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Different Aspects of Creativity

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Introduction

The essence of the lecture is described in the Abstract. But no theoretical description cannot replace clear examples. So, we will illustrate the ideas mentioned above with three examples taken from physics and mathematics.

Poets In Physics: Heat Exchanger

At the beginning we shall consider the heat exchanger. This device is described in some books, but usually the description is very complicated and involves calculus.

We are going to demonstrate certain considerations which show that the problem can be explained in a very clear way by using only several bricks: very basic physical ideas and laws.

Imagine that you have two equal portions of water, e.g. 1*l* each, at temperatures 0℃ and 100℃, respectively. You have also various vessels with arbitrary volumes and with various properties of their walls (adiabatic or/and isothermic).

We want to warm up the water initially cold with the heat of the water initially warm. Our aim is to find the maximum possible temperature of the water initially cold. We assume that the warm and cold waters cannot be mixed with each other. In order to make the problem more realistic you may assume that the warm water is somewhat dirty (it is taken e.g. from the power station) and the cold water is clean (it is taken e.g. from the city filters). For the sake of simplicity we assume that the specific heat of water is constant and we neglect the thermal expansion of water.

The answer is surprising: the final temperature of the water initially cold can practically be equal to 100℃! But at the beginning we shall describe how the cold water could reach 80℃.

At first we construct a "heat exchanger": We take 10 vessels of equal capacity whose total volume is negligible compared to the initial volume of the water. We fill 5 of them with the cold water and the remaining 5 with the warm water. Later we establish thermal contacts between the small vessels and the original portions of water (warm or cold) in order to obtain the temperature distribution shown in Fig.1.

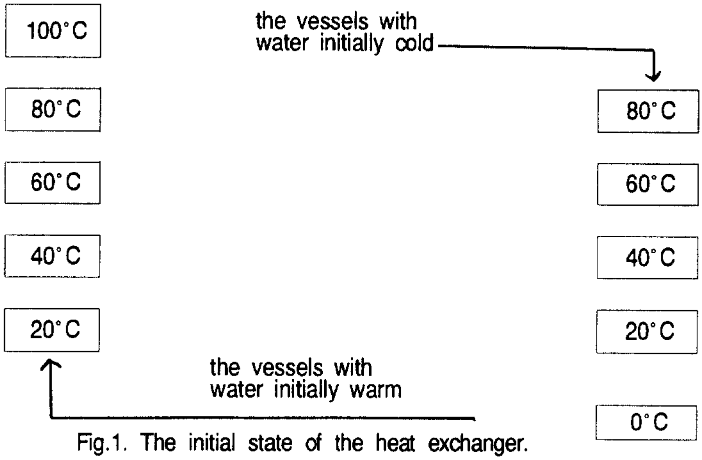


Fig.1. The initial state of the heat exchanger.

As the total volume of the vessels is negligible, the heat losses when constructing the heat exchanger and the volumes of the initial portions of water are negligible too.

Now we perform the operations described below the Fig.2, which shows the final temperature distribution in the system after completing all the operations. Next we establish thermal contacts shown in Fig.3. In this way we reconstruct the initial temperature distribution of the heat exchanger, but in result of the operations described in Fig.2 in the containers for "final" waters there are: one portion of water warmed up (with temperature 80℃) and one portion of water cooled down (with temperature 20℃). The operations from Fig. 2 can be repeated many times. Always the state of the heat exchanger is reconstructed, but the amount of water in the "final" containers increases by one portion.

In this way we can warm up the whole initial volume of water initially cold up to 80℃. (Of course, at the same time the water initially warm is cooled down to 20℃.)

