Semantics of Cooperative Dialogues

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Abstract

In this paper, I propose a semantic framework of the theory for cooperative dialogues in [18]. This framework is called SCD (Semantics of Cooperative Dialogues). It consists of a combination of DRT (Discourse Representation Theory) [2, 11, 13] and the Situation Semantics [5]. It also concludes a revision algorithm of shared belief in dialogues. Then I present some formal properties of SCD: possible situations, belief-sharing and self-referential part, and ontology of discourse referents. Next, some linguistic applications of SCD are discussed: negation and denial, Quantifying-In, Hob-Nob sentence, and Conway paradox.

1 Introduction

The semantics of cooperative dialogues must satisfy the following conditions:

(1) 1. partiality
2. multi-model
3. model transformation

1. should capture the epistemic partiality of participants of dialogues. For this, I make use of DRT (Discourse Representation Theory) [11, 13, 2] at the syntactic level, and Situation Semantics [5] at the semantic level.

Participants of dialogues are indicated with a, b. An utterance can be differently interpreted by a and b. And their interpretations can be different from the fact. To explain this, I assume the model of a, b, and the fact. (Cf. [15, 17].)

The semantic status of a dialogue changes along with a series of utterances. 3. explains the dynamic feature of dialogues.

I call the present framework Semantics of Cooperative Dialogues (SCD). In SCD, an utterance made by a participant is transformed into the corresponding logical formula, and added to the conditions in Discourse Representation Structure (DRS). DRS is interpreted in the situation-semantically constructed model to explain the semantic character of the dialogue.

An utterance transforms DRS_n to DRS_{n+1}. In this sense, it is a function which maps DRS_n to DRS_{n+1}.

2 Theoretical Framework

2.1 Syntax

2.1.1 Language $\mathcal{L}$

The syntax of DRSs and the language $\mathcal{L}$ for DRSs is defined as follows.

Vocabulary The vocabulary $Voc(\mathcal{L})$ of $\mathcal{L}$ consists of the following elements:

(i) Set of individual constants $Con(\mathcal{L}) \ni a, b, c, \cdots, r$ (sometimes with indices).
(ii) Set of individual variables $Var(\mathcal{L}) \ni x, y, z$ (sometimes with indices).
(iii) Set of $n(\geq 0)$-ary predicates $Pred^n(\mathcal{L}) \ni p_1^n, p_2^n, \cdots$. Set of predicates $Pred(\mathcal{L}) = \bigcup_n Pred^n$.
(iv) logical connectives $\neg, \vee, \wedge, \rightarrow, \equiv$.
(v) quantifiers $\forall, \exists$.

If $\mathcal{L}$ is obvious, $Con(\mathcal{L})$, $Var(\mathcal{L})$, $Pred^n(\mathcal{L})$, $Pred(\mathcal{L})$ are abbreviated to $Con, Var, Pred^n$, $Pred$ respectively. (Likewise $Form(\mathcal{L})$, $\mathcal{DRS}(\mathcal{L})$ below.)

Set of terms $Term = Con \cup Var$.

The elements of $Pred$ are normally denoted using mnemonics '*walk*' etc.

$Pred^2$ contains Bel', '='.

Formation Rules The class of $\mathcal{L}$ formulas $Form(\mathcal{L})$ is the greatest class such that if $p \in Form(\mathcal{L})$, then p is one of the following forms[1]:

(i) If $\alpha_1, \cdots \alpha_n \in Term$, then $p_i^n \in Pred^n \backslash \{Bel\}$, $p_i^n(\alpha_1, \cdots, \alpha_n) \in Form$.

(ii) If $\alpha \in Term$, $p \in Form$, then $Bel(\alpha, p) \in Form$.

(iii) If $p, q \in Form$, then $\neg p$, $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$, $(p \equiv q) \in Form$.

(iv) If $\alpha \in Var$, $p \in Form$, then $\forall \alpha p$, $\exists \alpha p \in Form$.

In obvious cases, parentheses in $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$ are omitted.

$(p \rightarrow q)$, $(p \equiv q)$ abbreviate $(\neg p \vee q)$, $(p \rightarrow q) \wedge (q \rightarrow p)$ respectively.

$=(\alpha_1, \alpha_2)$ are normally designated by $\alpha_1 = \alpha_2$

The elements of $Form(\mathcal{L})$ without free variables are called sentences, and their class are denoted by $Sent(\mathcal{L})$.

2.1.2 Discourse Representation Structure

The class of discourse representation structures $\mathcal{DRS}(\mathcal{L})$ constructed from $\mathcal{L}$ is the greatest class such that if $DRS(\mathcal{L}) \in \mathcal{DRS}(\mathcal{L})$, $DRS(\mathcal{L})$ is the following form:

> If $\alpha \in Term$, $C_{K(\alpha)} \subseteq Form \cup \mathcal{DRS}(\mathcal{L})$, $U_{K(\alpha)}$ is the set of individual constants and variables in $C_{K(\alpha)}$, then $< K(\alpha); U_{K(\alpha)}, C_{K(\alpha)} > \in \mathcal{DRS}(\mathcal{L})$.

The discourse representation structure $DRS < K(\alpha); U_{K(\alpha)}, C_{K(\alpha)} >$ is also designated by $K(\alpha) = < U_{K(\alpha)}, C_{K(\alpha)} >$, and $K(\alpha)$ represents the DRS.

