# Time-Frequency Domain Analysis of Acoustic Signatures Using Pseudo Wigner-Ville Distribution

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ABSTRACT Acoustic signal such as speech and scattered sound, are generally a nonstationary process whose frequency contents vary at any instant of time. For time-varying signal, whether a nonstationary or a deterministic transient signal, a traditional frequency domain representation does not reveal the contents of signal characteristics and may lead to erroneous results such as the loss of desired characteristics features or the mis-interpretation for a wrong conclusion. A time-frequency domain representation is needed to characterize such signatures. Pseudo Wigner-Ville distribution(PWVD) is ideally suited for portraying nonstationary signal time-frequency domain and carried out by adapting the fast Fourier transform algorithm.
In this paper, the important properties of PWVD were investigated using both stationary and nonstationary signatures by numerical examples PWVD was applied to acoustic signatures to demonstrate its application for time-frequency domain analysis.

## 1. INTRODUCTION

The physical condition and state of the objects those operate in transient mode or those are in the scattered sound field, are difficult to predict with any degree of accuracy. The conventional spectrum analysis of a signal provides averaged spectral values which are independent of time and portray the phenomena under the assumption that the signal is a stationary process. However many signatures such as speech, scattered sound and the radiated noise generated by mechanical vibration source with faults are generally a nonstationary process whose frequency content varies at any instant of time. For time varying signal, a traditional spectrum representation does not reveal the dynamic features of the signal and may lead to erroneous results such as the misinterpretation for a wrong conclusions. A time-frequency domain representation is needed to characterize such signatures. The PWVD is a three dimensional(time, frequency, magnitude) representation of an input signal and is ideally suited for describing transient or other nonstationary phenomena. The Wigner distribution function(WDF) has been used in the areas of optics and speech analysis[1,2,3]. Wahl and Bolton[4,5] used it to identify structure-borne noise components. The Wigner distribution function recently is proposed in the area of machinery condition monitoring and its diagnostics such as gear fault detection.

In this paper, the important properties of PWVD were investigated using both stationary and nonstationary signatures by numerical examples PWVD was applied to acoustic signatures to demonstrate its application for time-frequency domain analysis.

## 2. PSEUDO WIGNER-VILLE DISTRIBUTION FUNCTION

The WDF is a three dimensional(time, frequency, amplitude) representation of an input signal and is ideally suited describing transient or other nonstationary phenomena.

The Wigner distribution function is given as[6]

$$w(t,\omega) = \int_{-\infty}^{\infty} s^{*}(t-\tau/2)\ s(t+\tau/2)\ e^{-j\tau\omega} d\tau \tag{1}$$

where s(t) is the complex time signal, * denotes complex conjugate. This formula has a pattern of a instantaneous power spectrum[7]. There are two distinct advantages for the calculation of the WDF. First, it has the form of the Fourier transform and the existing FFT algorithm can be adaped for its computation. Second, for a finite time signal, its integration is finite within the record length of the existing signal.

The discrete type WDF developed by Classen and Mecklenbrauker[8] is expressed by,

$$w(t,\omega) = 2\sum_{\tau=-\infty}^{\tau=\infty} e^{-j2\omega\tau}\ s(t+\tau)\ s^{*}(t-\tau) \tag{2}$$

The discrete version of Eq.(2) for a sampled signal s(n) has the form

$$w(l,k) = \frac{1}{N}\sum_{n=0}^{N-1} s(l+n)\ s^{*}(l-n) e^{-j(4\pi/N)nk}, k=0,1,2,\ldots,N-1 \tag{3}$$

Eq.(3) indicates the WDF has a periodicity of N/2 [9]. Hence, even when the sampling of s(t) satisfies the Nyquist criteria, there are still aliasing components in WDF. A simple approach to avoid aliasing is to use an analytic signal before computing the WDF. The analytic signal is obtained by Hilbert transform[1]. A practical signal is a real value.

