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주파수변조 구동에 의한 가변릴럭턴스 스텝핑모우터의 불안정 해석

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Analysis of Dynamic Instability in a Variable-reluctance Stepping Motor Operated on Frequency-modulated Supply

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Abstract - A comprehensive analytical study of frequency-modulated supply of the dynamic instability in a variable-reluctance stepping motor, is described. It is shown that stability can be achieved by frequency modulation provided that the phase displacement between the modulating signal and the rotor velocity oscillation lies between certain limits. A simplified expression is derived, based on the assumption of high inertia. This model is used to obtain a qualitative understanding of how frequency modulation influences the dynamic stability of the variable -reluctance motor.

1. Introduction

A number of studies have been published on the analysis of the oscillation behaviour which occurs in the mid-frequency instability band. However as far as the development of a stabilisation scheme is concerned most of the papers provide insufficient physical understanding of the instability. In an earlier contribution [1] a stabilisation scheme for a variable-reluctance stepping motor, based on the frequency modulation approach, was presented, although no detailed analysis was given. In this work a comprehensive analytical study of frequency-modulated supply on the characteristics of dynamic instability in a variable-reluctance stepping motor, therefore, is described. Modifications to the characteristics of electromagnetic damping torque coefficient, resulting from the modulation of the supply frequency, are analysed to determine the constraints involved in the development of a satisfactory stabilisation scheme. The way in which the phase angle of a modulating signal can affect the dynamic instability characteristics is investigated by means of the anaysis of damping torque coefficient characteristics. A simplified expression is derived, based on the assumption of high inertia. This model is used to obtain a qualitative understanding of how frequency modulation influences the dynamic stability of variable-reluctance stepping motors.

2. Evaluation of the electromagnetic damping torque coefficient

Dynamic instability is considered for the case where the frequency of the electrical supply to the motor is modulated at the frequency of the rotor oscillation α, the depth of frequency modulation of the frequency being proportional to the amplitude of the oscillation in rotor velocity. The phase current solution applied to calculation of damping torque coefficient, is concluded the first correction term[4].

The resulting expression obtained for K_{df} can be separated conveniently into eight individual terms as specified by equation (1).

$$K_{df} = K_d + (K_{df1} + K_{df2} + K_{df3} + K_{df4} + K_{df5} + K_{df6} + K_{df7}) \qquad (1)$$

where

$$K_{df1} = + \frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\, 2I_o\{I_{u1}\sin(\varepsilon_2 - N_r\delta - \varphi) - I_{L1}\sin(\varepsilon_3 - N_r\delta + \varphi)\}$$

$$K_{df2} = + \frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\, A_1i_1\{I_{u1}\cos(\phi_o + \phi_1 - \varepsilon_2 + \varphi) - I_{L1}\cos(\phi_o + \phi_1 - \varepsilon_3 - \varphi)\}$$

$$K_{df3} = + \frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\, A_1i_o\{I_{u2}\cos(\phi_o - \varepsilon_2 - \phi_2 + \varphi) - I_{L2}\cos(\phi_o - \varepsilon_3 - \phi_3 - \varphi)\}$$

$$K_{df4} = - \frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\, I_1\{I_{u2}\sin(\phi_o - \varepsilon_2 - \phi_2 + N_r\delta + \varphi) - I_{L2}\sin(\phi_o - \varepsilon_3 - \phi_3 + N_r\delta - \varphi)\}$$

$$K_{dr5} = -\frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\,A_1i_o\{I_{\alpha1}\cos(\phi_o - \varepsilon_2 - \psi_1 + \varphi) + I_{\alpha1}\cos(\phi_o - \varepsilon_3 + \psi_1 - \varphi)\}$$

$$K_{dr6} = +\frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)\,A_1i_o\{I_{\alpha1}\cos(\phi_o + \varepsilon_2 + \psi_1 - 2N_r\delta - \varphi) + I_{\alpha1}'\cos(\phi_o + \varepsilon_3 - \psi_1 - 2N_r\delta + \varphi)\}$$

$$K_{dr7} = -\frac{PL_1N_r}{4\theta_o\alpha}(J_o + J_2)2I_1\cos\phi_o\{I_{\alpha1}\sin(\varepsilon_2 + \psi_1 - N_r\delta - \varphi) + I_{\alpha1}'\sin(\varepsilon_3 - \psi_1 - N_r\delta + \varphi)\}$$