$K(\alpha)$ is called the label of DRS or simply the DRS. $U_{K(\alpha)}$ is called the domain of DRS, its elements are called discourse referents (drs). The set of drs are designated by DR. $C_{K(\alpha)}$ are called the condition part of $K(\alpha)$, its elements are called conditions of $K(\alpha)$. Conditions which are elements of $Form$ are called simple conditions, those which are elements of $\mathcal{DRS}$ complex conditions, and they are designated by $SimpC_{K(\alpha)}$ and $CompC_{K(\alpha)}$ respectively. $SimpC_{K(\alpha)} \cup CompC_{K(\alpha)} = C_{K(\alpha)}$. The set of vocabulary in $K(\alpha)$ is designated by $Voc(K(\alpha))$.

If $K(\alpha) \ni \cdots \ni K(\beta)$, $K(\alpha)$ is called the superDRS of $K(\beta)$, and $K(\beta)$ a subDRS of $K(\alpha)$. A subDRS $K(\gamma)$ of $K(\alpha)$ such that there is no $K(\beta)$ such that $K(\alpha) \ni \cdots K(\beta) \ni \cdots K(\gamma)$ is called an immediate subDRS of $K(\alpha)$, and $K(\alpha)$ the immediate superDRS of $K(\gamma)$. If $K(\alpha)$ has no superDRSs, $K(\alpha)$ is called the starting DRS.

If there is a sequence of immediate subDRSs $K(\alpha 0), \cdots, K(\alpha n\text{-}1), K(\alpha n)$ starting with $K(\alpha 0)$, $K(\alpha n)$ is also designated by $K_{\alpha 0 \cdots \alpha n-1}(\alpha n)$ or $K(\alpha 0 \cdots \alpha n\text{-}1 \alpha n)$

The intuitive meaning of $K(\alpha)$ is that the epistemic subject α has the ontololgy of individuals $U_{K(\alpha)}$, and, of these individuals, believes that $C_{K(\alpha)}$.

$Exp(\mathcal{L}) = Voc(\mathcal{L}) \cup Form(\mathcal{L}) \cup \mathcal{DRS}(\mathcal{L})$ is called the class of expressions of $\mathcal{L}$.

2.1.3 DRS of dialogues

The epistemic status of a dialogue by a, b at a time point i is illustrated as follows:

$K(fi)$: [$a, b, c_1, c_2, c_3, \ldots$ | $A(c_1), B(c_2), C(c_3), \ldots$; $K(a)$: [ ] ; $K(b)$: [ ]]

Fig. 1

$K(fi)$ represents the epistemic status at the time point i of an abstract individual f. $K(fi)$ consists of the domain $U_{K(fi)} = \{a, b, c_1, c_2, c_3, \ldots\}$, and the condition part $C_{K(fi)}$. $C_{K(fi)}$ consists of simple conditions $A(c_1), B(c_2), C(c_3), \ldots$, and complex conditions $K(a), K(b)$, which represent the epistemic status of a and b.[2]

Let one of the participants of a dialogue be represented by α, the other by β. Then $K(\alpha)$ has the following internal structure:

$K(\alpha)$:
- < domain > $a, b, c_1, c_2, c_3, \ldots$
- < background knowledge >
  - $rank0$: ......
  - $rank1$: ......
  - $rank2$: ......
  - ....
- $K(a, b)$:
  - < domain > $a, b, d_1, d_2, \ldots$
  - < SRP > $K(a, b)$
  - < IKSP >
    - $d_1 = d_2$
    - ......
  - < EKSP >
    - $Bel(a, p)$
    - $Bel(b, p)$
    - $Bel(a, q)$
    - ......

Fig. 2

The rank in the background knowledge represents α's certainty of it. Rank 0 has the highest certainty.

$K(a, b)$ abbreviates $K(a)$ and $K(b)$ with the same domain and conditions.

SRP (Self-referential Part) represents the infinite embedding of shared belief for a. E.g., $K(a)$ contains the correspondence of the following statements:

(2) a believes that $d_1 = d_2$.
a believes that b believes that $d_1 = d_2$.
....

IKSP (Implicit Knowledge Sharing Part) consists of the conditions which α assumes are the common knowledge between α and β.

An utterance of α (β) is transformed into the corresponding first-order formula p, and added to EKSP (Explicit Knowledge Sharing Part) in the form $Bel(\alpha,p)$ ($Bel(\beta,p)$). I.e., $Bel(\alpha,p)$ ($Bel(\beta,p)$) is a shared belief between α and β. But p isn't, until her partner agrees to p so that p becomes a shared belief and a condition of EKSP.

2.2 Semantics

2.2.1 Model structure

$MS =< Rel, Ind, Par, Pol >$ is called a model structure.

Rel: set of relations,
Ind: set of individuals,
Par: set of parameters,
Pol: $\{0,1\}$ (set of polarities).

$Rel = \bigcup_n Rel^n$. Rel^n is called a set of $n(\geq 0)$ place relations. Rel^n consists of n place relations $rel_1^n, rel_2^n, \ldots$.

Rel^1 contains '$\forall\alpha$', '$\exists\alpha$' ($\alpha \in Par$). Rel^2 contains 'Bel', '$\vee$', '$\wedge$', '$\rightarrow$', '$\equiv$'. They are called special relations, their set is represented by $SpecRel$. Rel^2 contains '='.

The universe of hypersets[3] with the set of atoms $Rel \cup Ind \cup Par \cup Pol$ is called the class of entities of the model structure MS, and represented by $ENT(MS)$.

If for two model structures MS_1, MS_2, $Rel_1 \supseteq Rel_2, Ind_1 \supseteq Ind_2, Par_1 \supseteq Par_2$, MS_1 is called a super-model structure of MS_2, MS_2 a sub-model structure of MS_1.