To calculate the Wigner distribution of the sampled data, it is necessary that Eq.(3) be modified to Eq.(4), because the WDF has N/2 periodicity.

$$w(m\Delta t,\ k\Delta\omega) = 2\Delta \sum_{n=0}^{2N-1} s[(m+n)\ \Delta t]\ s^{*}[(m-n)\ \Delta t]\ e^{-j2\pi nk/(2N)} \tag{4}$$

where $\Delta\omega = \pi/(2N\Delta t)$ and $\Delta t$ the sampling interval. The algorithm used in this paper is based on one written by Wahl and Bolton and can be expressed as:

$$\begin{aligned} w(m\Delta t,\ k\Delta\omega) &= RE\{2\Delta t\ FFT[corr(i)]\} \\ corr(i) &= s(m+i-1)\ s^{*}(m+i+1), \qquad m \geq i \\ &= 0, \qquad m < i \end{aligned} \tag{5}$$

where $1 \leq i \leq N+1$, $corr(2N-i+2) = corr^{*}(i)$, $2 \leq i \leq N$

The frequency resolution in Eq.(5) is one forth the resolution of an ordinary power spectrum density function, that is, it has a high resolution than the result of FFT of the original N point time record.

There are two methods to suppress the interference components of the WDF. Classen and Mecklenbrauker desribe the application of a sliding window in the time domain before calculating WDF. The WDF obtained with a window function is called the pseudo-WDF. A second option is to smooth the WDF

with a sliding averaging window in the time-frequency plane. In both case the result is to deemphasize components arising from calculations and to emphasize deterministic components. Obviously, averaging a Wigner-Ville distribution will result in a PWVD.
In this paper, a sliding exponential window in the time-frequency domain was chosen. That is, a Gaussian window function, $G(t,\omega)$ is selected to reduce the interference and to avoid the negative values as follows: let

$$G(t,\ \omega) = \frac{1}{2\pi\sigma_t\sigma_\omega} e^{-[(t^2/2\sigma_t^2)+(\omega^2/2\sigma_\omega^2)]}, \tag{6}$$

then

$$w(t,\ \omega) = \frac{1}{2\pi}\int\int w(t',\ \omega')\ G(t-t',\ \omega-\omega')\ dt'\ d\omega' > 0 \tag{7}$$

where $\sigma_\tau$ , $\sigma_\omega > 0$ and $\sigma_\tau\sigma_\omega \geq 1/2$. The time and frequency resolution's $\Delta t$ and $\Delta\omega$ of this Gaussian window are related by

$$\sigma_t = j\Delta t, \quad \sigma_\omega = k\Delta\omega \tag{8}$$

in the discrete form. Selecting t and $\omega$ to be the multiple of time and frequency steps, the sampled Gaussian window function is expressed by,

$$G(p,\ q) = \frac{1}{2\pi jk\Delta t\Delta\omega} e^{-[(p^2/2j^2)+(q^2/2k^2)]} \tag{9}$$

where p and q are integer numbers in the range $\pm 2j$ and $\pm 2k$, respectively. The convolution of the sampled WDF and the Gaussian window function can be evaluated as follows:

$$w'(l,\ m) = \frac{\Delta t\Delta\omega}{2\pi}\sum_{p=l-j}^{l+j}\sum_{q=m-k}^{m+k} w(p,q)\ G(p-l,\ q-m) \tag{10}$$

## 3. EXAMPLES AND DISCUSSIONS

A signatures generated by machinery and scattered sound involve many information about operating condition and the current status of target. It can be obtained the information from the measured signatures by applying the analysis tools appropriate to the characteristics of signal for the time records. As discussed in the previous sections, Wigner distribution is a signal transformation that is particularly suited for the time-frequency analysis of nonstationary signals. There are many advantages of using PWVD for both steady and transient signals[10, 11].