with $A_1 = J_o(G_r\theta_o\cos\gamma)\,J_o(G_r\theta_o\sin\gamma)$

$A_2 = 2J_1(G_r\theta_o\cos\gamma)\,J_o(G_r\theta_o\sin\gamma)$

$A_3 = 2J_o(G_r\theta_o\cos\gamma)\,J_1(G_r\theta_o\sin\gamma)$

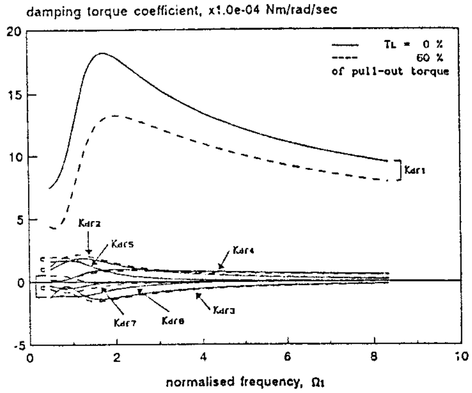
Here, P denotes the number of phases; Bessel function coefficients with unspecified arguments (J_o, J_1, ..., J_m) are functions of $N_r\theta_o$.; and the expression for K_d was given in a reference[2]. It should be noted, however, that the term K_d is modified by the effect of frequency modulation on certain current amplitudes. The changes to these currents are indicated in (2)

$$i_o \to A_1i_o,\quad i_1 \to A_1i_1,\quad i_{bm} \to A_1i_{bm}$$

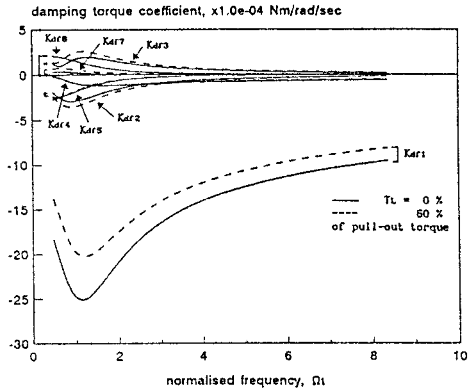
$$\left.\begin{array}{l} i_{a(2m)} \\ i_{a(2m+1)} \end{array}\right\} \to \left\{\begin{array}{l} A_1i_{a(2m)} \\ A_1i_{a(2m+1)} \end{array}\right. \qquad (2)$$

Figure 1 gives example of the characteristics of electromagnetic damping torque coefficient in terms of the components $K_{dr1} \to K_{dr7}$, for a five phase variable-reluctance motor operated on the three → two phases-on excitation mode(N.B. Evaluation of K_d components is shown in reference 2). In order to illustrate how the characteristics are affected by the phase angle γ of the frequency modulating signal, two values of γ (= -90°, 90°) are adopted.

A number of points emerge from consideration of Figure 1. For $\gamma = -90^\circ$ and $\gamma = 90^\circ$ it appears that K_{dr1}, K_{dr2}, K_{dr3} and K_{dr4} are the most important in determining the overall characteristics of the damping coefficient, with K_{dr1} being of considerably greater magnitude than the others. Although the terms K_{dr5} and K_{dr6} are not small enough to be neglected in the region of the stability boundary(approximately Ω_1 = 1) the magnitudes become insignificant with increasing frequency. The effect of load torque on the terms K_{dr2}, K_{dr3} and K_{dr5} is small, because δ does not enter directly into the expressions for these components. On the other hand K_{dr1} and K_{dr6} are influenced significantly by the load variation through the presence of the load angle.