2.2.2 Infon, situation, truth, and parameter

The class of infons and situations are defined in a simultaneous co-inductive manner as follows:

Infon The class of infons Inf is the largest class such that if $inf \in Inf$, then inf is one of the following form:

If $rel_i^n \in Rel^n \backslash SpecRel$, $\alpha_1, \ldots, \alpha_n \in Ind \cup Par$, $pol \in Pol$, then $< rel_1^n, \alpha_1, \ldots, \alpha_n; pol > \in Inf$.

If $\tau \in Ind \cup Par$, $\alpha \in Par$, $inf_1, inf_2 \in Inf$, $pol \in Pol$, then
$< \forall\alpha, inf_1; pol >$,
$< \exists\alpha, inf_1; pol >$,
$< \vee, inf_1, inf_2; pol >$,
$< \wedge, inf_1, inf_2; pol >$,
$< \rightarrow, inf_1, inf_2; pol >$,
$< \equiv, inf_1, inf_2; pol >$,
$< Bel, \tau, inf; pol > \in Inf$.[4]

The infon $\overline{inf}$ with the inverted polarity of inf is called the conjugate of inf.

Situation The set of situations $Sit = \mathcal{P}(Inf)$.

Parameters If $Q \in \{\forall, \exists\}$, $\alpha \in Par$, $Q\alpha$ is called the quantificational relation. For an occurence of α in inf, if $Q\alpha$ occurs in inf along the construction of inf from the occurence of α upwards, the occurence of α is in the scope of the first occurence of such $Q\alpha$. And the occurence of such an α is called the bound parameter of inf. If α doesn't occur in the scope of any $Q\alpha$, the occurence of the α is called a free parameter of inf.

If inf contains no free parameters, inf is called a pure infon, and its class is represented by $Pure(Inf)$. If inf contains free parameters, inf is called a parametrized infon, and its class is represented by $Par(Inf)$.

Likewise, the class of pure situations $Pure(Sit)$, parametrized situations $ParSit$ are defined.

$Pure(Inf) \cup Par(Inf) = Inf$,
$Pure(Sit) \cup Par(Sit) = Inf$.

If α is a free parameter or an individual in inf, the result of replacing all occurences of α with $a \in Ind \cup Par$ is represented by inf_a^α, and is called the (α, a)-variant of inf.

Truth inf is true in the situation s iff $inf \in s$. It's designated as $s \models inf$.

inf is false in the situation s iff $inf \notin s$ It's designated as $s \not\models inf$

$s \models inf$ ($s \not\models inf$) is also called that s supports (does not support) inf.

2.2.3 Constraints on situations

In the following, $i, j, k \in Ind \cup Par$, $\alpha \in Par$, $inf, inf_1, inf_2 \in Inf$, $s, t, u \in Sit$

Consistent situation The situation s which satisfies the following condition is called a consistent situation:

(3) For an arbitrary infon inf, it's not the case that $inf, \overline{inf} \in s$.

Contradictory situation The situation s which satisfies the following condition is called a contradictory situation:

(4) There is some infon inf such that $inf, \overline{inf} \in s$.

Basic situation The situation s which satisfies the following condition is called a basic situation:

(5) (i) $s \models < Bel, i, inf; 1 > \Leftrightarrow s(sit(i)) \models inf$
$s \models < Bel, i, inf; 0 > \Leftrightarrow s(sit(i)) \not\models inf$
$sit(i)$ is the situation which corresponds to i, is normally abbreviated to i. si is the situation i from the viewpoint of s. (Likewise for sij etc.)

(ii) For $s^*, u^* \in Sit^*$, $s^*ttu^* \models inf \Leftrightarrow s^*tu^* \models inf$

Reduction axiom (5ii) means that 'a believes that a believes that p.' is identified with 'a believes that p.', which is formulated by the following reduction axiom:

For $\alpha \in Term, p \in Form$,
$Bel(\alpha, Bel(\alpha, p)) \equiv Bel(\alpha, p)$.

Canonical situation The basic situation s which satisfies the following condition is called a canonical situation:

(6) (i) $s \models inf \Leftrightarrow inf \in \{inf' \mid \bigwedge s \leq inf'\}$

(ii) $s \models < \forall\alpha, inf; 1 > \Leftrightarrow$ for all $a \in Ind$, $s \models inf_a^\alpha$
$s \models < \forall\alpha, inf; 0 > \Leftarrow$ for an $a \in Ind$, $s \models \overline{inf_a^\alpha}$

(iii) $s \models < \exists\alpha, inf; 1 > \Leftarrow$ for an $a \in Ind$, $s \models inf_a^\alpha$
$s \models < \exists\alpha, inf; 0 > \Leftrightarrow$ for all $a \in Ind$, $s \models \overline{inf_a^\alpha}$

(iv) $s \models <=, i, i; 1 >$
$s \models <=, i, j; 1 > \Rightarrow s \models <=, j, i; 1 >$
$s \models <=, i, j; 1 >, s \models <=, j, k; 1 > \Rightarrow s \models <=, i, k; 1 >$.
$s \models <=, i, j; 1 >, s \models inf \Rightarrow s \models inf_j^i$.

(6i) means that the canonical situation s is a filter with the generator $\bigwedge s$. So the canonical situation has the following property:

(7) (i) $s \models < \wedge, inf_1, inf_2; 1 > \Leftrightarrow s \models inf_1$, and $s \models inf_2$.
$s \models < \wedge, inf_1, inf_2; 0 > \Leftarrow s \models \overline{inf_1}$, or $s \models \overline{inf_2}$.