### A. Sweeping-up Harmonic Wave

Fig.1 is PWVD of the signal which sweeps up along the frequency with a logarithmic rate, that is, the sweep rate is propotional to the square root of time. It was found that the maximum magnitude of the PWVD increase with incresing the frequency. This fact is shown that the PWVD of the stable signal has a larger magnitude than the unstable signals although having the same magnitude in time domain and PWVD is the good tool for the analysis of the stability of the signal. The following functions were used to generate the desired signal:

$$s(t) = 4\ \cos\ \{\ 2\pi(30+60\ t^{1/2})\ t\ \} \tag{11}$$

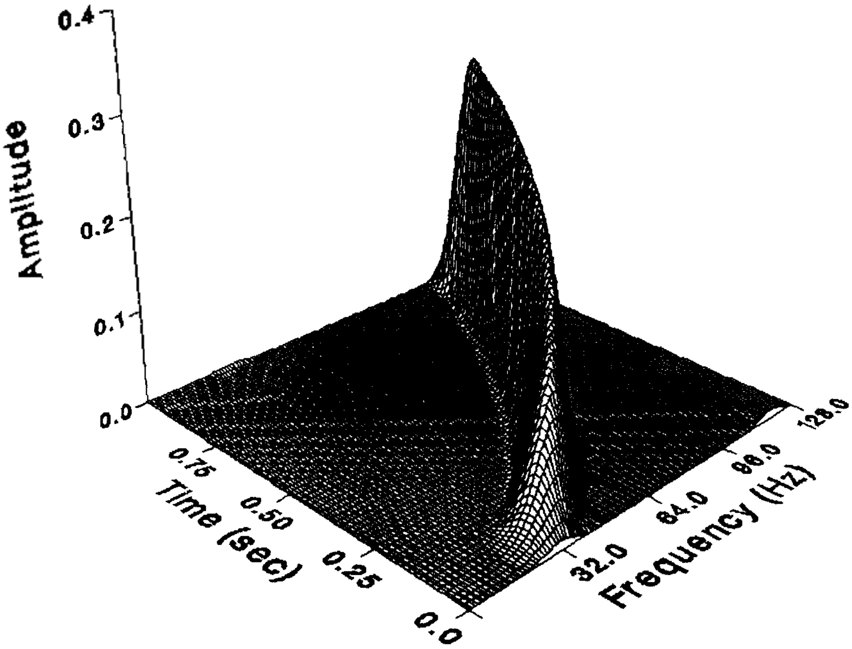



Figure 1. PWVD of a signal of sweeping-up with a logarithmic rate with time. ($f_s$=256 Hz, N=256 and smoothing window size = 10x10).

B. Scattering form a Cylindrical Shell

Scattering from a finite object provides many interesting subjects for the analysis of wave propagation phenomena in elastic body. Figure 2 and 3 are the measured scattered ultrasound from a cylindrical shell with and without a stiffener at middle point in longitudinal direction, respectively. The cylindrical shell is made of 0.6 cm thick steel with 22.12 cm in diameter, 131.3 cm long and has a ring stiffener, located at middle 68 cm from the front. The source signal used in the experiment was the continuous sine wave with pulse length 0.1 msec and center frequency 50 kHz. The data used here were from the backscattered sound coming from the test cylinder. An angle is 30° in figure 2 and 40° in figure 3 between the longitudinal axis of test cylinder and the acoustic axis of transducer or receiver. The scattered sound consists of components whose sources include specular reflection, creeping wave reradiation and so on. Each of them has different phase velocities and frequency components. The first peak of time pattern in figure 2(a) is by specular reflection included an elongation effect by front end of cylinder and the second one is by reradiation due to stiffener.

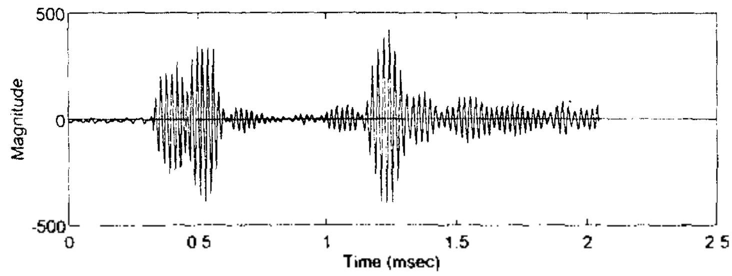
Figure 2(b) indicates the different frequency components of the first and second peaks for the same sound source. From this figure, we can see the frequency change by the characteristics of structure. If we applied the traditional spectral method to figure 2(a), we can see the frequency change by structure. WDF well represents the global feature about signatures including the transient components.