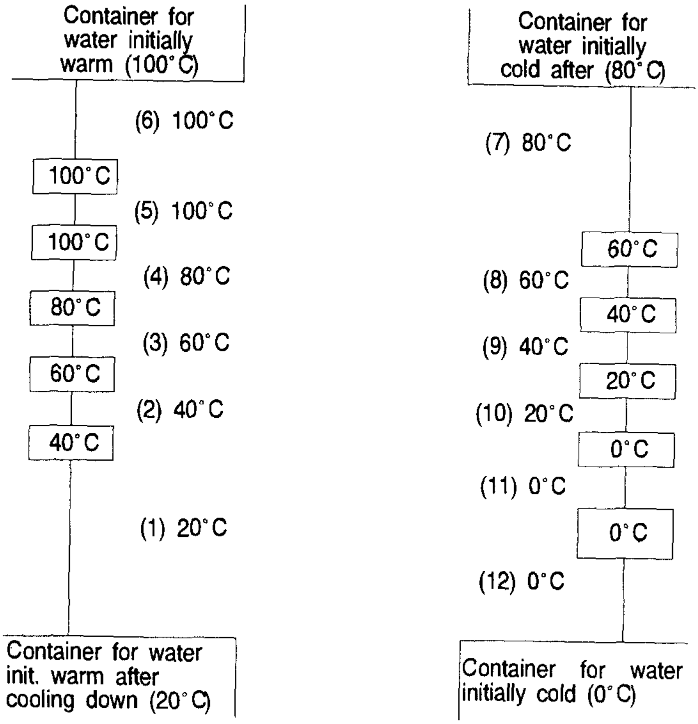


Fig.2. Operations at the left side: The water with temperature 20℃ is poured out into the "container for water initially warm after cooling down". The water with 40℃ is poured out into the vessel which previously contained water with 20℃. The water with 60℃ is poured out into the vessel which previously contained water with 40℃. The water with 80℃ is poured out into the vessel which previously contained water with 60℃. The water with 100℃ is poured out into the vessel which previously contained water with 80℃. At the end we refill the empty upper vessel with a new portion of water with 100℃. The figure shows the final temperature distribution. The numbers in brackets correspond to the order of the operations described above. Similar operation are performed at the right side (in the order corresponding to the numbers in brackets). The temperatures in brackets (at the arrows) are the temperatures of the water that is poured out of one vessel into another.

It is clear that by taking 100 vessels with the total volume negligible compared to $1l$ we could worm up the cold water up to 98℃. For 1000 vessels with the total volume << $1l$ the final temperature of the water initially cold would be 99.8 ℃, for 10,000 vessels - 99.98℃, etc.

The above considerations illustrate how the real heat exchanger works. The only difference is that the real heat exchanger, schematically shown in Fig.4, works continuously, while our heat exchanger works in a discrete manner.

As you see the very serious and difficult problem has been explained here in the way that can be understood even by the elementary school pupils. The explanation makes use of elementary knowledge on the heat exchange and thermal equilibrium only. The number of "bricks" used here is very small, but the construction is very beautiful!

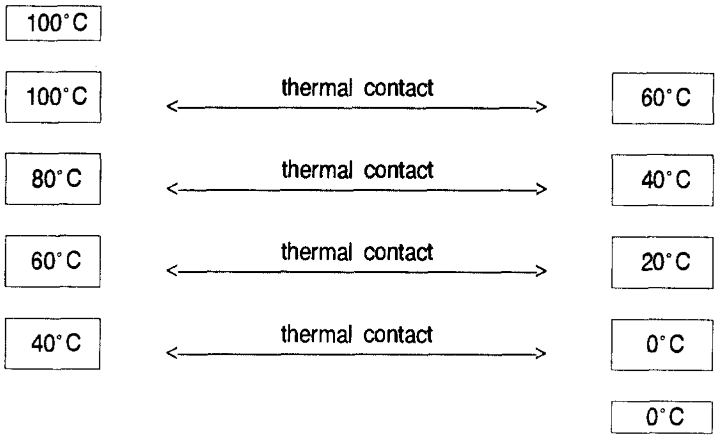


Fig.3 Now we establish thermal contacts between the vessels "on the same level". In result the temperature distribution shown in Fig.1 is reconstructed. But now in the containers for "final" waters there are: one portion of water warmed up (with temperature 80℃) and one portion of water cooled down (with temperature 20℃). The operations shown in Fig.2 can be repeated many times. Always the initial temperature distribution of the heat exchanger (Fig.1) is reconstructed and the amounts of the "final" water increases by one portion.