(ii) $s \models < \vee, inf_1, inf_2; 1 > \Leftarrow s \models inf_1$, or $s \models inf_2$,
$s \models < \vee, inf_1, inf_2; 0 > \Leftrightarrow s \models \overline{inf_1}$, and $s \models \overline{inf_2}$.

(iii) $s \models < \wedge, inf_1, inf_2; 0 > \Leftrightarrow$
$s \models < \vee, \overline{inf_1}, \overline{inf_2}; 1 >$,
$s \models < \vee, inf_1, inf_2; 0 > \Leftrightarrow$
$s \models < \wedge, \overline{inf_1}, \overline{inf_2}; 1 >$. (de Morgan)

(iv) $s \models < \wedge, inf_1, inf_2; 1 > \Leftrightarrow$
$s \models < \wedge, inf_2, inf_1; 1 >$,
$s \models < \vee, inf_1, inf_2; 1 > \Leftrightarrow$
$s \models < \vee, inf_2, inf_1; 1 >$. (Commutativity)

(v) $s \models < \rightarrow, inf_1, inf_2; 1 >$.
$s \models inf_1 \Rightarrow s \models inf_2$. (Modus Ponens)

In the following, I consider only canonical situations.

Collapse axiom

(8) For $\alpha \in Term$, $p \in Form$,

$$Bel(\alpha, p) \rightarrow p$$

is called the collapse axiom.

The situation s such that

$$s \models < \rightarrow, < Bel, \alpha, inf; 1 >, inf; 1 >$$

is called a collapsed situation.

2.2.4 Model

$\mathcal{M} =< MS, f >$ is a model of DRS. f is a partial function from $Exp(\mathcal{L})$ into $Ent(MS)$ defined as follows:

(9) For $Voc(\mathcal{L})$:

(i) If $\alpha \in Con$, then $f(\alpha) \in Ind$.

(ii) If $\xi \in Var$, then $f(\xi) \in Par$.

(iii) If $pred_i^n \in Pred^n$, then $f(pred_i^n) = rel_i^n$.

(For $\tau \in Term(\mathcal{L}) \cup Pred(\mathcal{L})$, $f(\tau)$ is normally represented by τ.

(10) For $Form(\mathcal{L})$:

(i) For $\alpha_1, \cdots, \alpha_n \in Term$,
$p_i^n \in Pred^n \backslash \{Bel\}$,
$p_i^n(\alpha_1, \cdots, \alpha_n) \in Form$,
$f(p_i^n(\alpha_1, \cdots, \alpha_n)) =$
$< f(p_i^n), f(\alpha_1), \cdots, f(\alpha_n); 1 >$.

(ii) For $p, q \in Form$, $\neg p$, $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$, $(p \equiv q) \in \underline{Form}$,
$f(\neg p) = \overline{f(p)}$, $f(p \vee q) = < \vee, f(p), f(q); 1 >$,
$f(p \wedge q) = < \wedge, f(p), f(q); 1 >$,
$f(p \rightarrow q) = < \rightarrow, f(p), f(q); 1 >$,
$f(p \equiv q) = < \equiv, f(p), f(q); 1 >$.

(iii) For $\alpha \in Var$, $p \in Form$, $\forall \alpha p$, $\exists \alpha p \in Form$,
$f(\forall \alpha p) = < \forall f(\alpha), f(p); 1 >$,
$f(\exists \alpha p) = < \exists f(\alpha), f(p); 1 >$.

(iv) For $\alpha \in Term, p \in Form$,
$Bel(\alpha, p) \in Form$,
$f(Bel(\alpha, p) = < f(Bel), f(\alpha), f_{f(\alpha)}(p); 1 >$.
($f_{f(\alpha)}$ is the interpretation function of $\mathcal{M}_{f(\alpha)}$.)

(11) For $\mathcal{DRS}(\mathcal{L})$:

For $\alpha \in Term$, $K(\alpha) \in \mathcal{DRS}(\mathcal{L})$
$f(K(\alpha)) = < \mathcal{M}_\alpha, sit(\alpha), \{f_\alpha(K_i)\}_{i \in \mathbf{N}} >$.
$f_\alpha \lceil DR = U_{K(\alpha)}$. ($f_\alpha \lceil DR$ is the restriction of f_α's domain to DR.)
f_α is a bijection such that $f_\alpha[Con \cup U_{K(\alpha)}] = Ind_\alpha$
f_α is a bijection such that $f_\alpha[Var \cup U_{K(\alpha)}] = Par_\alpha$
$sit(\alpha)$ is the minimal canonical situation such that
$sit(\alpha) \supseteq \{f_\alpha(c) \mid c \in SimpC_{K(\alpha)}\}$.
K_i is an immediate subDRS of $K(\alpha)$.

If $K(\alpha)$ contains a series of embedded subDRSs, the above interpretation produces a series of subinterpretations and subsituations $f_\alpha, f_{\alpha\beta}, f_{\alpha\beta\gamma}, \cdots$; $\alpha, \alpha\beta, \alpha\beta\gamma, \cdots$. As in (5ii), $f_{s^*ttu^*}$ and s^*ttu^* are identified with $f_{s^*tu^*}$ and s^*tu^*.

The interpretation of a dialogue at the time point i is given by $f(K(fi))$.