Figure 3 is the measured scattered ultrasound from a cylindrical shell without a stiffener at middle point in longitudinal direction, that is, homogeneous shell. From figure 3, we can see the incident angle is the angle around the coincident one. Figure 3(b) indicates the change of frequency contents and the sound radiation by coincident effects. And the last peak is reradiated by the end of cylidrical shell. PWVD well represents the global features along the time axis including the transient components.

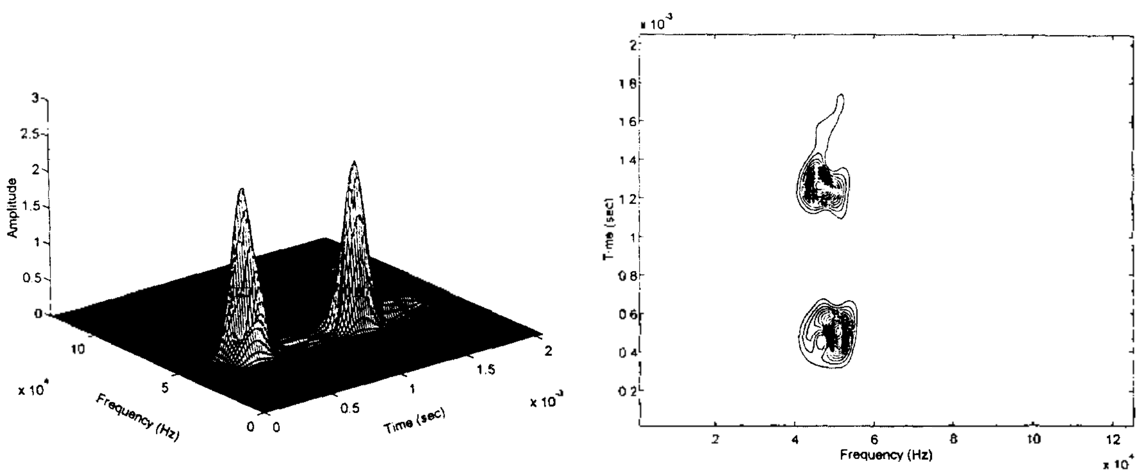
4. CONCLUSIONS

The pseudo Wigner-Ville distribution has been investigated and applied to analyzing nonstationary signals typical of transient acoustical signatures. The results of this research will be a valuable analysis for condition monitoring of an objects by scattered sound. The following conclusion can be drawn:

(1) The PWVD is ideally suited for portraying nonstationary time signals as well as stationary signals.
(2) The use of analytic signal in calculating the Wignerdistribution eliminates aliasing problems.
(3) The Gaussian window function for smoothing the Wigner-Ville distribution is very effective and the presence of cross-terms is significantly reduced.
(4) The PWVD characterizes the time-frequency domain distribution of the signal well and may be a useful tool for analysis of acoustic signatures with transient conditions.

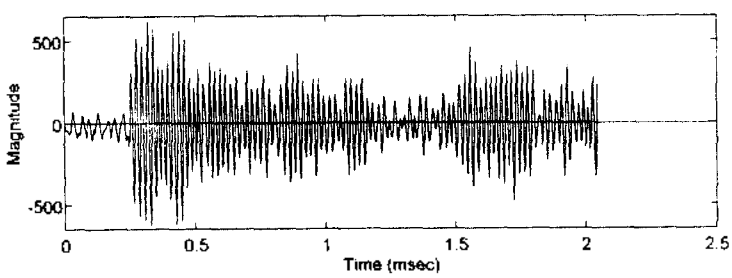



(a)

