phi}_k \log_{10} e)$. In this experiment, the sampling time was 1/20 s and true value of reverberation time, T, was measured to be 4.3 s($\Phi = 0.16$) in advance without a background noise. The estimated results are shown in Fig.1.

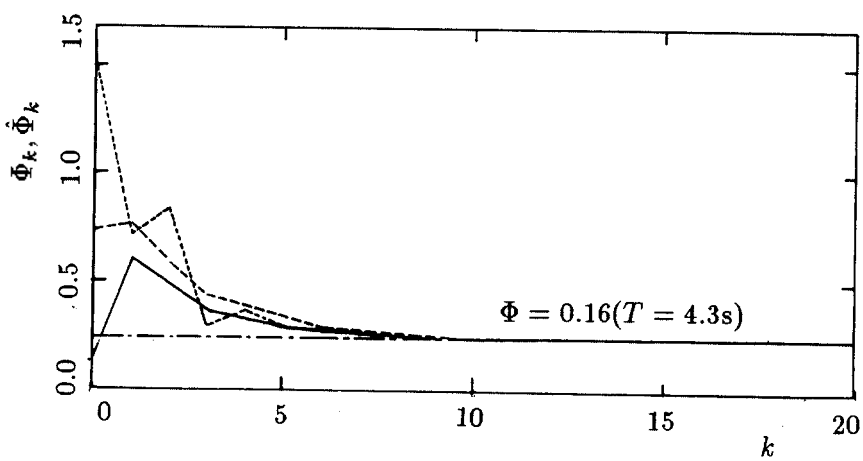

Fig.1 Estimated results of a reverberation time in a room contaminated by a background noise for three initial conditions.

4. ON-LINE ESTIMATION OF REVERBERATION CURVE BASED ON NOISY OBSERVATION

Suppose that the estimation algorithm of digital filter is first given by the following form with use of the successive observation for purpose of restricting artificially the expression framework of estimation algorithm in advance for practical use:

$$\hat{f}(x_k) = \sum_{n=0}^{\infty} \alpha_n^{(f)}(k) \varphi_n(y_k). \tag{13}$$

The problem here is how to evaluate the parameter, $\alpha_n^{(f)}(k)$. $\{\varphi_n(y_k)\}$ is a sequence of linearly independent polynomials which can be determined according to the engineering need and are not necessarily orthogonalized. Now, a sequence of orthogonal polynomials, $\{\theta_n(y_k)\}$, can be constituted based on $\{\varphi_n(y_k)\}$ by using the Schmidt's orthogonalizing technique, as follows:

$$\theta_n(y_k) = \sum_{j=0}^{n} \lambda_{nj}\varphi_j(y_k), \qquad \int \theta_m(y_k)\theta_n(y_k)\rho(y_k)dy_k = B_n\delta_{mn}, \tag{14}$$

where the weighting function, $\rho(y_k)$, is given in advance by considering the properties for experimental data and data processing. Now, $P(y_k|x_k, Y_{k-1})$ can be expanded with use of $\theta_n(y_k)$, as follows:

$$P(y_k|x_k, Y_{k-1}) = P(y_k|Y_{k-1}) \sum_{n=0}^{\infty} \bar{D}_n(x_k)\theta_n(y_k), \tag{15}$$

$$\bar{D}_n(x_k) = \frac{1}{B_n} \int \frac{P(y_k|x_k, Y_{k-1})}{P(y_k|Y_{k-1})}\theta_n(y_k)\rho(y_k)dy_k. \tag{16}$$

We finally obtain, $\alpha_n^{(f)}(k)$:

$$\alpha_n^{(f)}(k) = \langle f(x_k)D_n(x_k)|Y_{k-1}\rangle, \qquad D_n(x_k) \equiv \sum_{j=n}^{\infty} \lambda_{jn}\bar{D}_j(x_k). \tag{17}$$