2.3 Revision of DRS

The DRS and the situation it represents is dynamically changed by a series of utterances. The algorithm is formulated as follows:[5]

(12)

(i) (ia) If α utters P at $K(fi)$, then $Bel(\alpha, p)$ is added to EKSP of $K_{fi\alpha}(a, b)$, and $K_{fi\beta}(a, b)$. If necessary, α and β add to their IKSPs the conditions of which α and β think as mutual knowledge respectively. With the resulting DRS $K(fi+1)$, the dialogue is terminated, or goes on the the next stage. (I.e., α doesn't tell a lie, nor say a contradiction of her belief.)

(ib) If β utters P in $K(fi)$, then α constructs $K(fi+1)$ by the same procedure as (ia), and executes (ii).

(ii) α constructs $K_{fi+1}(\alpha')$ which is $K_{fi+1}(\alpha)$ with the collapse axiom in $C_{K_{fi+1}(\alpha)}$.

(iia) If the situation $fi+1\alpha'$ is consistent, the dialogue terminates with $K(fi+1)$ (i.e., $K(fi)$ with $Bel(\beta, p)$ in EKSP of $K_{fi\alpha}(a, b)$ and $K_{fi\beta}(a, b)$), or goes on to the next stage.

(iib) In case the situation $fi + 1\alpha'$ is contradictory, if α finds the certainty of $Bel(\beta, p)$ lower than her IKSP or background knowledge, then α tries to abolish $Bel(\beta, p)$ in $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$. If it succeeds, the dialogue terminates with $K(fi + 2)$ (i.e., $K(fi + 1)$ without $Bel(\beta, p)$ in EKSP of $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$, or goes on to the next stage. Otherwise, do (iic).

(iic) Insofar as there's a candidate q for consistency recovery in IKSP of $K_{fi+1}(\alpha')$, α tries to abolish q in $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$. If it succeeds, the dialogue terminates with $K(fi+2)$ which is $K(fi+1)$ without q in $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$, or goes on to the next stage. Otherwise, do (iid).

(iid) $n := 2$. (*) Insofar as there's a candidate q for consistency recovery in the background knowledge of $K_{fi+1}(\alpha')$, α tries to abolish q in $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$. If it succeeds, the dialogue terminates with $K(fi + 2)$ which is $K(fi + 1)$ without q in the background knowledge of $K_{fi+1\alpha}(a, b)$ and $K_{fi+1\beta}(a, b)$, or goes on to the next stage. Otherwise, $n := n - 1$. If $n = 0$, the dialogue fails. Otherwise, go to (iid*).

3 Some properties of SCD

3.1 possible situations

SCD considers only canonical situations, and they satisfy (5ii). So, only the situations $fi(a)(ba)^*$, $fi(b)(ab)^*$ are possible.

3.2 Shared belief

If both $Bel(a, p)$ and $Bel(b, p)$ are added to EKSP, then it is semantically equal to adding p. It's certified as follows:

Due to 3.1 and the existence of SRP in $K_{fi\alpha}(a, b)$, if p is added to $EKSP$, the situations which support $f(p)$ are reduced to

(13) $(a)b(ab)^*$, $(b)a(ba)^*$. (fi on the top is omitted.)

On the other hand, if $Bel(a, p)$ and $Bel(b, p)$ are added to $EKSP$, the situations which support $f(p)$ are reduced to

(14) $(a)b(ab)^*a$, $(b)a(ba)^*a$.
$(a)b(ab)^*b$, $(b)a(ba)^*b$.

But this is reduced to

(15) $(a)b(ab)^*a \subseteq (b)a(ba)^*$.
$(b)a(ba)^*a = (b)a(ba)^*$.
$(a)b(ab)^*b = (a)b(ab)^*$,
$(b)a(ba)^*b \subseteq (a)b(ab)^*$.

I.e., (13) and (15) are equivalent, which gives a semantic foundation of the Axiom of Shared Belief in [18].

3.3 DRS and belief sentences

(10iv), (11) reveal that DRSs and belief sentences can be semantically identified. In particular, $< U, \{Bel(\alpha, p)\} >$ equals $< U, K(\alpha) >$, where $K(\alpha) =< U', p >$.

3.4 Negation and denial

In SCD, the negation and the denial of a statement p are distinguished from each other as follows:

(16) negation: $Bel(\alpha, \neg p)$,
denial: $\neg Bel(\alpha, p)$[6].

The distinction is semantically explained as follows:

(17) E.g., let p be '$walk(j)$', i.e., the DRS condition corresponding to 'John walks.' Then:

$s \models f(Bel(\alpha, \neg p))$
$\Leftrightarrow s \models < \underline{Bel, \alpha}, f_\alpha(\neg p); 1 >$
$\Leftrightarrow s\alpha \models \overline{f_\alpha(p)}$
$\Leftrightarrow s\alpha \models < walk, j; 0 >$.

$s \models f(\underline{\neg Bel(\alpha, p))}$
$\Leftrightarrow s \models \overline{f(Bel(\alpha, p))}$
$\Leftrightarrow s \models < Bel, \alpha, f_\alpha(p); 0 >$
$\Leftrightarrow s\alpha \not\models < walk, j; 1 >$.

I.e., $Bel(\alpha, \neg p)$ means that α explicitly asserts that John doesn't walk, but $\neg Bel(\alpha, p)$ means that α doesn't know that John walks (and that he doesn't walk, too).

3.5 Disjunction in canonical situations

Canonical situations have the following property (cf. [16, 22]):

(18) $s \models < \vee, inf_1, inf_2; 1 > \Leftarrow s \models inf_1$, or $s \models inf_2$.

It explains a semantic intuition about disjunction in natural language. E.g.: John put his passport in one of the drawers a, b of his desk. But their locks are broken, and he cannot remember the drawer with the passport in it. Then he cannot identify the drawer, although it's certain that the passport is either in a or b.