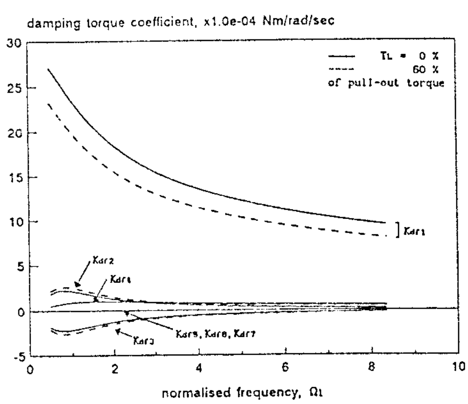
(a)



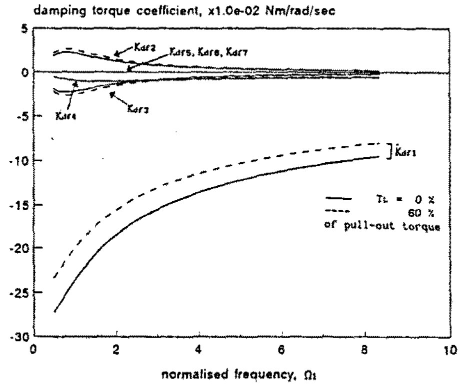
(b)

Fig. 1 Comparision of additional damping torque coefficient terms (V/r = 3.0A, G_r = 10)
(a) $\gamma = -90^\circ$ (b) $\gamma = 90^\circ$
(r = 6.5Ω, $J_{rL} = J_r$, L_o = 6.17mH, L_1 = 2.99mH, θ_o = 0.01°)

The characteristics of $K_{dr1} \to K_{dr7}$ for 'high' inertia operation are displayed in Figure 2. In this case K_{dr1} and K_{dr4} are the principal components introduced by frequency modulation, since K_{dr2} and K_{dr3} cancel exactly (shown analytically in Section 3) and K_{dr5}, K_{dr6} and K_{dr7} are of negligible magnitude. The variation of the various components with load torque is similar to that observed in Figure 1(for rotor inertia only).



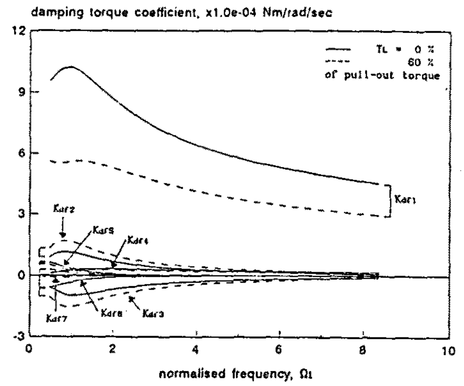
(a)



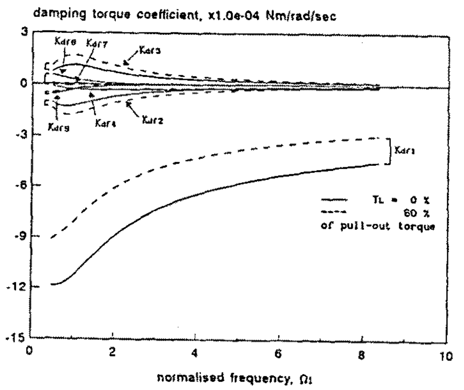
(b)

Fig. 2 Comparision of additional damping torque coefficient terms (V/r = 3.0A, Gr = 10)
(a) $\gamma = -90^\circ$ (b) $\gamma = 90^\circ$

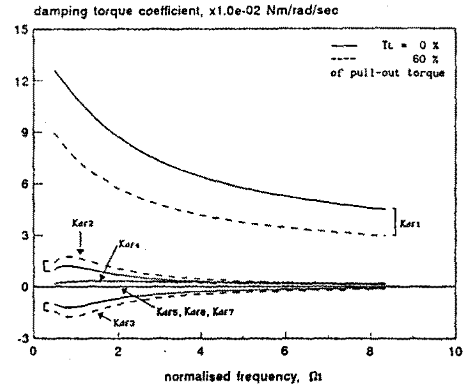
Figure 3 gives the characteristics of $K_{dr1} \rightarrow K_{dr7}$, in the different case of the excitation mode and oscillation amplitude, for both rotor inertia only and 'high' inertia load. Although the individual terms are somewhat decreased in magnitude the characteristic behavior displayed in Figures 1 and 2, in terms of the relative importance of the various components, remains fundamentally unaltered.