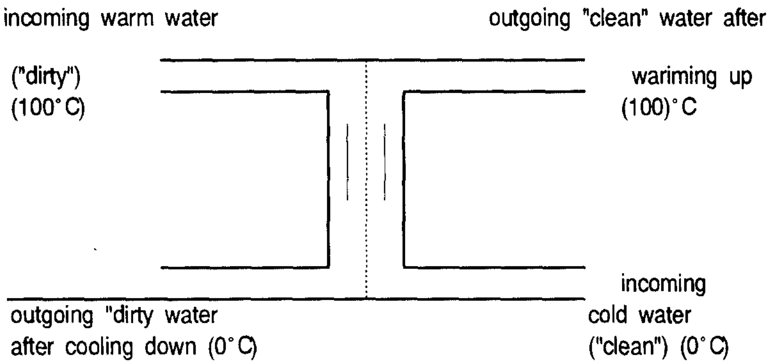


Fig.4. Scheme of a real heat exchanger. The heat exchange takes place along the contact of pipes with the "dirty" and "clean" waters (broken line).

Free Hunters: Circuit with Passive Elements

Now we shall give an example of certain way of thinking that are far from any typical approach. Let us consider the following circuit:

Our aim is to show that the part of the circuit between the points A and B (marked in Fig.5) for $R^2 = L/C$ behaves like an ohmic resistors (the voltage E should be equal to the current I flowing from A to B multiplied by some real constant; no phase shift between the current I and the voltage E).

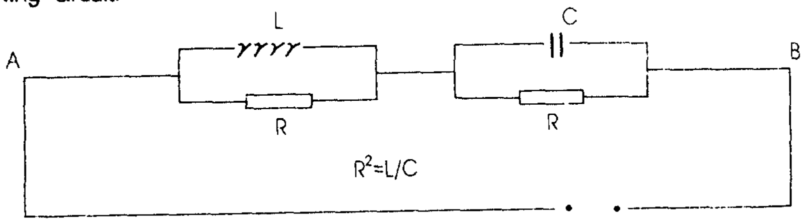
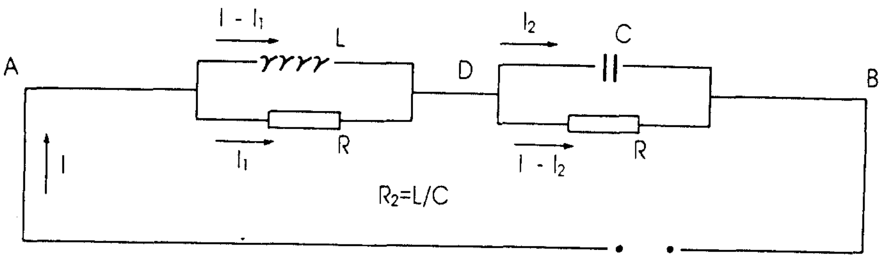


Fig. 5.

The circuit considered here is very typical and there is a very well known algorithm which allows to solve our problem. We think here about the so called impedances. If the concept of impedances is used, the problem can be solved by everybody without any difficulties, even by non-clever pupils. But the impedances are not commonly known (they are not present in our syllabus for the high schools). See how the problem is solved by one of the "free hunters".