4 Linguistic applications of SCD

4.1 Ontology of discourse referents

In the folloging DRS, a in $U_{K(fi)}$ and a in $U_{K(b)}$ are different:

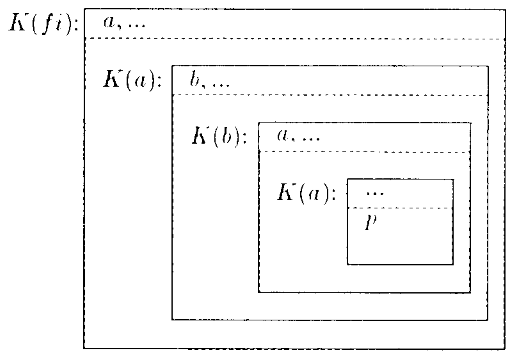

Fig.3

The former is the individual a in the factual situation, the latter is a in $K(b)$ in the factual $K(a)$. In order to distinguish them in $K(fi)$, the latter is designated by the "relative path" from $K(fi)$ as aba. Although a and aba are different, fi can identify both of them in her ontology, which generally means that U_K contains every discourse referents (for short: drs) of its subDRSs.[7] drs which stem from subDRSs of K are called subdrs of K, those proper to K are called proper drs of K.

On the other hand, a isn't totally different from aba, but aba is a counterpart of a, which is formulated as the DRS condition $c\text{-}part(a, aba)$.

In the following, $\alpha, \beta, \gamma \in DR$ (set of drs), $\tau, \nu, \chi \in DR^*$, $i, j, k \in Ind$, then the relation $c\text{-}part$ has the following syntactic and semantic properties:

(19) $\alpha \in U_{K(\tau\nu)}$ and $\nu\alpha \in U_{K(\tau)}$ are notational variants of the same dr.
If $\alpha \in U_{K(\tau\nu)}$, then $\nu\alpha \in U_{K(\tau)}$.
If $c\text{-}part(\alpha, \beta) \in C_{K(\tau)}$, then $\alpha \in U_{K(\tau\nu)}$, $\beta \in U_{K(\tau\nu\chi)}$.

(20) a) $s \models < c\text{-}part, i, i; 1 >$ (Reflexivity)
b) $s \models < c\text{-}part, i, j; 1 >$,

$s \models <c\text{-}part,j,k;1> \Rightarrow$
$s \models <c\text{-}part,i,k;1>.$ (Transitivity)

But it's not the case that
$s \models <c\text{-}part,i,j;1> \Rightarrow$
$s \models <c\text{-}part,j,i;1>$ (Symmetry),
because α of $K(\alpha)$ can identify its counterpart in its subDRSs, but no vice versa.

Next, we consider a notational problem of *drs* in the couterpart relation. If $\alpha \in U_{K(\tau)}$ and $\alpha \in U_{K(\tau\upsilon)}$ correspond 1:1, i.e.:

(21) $\forall \xi \in U_{K(\tau\upsilon)}, \zeta \in U_{\tau}$:
$c\text{-}part(\alpha, \upsilon\alpha),\ c\text{-}part(\alpha, \xi) \in C_{K(\tau)} \Rightarrow$
$\upsilon\alpha = \xi,$
$c\text{-}part(\alpha, \upsilon\alpha),\ c\text{-}part(\zeta, \upsilon\alpha) \in C_{K(\tau)} \Rightarrow$
$\alpha = \zeta,$

there's no problem. But in (22), an individual splits up in two individuals, and in (23), two individuals merge together to one individual:

(22) Ralph believes that a man in a brown hat is a spy. On the other hand, he doesn't believe that a man seen at the beach is not a spy. But in fact, they are one and the same man called Ortcutt. [20. 14]

(23) Bill knows Claire, a university student. But in fact, he believes that the twins Claire and Anna are one and the same person called Claire.

Let $K(r), K(bill)$ be the DRSs which represent the epistemic status of Ralph and Bill respectively. Then we index the above-mentioned *drs* in $K(r), K(bill)$: $ort1, ort2, cl1$, or designate them with mnemonics: mbh, msb for $ort1, ort2$. If $K(\alpha)$ contains $K(r)$ or $K(bill)$, then the above-mentioned individuals are designated as $\tau ort1$, $\tau ort2$, and $\tau cl1$ in $U_{K(\alpha)}$, where τ is r or $bill$ respectively.

But it should be noticed that sub*drs* don't represent actual entities in K. So we assume that K contains the condition $\neg exist(sd)$ for an arbitrary sub*dr* *sd* of K. We also call sub*drs* non-actual *drs*.

4.2 Quantifying-In

The situation of (22) is represented by the following *DRS*:

```
K(fi): | ort; mbh, msb
       |------------------------------
       | Ortcutt(ort)
       | Repr(ort, mbh)
       | Repr(ort, msb)
       | spy(ort)
       | K(r): | mbh, msb;
       |       |----------------
       |       | ¬(mbh = msb)
       |       | spy(mbh)
       |       | ¬spy(msb)
```

Fig.4

In this case, we assume that a special kind of couterpart relation called representation relation exists between *ort* and *mbh*, and *ort* and *msb*. It's represented as $Repr(ort, mbh), Repr(ort, msb)$. In general, $Repr(\alpha, \beta)$ means that β is α only with a special kind of α's properties.