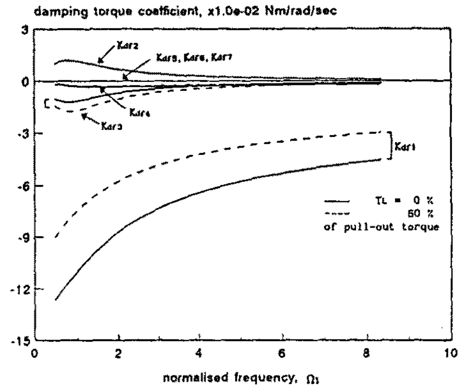
(a) $\gamma = -90^\circ$



(b) $\gamma = 90^\circ$



(c) $\gamma = -90^\circ$



(d) $\gamma = 90^\circ$

Fig. 3 Comparision of additional damping torque coefficient terms of one-phase-on mode (V/r = 3.0A, Gr = 10)
(a), (b) $J_{rL} = J_r$ (c), (d) $J_{rL} = 10^4 J_r$
($r = 6.5\Omega$, $L_o = 6.17mH$, $L_1 = 2.99mH$, $\theta_o = 3.0^\circ$)

3. Stability Characteristics

To obtain a qualitative understanding of the way in which frequency modulation influences the dynamic stability of the motor, it is advantageous to consider the additional terms in the expression for the damping torque coefficient K_{df}, in isolation. In order to simplify the problem and obtain an analytical expression for the frequency modulation components it is convenient to consider the 'high inertia' case.

For practical values of the depth of modulation, $A_1 = 1$, $A_2 = G_r\theta_o\cos\gamma$ and $A_3 = G_r\theta_o\sin\gamma$. Also, when θ_o is sufficiently small, $J_o = 1$ and $J_2 = 0$. Thus. subject to the assumption of 'high inertia' so that $\omega_1 \gg \alpha$, the additional damping coefficient K_{df} may be rearranged in terms of K_v ($= V_o/V_1$), K_L ($= L_o/L_1$) and Ω_1 ($= \omega_1/(r/L_o)$) as

$$K_{df}' = \frac{PL_1N_rV_1^{\ 2}}{4\theta_o\alpha}\left[- \frac{2K_v\ G_r\theta_o\ \sin\gamma}{r^2(1+\Omega_1^{\ 2})^{1/2}}\cos(N_r\delta - \phi_o)\right.$$

$$- \frac{\Omega_1\ G_r\theta_o\ \sin\gamma}{r^2K_L(1+\Omega_1^{\ 2})\ \{1+(2\Omega_1)^2\}^{1/2}}\sin\phi_1$$

$$+ \frac{\Omega_1 \; G_f \theta_o \sin\gamma}{r^2 K_L (1 + \Omega_1^2) \, \{1 + (2\Omega_1)^2\}^{1/2}} \sin \phi_1$$

$$- \frac{K_v \Omega_1^2 \; G_f \theta_o \sin\gamma}{r^2 K_L^2 (1 + \Omega_1^2) \, \{1 + (2\Omega_1)^2\}^{1/2}} \cos(N_r\delta - \phi_1)$$

$$- \frac{\alpha L_1 \; G_f \theta_o \cos\gamma}{2r^3(1 + \Omega_1^2)} + \frac{\alpha L_1 \; G_f \theta_o \cos\gamma}{2r^3(1 + \Omega_1^2)} \cos(2N_r\delta - 2\phi_o)$$

$$- \left. \frac{\alpha K_v \Omega_1 L_1 \; G_f \theta_o \cos\gamma}{r^3 K_L (1 + \Omega_1^2)^{3/2}} \sin(N_r\delta - \phi_o) \right] \qquad (3)$$

Inspection of equation (3) shows that $K_{df1} \rightarrow K_{df4}$ are zero when $\gamma = 0^\circ$ and have a peak magnitude for any particular set of operating conditions when $\gamma = \pm 90^\circ$. $K_{df5} \rightarrow K_{df7}$ on the other hand are zero for $\gamma = \pm 90^\circ$ and at a peak value value for $\gamma = 0^\circ$. It should be noted, however, that for the value of γ of principal interest K_{df5}, K_{df6} and K_{df7} have magnitudes which are insignificantly small, in comparison with those of the other terms ($K_{df1} \rightarrow K_{df4}$), for all values of load torque. This may be expected since the expressions for $K_{df5} \rightarrow K_{df7}$ in equation (3) involve the angular frequency of the rotor oscillation(α) in the numerator, and $\alpha \rightarrow 0$ as $J_{rL} \rightarrow \infty$. In addition, it may be observed that the K_{df2} term is identical to that of K_{df3}, but with opposite sign, and thus K_{df2} and K_{df3} cancel exactly. As a result the findings of Section 2 are confirmed and equation (3) may be approximated in the form