Using the notation of Fig.6, we may write the following equations :



(1) for the mesh ARDRBEA the Kirchhoff's law gives:

$$E = RI_1 + R(I - I_2);$$

(2) the voltages on the coil and the left-hand resistor are equal:

$$RI_1 = L\frac{d}{dt}(I - I_1);$$

(3) the voltages on the capacitor and the right-hand resistor are equal:

$$\frac{Q}{C} = R(I - I_2);$$

(4) in the last formula Q denotes the charge accumulated on the capacitor. We have:

$$I_2 = \frac{dQ}{dt}.$$

By differentiating the equation (3), and making use of the equation (4) and relation RC = L/R, we obtain the equation:

$$RI_2 = L\frac{d}{dt}(I_1 - I_2).$$

Subtracting this equation from the

equation (2), we get the differential equation:

$$(I_1 - I_2) = -RC\frac{d}{dT}(I_1 - I_2).$$

The differential equations are not present in the high schools. What to do? The situation seems to be hopeless.

But not for free hunter:

Remark that the last equation does not include E. The variation with time of $I_1 - I_2$ is therefore independent of variation with time of E. So, one can consider an arbitrary E, e.g. $E = 0$. In this case there are no currents in the circuit.

Thus:

$$I_1 - I_2 = 0,$$

i.e.

$$I_1 = I_2.$$

Inserting this into the equation (1) we get:

$$E = RI.$$

q.e.d.

We see that the problem has been solved in a very untypical way. Due to a genius remark on independence of E all the difficulties connected with solving the differential equation have been avoided!

Small Discoverers: Pythagorean Theorem

The methods based on symmetry or dimensional analysis are absent in the high schools. These methods are, however, very useful and allow to simplify many considerations. Due to them some problems can be solved without any calculations. Certain pupils discover them by themselves. Now we present a proof of the Pythagorean theorem based on dimensional analysis.

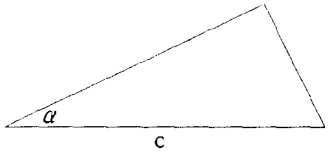


Fig.7.

Let us consider a rectangular triangle shown in Fig.7. The triangle is well determined by two such elements as the edge c and the angle α. It means that if we know c and α then all other parameters of the triangle like the edge a, the edge b, the height, the perimeter, etc. can be determined.

In particular, the area S of the triangle should be expressed by c and α. The dimension of c is cm, the dimension of α is 1 (the angle is a dimensionless quantity). The dimension of S is cm^2. The only quantity with dimension cm^2 that can be built up from c and α (and that is not identically equal to zero) is $f(\alpha)c^2$, where f is a dimensionless function of α, not identically equal to zero. Thus, we may write:

(1) $S = C^2 f(\alpha)$

Now we divide the triangle into two parts I and II with the height h as in Fig.8. The angles denoted with the same symbol α are equal (their arms are perpendicular). The triangles I and II are rectangular and they have the same angle α at the hypotheses.
Thus,

(2) $S_I = a^2 f(\alpha)$ and

$S_{II} = b^2 f(\alpha)$.

From additivity of area we have: -

(3) $S = S_I + S_{II}$

The last relation, in virtue of the equations (1) and (2), can be written in

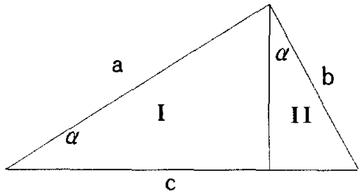


Fig.8

the form:

$$c^2 f(\alpha) = a^2 f(\alpha) + b^2 f(\alpha).$$

After dividing by $f(\alpha)$, we get:

$$c^2 = a^2 + b^2$$

We obtained the Pythagorean theorem without any calculations, we have got it in result of applying the dimensional analysis and several basic geometric properties.

Final Remarks

Creativity has many aspects, many faces. It is not any discovery. But usually we discuss the problem theoretically only. The aim of this lecture was to show different faces of creativity in practice by presenting several concrete examples. The examples are of quite different character and on different levels of difficulty. Our aim was to describe some criteria that may be used in practice.