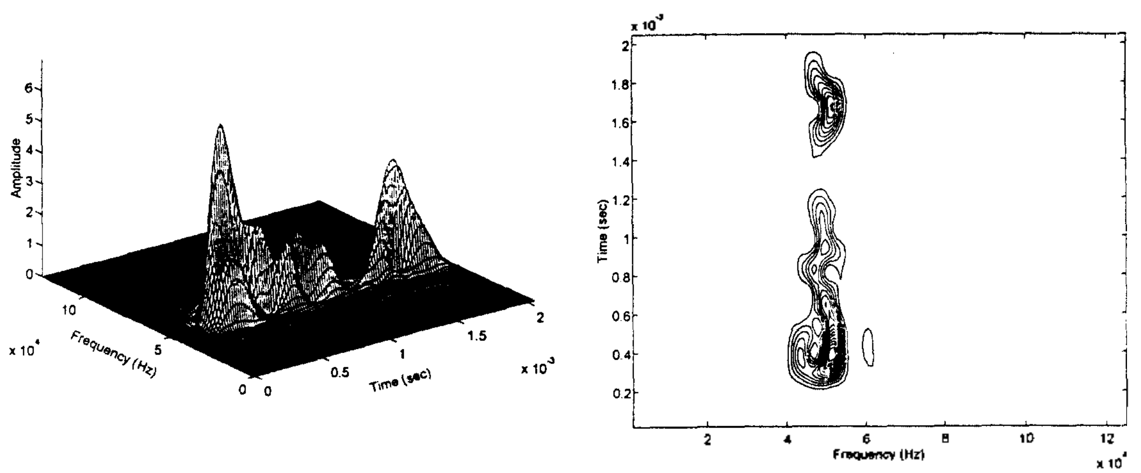



(b)

Figure 2. Time-frequency localization of WDF, (a) time signal, (b) 3-D and contour plots of its PWVD ($f_s$=250 kHz, N=512).




(a)




(b)

Figure 3. Time-frequency localization of PWVD, (a) time signal, (b) 3-D and contour plots of its PWVD ($f_s$=250 kHz, N=512).

REFERENCES


1. Bastiaans, M. J.,"The Wigner Distribution Function Applied to Optical Signals and Systems," Optics Communications 25(1), 26-30 (1978).
2. Bastiaans, M. J.,"Wigner Distribution Function and its Application to First-Order Optics," J. Optical Society Am. 69(12), 1710-1716(1979).
3. Bartelt, H. O., Brenner, K. H. and Lohmann, A. W.,"The Wigner Distribution function and its Optical Production, Optics Communications 32(1), 32-38(1980).
4. Wahl, T. and Bolton, J., 'The use of the Wigner Distribution to Anayze Structural Impulse Response,' International Congress on Recent Developments in Air and Structure Borne Sound, March, 1990.
5. Wahl, T. and Bolton, J., 'The application of the Wigner Distribution to the Identification of Structure-Borne Noise Components,' J. Sond & Vib. 163(1), 101-122 (1993).
6. Wigner, E., 'On the Quantum Correction for Thermodynamic Equilibrium,' Physics Review 40, 749-759 (1932).
7. Mark, W. D., 'Spectral Analysis of the convolution and filtering of Non-stationary Stochastic Processing, ' J. Sound & Vib. 11, 19-63 (1970).
8. Classen, T. and Mecklenbrauker, W.,'The Wigner Distribution - A Tool for Time Frequency Analysis,' Philips J. of Research 35, Part I: 217-250, Part II: 276-300, Part III: 372-389 (1980).
9. N. Yen, 'Time and Frequency Representation of Acoustic Signals by Means of the Wigner Distribution Function: Implementation and Interpretation,' J Acoust. Soc. Am. 81(6), 1841-1850(1987).
10. Y. S. Shin and J. J. Jeon, 'Pseudo Wigner-Ville Time Frequency Distribution and its Application to Machinery Condition Monitoring,' J. of Shock & Vibration 1(1), 65-76 (1993).
11. J. J. Jeon and Y. S. Shin, **Pseudo Wigner-Ville Distribution, Computation Program and its Application to Time-Frequency Domain Problem**, NPS-ME-93-002, March 1993.