$$K_{df}' = - \frac{P L_1 N_r V_1^2 \, K_v \sin\gamma}{4r^2 K_L^2 \, \alpha} \left[\frac{G_f \cos(N_r\delta - \zeta)}{(1 + \Omega_1^2)\{1 + (2\Omega_1)^2\}} \right] \qquad (4)$$

where

$$\zeta = \tan^{-1} \left[\frac{2K_L^2 \Omega_1 \{1 + (2\Omega_1)^2\} + 2\Omega_1^3}{2K_L^2 \{1 + (2\Omega_1)^2\} + \Omega_1^2} \right] \qquad (5)$$

In equation (4), the maximum value of K_{df}' occurs when $(\sin \gamma)/\alpha$ is at a negative peak provided that $(N_r\delta - \zeta) < \pi/2$. Furthermore, at any particular operating frequency, the steady-state pull-out torque is produced when $N_r\delta - \phi_{po} = \pi/2$ (see reference 2). Therefore for any load torque up to the pull-out value, $\cos(N_r\delta - \zeta) > 0$ and a positive damping coefficient can be obtained by appropriate choice of γ. The phase displacement γ between the modulating signal and the velocity variation is thus a very important factor in determining the dynamic stability. The general relationship between K_{df}' and γ indicated by equation (4) is that K_{df}' is positive for $-180^\circ < \gamma < 0^\circ$ and is negative for $0^\circ < \gamma < 180^\circ$. The variation with γ of the 'high inertia' damping torque coefficient characteristic for the motor operated in the three→two excitation mode, is shown in the Figure 4, for oscillation onset($\theta_o = 0.01^\circ$). The damping coefficient is dominated by the frequency-modulation components and the relationship between K_{df} and γ corresponds to that suggested by equation (4), i.e., K_{df} positive for $-180^\circ < \gamma < 0^\circ$ and negative for $0^\circ < \gamma < 180^\circ$.

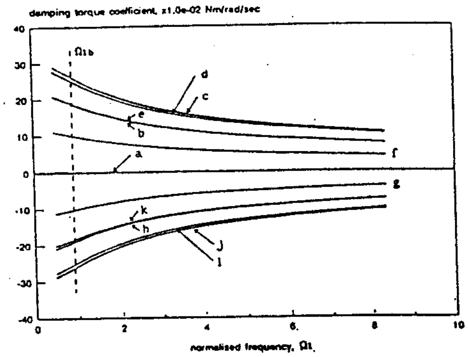


Fig. 4 Variation of damping torque coefficient(K_{df}) with phase angle γ of frequency-modulating signal ($V/r = 3.0A$, $G_f = 10$)
a $\gamma = 0^\circ$, 180° b -30° c -60° d -90° e -120°
f -150° g 150° h 120° i 90° j 60°
k 30°
($r = 6.5\Omega$, $J_{rL} = 10^4 J_r$, $L_o = 6.17mH$, $L_1 = 2.99mH$, $\theta_o = 0.01^\circ$, $T_L = 0Nm$)

Within the normal range of practical values of rotor-plus-load inertia, the assumption that $\omega_1 \gg \alpha$, upon which the derivation of the simplified expression for K_{df} (equation (4)) is based, no longer applies. Furthermore, because the existing damping coefficient torque K_d (shown in reference 2) is of a similar order of magnitude to the additional damping torque coefficient introduced from the frequency modulation, it is not instructive to consider the additional components only. Figure 5 shows the characteristic of damping torque coefficient K_{df} (at $\theta_o = 0.01^\circ$) for rotor inertia only. In Figure 6, the characteristic is presented in smaller increments of γ around the boundary values for which a positive damping coefficient is produced at any value of excitation frequency. The motor can be operated stably at all input frequencies provided that γ is at least in the range $-140^\circ \le \gamma \le -5^\circ$, for the three→ two phases-on excitation mode. Thus, if the supply frequency be modulated at the oscillation frequency, using a signal which is displaced from the rotor-velocity oscillation by an angle within the aforementioned range, the motor produces net positive electromagnetic damping and the oscillation decays to zero. If a value of γ is used outside of the band which produces a positive damping coefficient, stable operation may not be obtained over a wide range of stepping frequency and oscillation onset can occur at a lower frequency than when the machine is operating without frequency modulation (N.B. $\theta_o = 0.01^\circ$ is taken to be the oscillation amplitude at onset).