In order to demonstrate the actual effectiveness of this method, the application is made to the estimation problem of a reverberation curve in a room from the noisy observation data. Considering the additivity property of sound energy, the reverberation mechanism is given by

$$y_k = x_k + v_k, \tag{18}$$

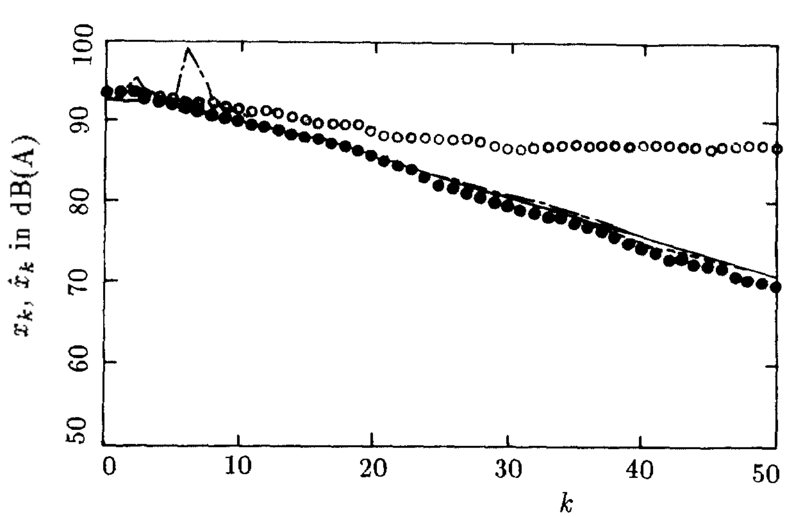

Fig.2 Estimated results of a reverberation curve in a room buried in a background noise. (•) denotes true values of a reverberation curve; (∘) observations; ——estimate by 1st approximation; -------estimate by 2nd approximation; —·—estimate by 3rd approximation.

where v_k is a background noise and y_k is a noisy observation. The reberveration curves estimated by the present method are shown in Fig.2.

The present method can be applied to the paremeter estimation problem for a reberveration time of the room. From Eq.(10), $x_k = x_0 \exp\{-\Phi_k k\}$ is obtained. Thus, the observation equation becomes as follows:

$$y_k = x_0 \exp\{-\Phi_k k\} + v_k. \tag{19}$$

In these experiments, the sampling time was 1/30 s and the true value of a reverberation time was measured to be 4.8 s without backgound noise. The estimated results are shown in Fig.3.

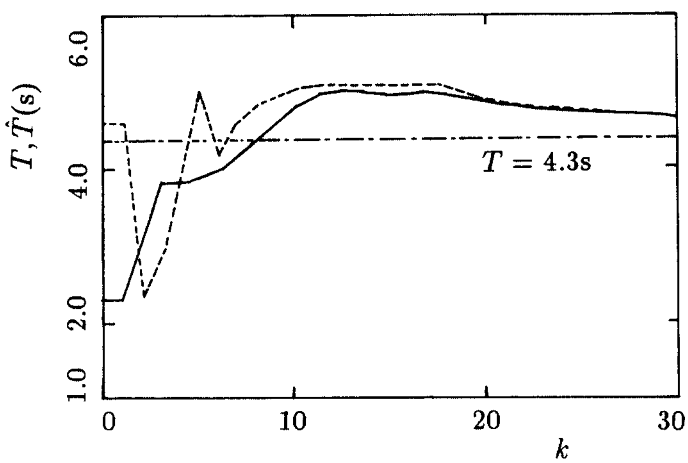

Fig.3 Estimated results of a reverberation time in a room buried in a background noise. ------- estimate by 2nd approximation; ——— estimate by 3rd approximation.

5. CONCLUSIONS

Two types of the state estimation methods for reverberation characteristics of the environmental acoustic systems have been discussed by putting emphasis on the principal viewpoint. Experimental results shows the practical effectiveness of the proposed methods. Detailed calculations for the derivations of the estimation and prediction algorithms have all been omitted here owing to the page limitation and our main attention was solely paid to its physical meanings and the essence of mathematical realization.

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