The semantic properties of *Repr* is formalized as follows:

(24) a) $s \models < Repr, i, i; 1 >.$
b) $s \models < Repr, i, j; 1 >, s \models < Repr, j, k; 1 > \Rightarrow$
$s \models < Repr, i, k; 1 >.$
c) $s \models < Repr, i, j; 1 >, s \models inf \Rightarrow s \models inf_i^j.$
d) $s \models < Repr, d1, e; 1 >,$
$s \models < Repr, d2, e; 1 > \Rightarrow <=, d1, d2; 1 >.$

a), b) are general conditions of counterpart relation. d) excludes the cases as in (23).

Now, we can make the following semantic inference:

(25) $fi \models < Repr, ort, mbh; 1 >,$
$fi \models < Bel, r, < spy, mbh; 1 >; 1 >$
$\Leftrightarrow fi \models < \wedge, < Repr, ort, mbh; 1 >,$
$< Bel, r, < spy, mbh; 1 >; 1 >; 1 >$
$\Rightarrow fi \models < \exists x, < \wedge, < Repr, ort, x; 1 >,$
$< Bel, r, < spy, x; 1 >; 1 >; 1 >; 1 >$

The DRS condition to which (25) corresponds is

(26) $\exists x(Repr(ort, x) \wedge Bel(r, spy(x))).$

Likewise,

(27) $\exists x(Repr(ort, x) \wedge Bel(r, \neg spy(x)))$

is inferred. Further, we can infer

(28) $\exists x(Ortcutt(ort) \wedge Repr(ort, x) \wedge$
$Bel(r, spy(x))),$
$\exists x(Ortcutt(ort) \wedge Repr(ort, x) \wedge$
$Bel(r, \neg spy(x))).$

If $Ortcutt(ort) \wedge Repr(ort, x)$ is abbreviated to $R(Ortcutt, ort, x)$, (28) amounts to

(29) $\exists x R(Ortcutt, ort, x) \wedge Bel(r, spy(x)))$.
$\exists x R(Ortcutt, ort, x) \wedge Bel(r, \neg spy(x)))$.

Their natural language expression is

(30) Ralph believes that Ortcutt is a spy, and he believes that Ortcutt is not a spy.

It's almost self-contradictory. But the real contradiction is

(31) Ralph believes that Ortcutt is a spy and is not a spy,

and not (30). SCD can also distinguish such cases.

4.3 Hob-Nob sentence

The so-called Hob-Nob sentence

(32) Hob believes that a witch killed his cow, and Nob believes that she poisoned his pig

has two problems. If (32) is formulated as

(33) $Bel(h, \exists x(witch(x) \wedge kc(x))) \wedge Bel(n, she(x) \wedge pp(x))$,

the existential quantifier cannot bind the occurrence 'x' in '$she(x)$'. ('kc', 'pp' stand for 'killed his cow', 'poisoned his pig' respectively.) On the other hand, if (32) is formulated as

(34) $\exists x(witch(x) \wedge Bel(h, kc(x)) \wedge she(x) \wedge Bel(n, pp(x)))$.

it implies that the witch actually exists. In SCD, (32) is formulated like (34). But the problem with (34) is solved by means of non-actual *drs*. Suppose that the original statements of Hob and Nob are the following:

(35) (a) Hob: Urgl killed my cow.
(b) Nob: Urgl poisoned my pig.

The DRS which infers a reading of (32) from (35) is illustrated as follows:[8]

$K(fi)$:

$h, n; d, e$
$Bel(h, Urgl(d) \wedge kc(d))$ $Bel(n, Urgl(e) \wedge pp(e))$ $witch(d)$, $\neg exist(d), \neg exist(e)$ $Bel(n, d = e)$.

Fig.5

(32) is inferred from (35) as follows:

(36) 1. $fih \models < Urgl, d; 1 > \wedge < kc, d; 1 >$. from $C_{K(fi)}$
2. $fin \models < Urgl, e; 1 > \wedge < pp, e; 1 >$. ditto
3. $fin \models < =, d, e; 1 >$. ditto
4. $fi \models < witch, d; 1 >$. ditto
5. $fih \models < kc, d; 1 >$. 1.
6. $fi \models < Bel, h, < kc, d; 1 >; 1 >$ 5.
7. $fi \models < Repr, d, d; 1 >$. (24a)
8. $fi \models < witch, d; 1 > \wedge < Repr, d, d; 1 > \wedge < Bel, h, < kc, d; 1 >; 1 >$. 4, 6, 7.
9. $fin \models < pp, e; 1 >$. 2.
10. $fin \models < pp, d; 1 >$. 3, 9
11. $fi \models < Bel, n, < pp, d; 1 >; 1 >$. 9.
12. $fi \models < she, x; 1 >$. intr. of anaphora
13. $fi \models < =, x, d; 1 >$. ditto
14. $fi \models < she, d; 1 >$. 12, 13.
15. $fi \models < she, d; 1 > \wedge < Repr, d, d; 1 > \wedge < Bel, n, < pp, d; 1 >; 1 >$. 7, 11, 14.
16. $fi \models < \exists x, << witch, d; 1 > \wedge < Repr, d, x; 1 > \wedge < Bel, h, < kc, x; 1 > ; 1 > \wedge < she, d; 1 > \wedge < Repr, d, x; 1 > \wedge < Bel, n, < pp, x; 1 >; 1 >; 1 >; 1 >$. 8, 15.

The $K(fi)$ condition corresponding to (36.16) is

(37) $\exists x(R(witch, d, x) \wedge Bel(h, kc(x)) \wedge R(she, d, x) \wedge Bel(n, pp(x)))$,

and the natural language sentence corresponding to (37) is (32).

'd', 'e' are non-actual *drs* which stem from Hob's and Nob's belief respectively, and $K(fi)$ contains '$\neg exist(d)$' and '$\neg exist(e)$', and the existential quantification is instantiated by 'd', which captures the non-actuality of the witch.

4.4 Conway paradox

Conway paradox is known as an example which shows a difference between the private and the common knowledge. In [3], it's stated as follows:

(38) Claire and Max play poker, and both of them have an ace. Then they believe

(i) Claire or Max has an ace.

Dana comes along, and asks them if they know whether the other has an ace. They answer

(ii) No.

Dana says:

(iii) "At least one of you has an ace",

and repeats the same question. They answer

(iv) No.

But then Claire infers as follows:

(inf.) If Max doesn't know that I have an ace, having heard that one of us does, he must have an ace.

At first sight, (i) and (iii) seem to carry the same information. But the former is a private knowledge, the latter a common one, which enables Claire (inf.). But (inf.) is by no means a paradox, but an adequate inference. In SCD, it's formulated as follows:

First, (38) from the viewpoint of Claire is represented by the following DRS conditions:

(39) (i′) $Bel(cl, cla \vee mxa)$
(ii′) $a)$ $\neg Bel(cl, mxa)$,
(ii′) $b)$ $Bel(cl, \neg Bel(mx, cla))$
(iii′) $a)$ $Bel(cl, cla \vee mxa)$
(iii′) $b)$ $Bel(mx, cla \vee mxa)$
(iv′) $a)$ $\neg Bel(cl, mxa)$,
(iv′) $b)$ $Bel(cl, \neg Bel(mx, cla))$

Further,

(40) $Bel(mx, mxa) \vee Bel(mx, \overline{mxa})$

is assumed. (Notation: cl:Claire; mx:Max; $has(Max,ace)$:mxa; $\neg has(Max, ace)$:$\overline{mxa}$, etc.) Then, the Claire's conclusion in (inf.) is semantically inferred from (39), (40) as follows:

(41) $cl \models f_{cl}((39\text{ii}'\text{b})) =$
$cl \models < Bel, cl, < Bel, mx, cla; 0 >; 1 >$
$\Leftrightarrow clcl \models < Bel, mx, cla; 0 >$
$\Leftrightarrow cl \models < Bel, mx, cla; 0 >$
$\Leftrightarrow clmx \not\models cla.$ ... (a)
(Notation: cla :< $has, cl, ace; 1$ >. Likewise $mxa, \overline{mxa}$.)
$cl \models f_{cl}((39\text{iii}'\text{b})) =$
$cl \models < Bel, mx, cla \vee mxa; 1 >$
$\Leftrightarrow clmx \models cla \vee mxa.$ ... (b)
$cl \models f_{cl}((40)) =$
$cl \models < \vee, < Bel, mx, mxa; 1 >,$
$< Bel, mx, \overline{mxa}; 1 >; 1 >$ ... (c)
(38inf.): Assume that $clmx \models \overline{mxa}$. ... (d)
From (b), $clmx \models < \rightarrow, \overline{mxa}, cla; 1 >$. ... (e)
From (d),(e) $clmx \models cla$. ... (f)
(f) contradicts (a). Therefore
$clmx \not\models \overline{mxa}$. ... (g)
Assume that $clmx \not\models mxa$. ... (h)
Under canonical situations, the following holds:
From (h), $cl \models < Bel, mx, mxa; 0 >$. ... (i)
From (g), $cl \models < Bel, mx, \overline{mxa}; 0 >$. ... (j)
From (i),(j), $cl \models < \wedge, < Bel, mx, mxa; 0 >,$
$< Bel, mx, \overline{mxa}; 0 >; 1 >$.
$\Leftrightarrow cl \models < \vee, < Bel, mx, mxa; 1 >,$
$< Bel, mx, \overline{mxa}; 1 >; 0 >$. ... (k)
(k) contradicts (c). Therefore $clmx \models mxa$.

Notice that (41b), i.e. 'Claire or Max has an ace.' as a common knowledge, plays an essential role in the inference.

5 Conclusion

In this paper, I formulated a semantic framework for [18] which satisfies the properties in (1), and proposed a solution of some linguistic problems using it.

But the following themes should be studied further:

i) Adequate systematization of knowledge in dialogues in order to make precise the mechanism of DRS revision.
ii) The ontological status of discourse referents must be clarified, not only in its absolute characters, but also in its relational characters such as counterpart relation, intentional identity.
iii) Refinement of the algorithm of belief revision.
iv) Clarification of degree of belief-sharing in a dialogue as in [19].

Notes

1 $Form(\mathcal{L})$ and $\mathcal{DRS}(\mathcal{L})$ below are co-inductively defined. As to the co-inductive definition, see [4].

2 Precisely, a, b must be ai, bi, i.e. a, b at the time point i. But they are normally represented by a, b.

3 As to hypersets, see [4].

4 Quantifiers and logical connectives are given a more sophisticated DRS-representation and interpretation in the DRT literature. But it's not the subject here.

5 It's considered as a semantic version of the revision algorithm presented in [18]. Further, cf. [7, 8, 10, 21].

6 The terminological distinction between 'negation' and 'denial' is based on [4].

7 It should not be confused with the accessibility of drs in a parent DRS from its subDRSs.

8 Precisely, 'd', 'e' must be 'he' and 'ne' in $U_{K(fi)}$. But for simplicity, I identify them. They are related via "horizontal" link between them. (Cf. "intentional identity" in [9], "internal anchor" in [12].), which contrasts with "vertical" link like counterpart relation. But this subject is to be considered on another occasion.

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