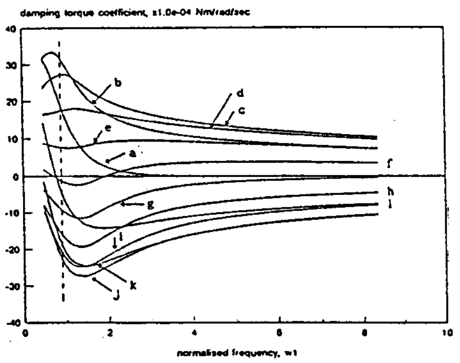


Fig. 5 Variation of damping torque coefficient(K_{df}) with phase angle γ of frequency-modulating signal ($V/r = 3.0A$, $G_r = 10$)
a $\gamma = 0^\circ$ b -30° c -60° d -90° e -120°
f -150° g 180° h 150° i 120° j 90°
k 60° l 30°
($r = 6.5\Omega$, $J_{rL} = J_r$, $L_o = 6.17mH$, $L_1 = 2.99mH$, $\theta_o = 0.01^\circ$, $T_L = 0Nm$)

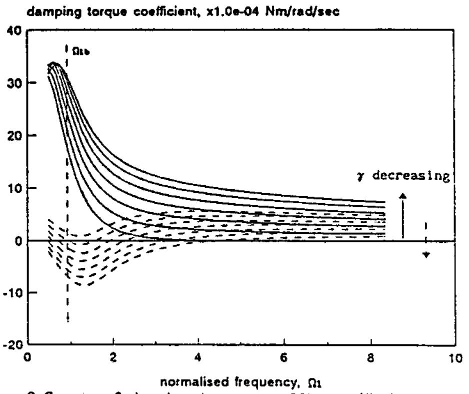


Fig. 6 Curves of damping torque coefficient(K_{df}) against frequency characteristic ($V/r = 3.0A$, $G_r = 10$)
—— : $\gamma = 0^\circ, -5^\circ, -10^\circ, -15^\circ, -20^\circ, -25^\circ, -30^\circ$
---- : $-140^\circ, -145^\circ, -150^\circ, -155^\circ, -160^\circ, -165^\circ, -170^\circ$
($r = 6.5\Omega$, $J_{rL} = J_r$, $L_o = 6.17mH$, $L_1 = 2.99mH$, $\theta_o = 0.01^\circ$, $T_L = 0Nm$)

4. Disscusion

It is of interest to compare the characteristics of damping torque coefficient with frequency modulation in variable-reluctance stepping motors with those for hybrid permanent-magnet stepping motors published recently by Pickup and Russell[3]. Although the characteristics of K_{df} described are obtained from analysis of a unipolar drive system, the fundamental patterns of the dynamic stability are very similar to those of a hybrid stepping motor. The small difference which appears in the mid-frequency range, is caused by the influence of the additional damping torque terms (K_{df2}, K_{df3} and K_{df4}) dependent on current components of frequencies $2\omega_1$ or $(2\omega_1 \pm \alpha)$. Another point of interest is the effect of K_{df5}, K_{df6} and K_{df7} on the damping coefficient characteristics; components which are related to the current oscillation frequency $m\alpha$. For rotor inertia only, the particularly significant influence of K_{df6} is observed in Section 2. It is important to note, however, that for the region of high stepping frequency the importance of the damping components $K_{df2} \to K_{df7}$ becomes much less and the overall damping coefficient characteristics are identical to those of the hybrid motor.

The characteristics given by equation (4), derived on the basis of high inertia and small oscillation amplitude, are fundamentally identical to those of the hybrid motor. The resulting relationship between damping torque coefficient K_{df} and phase displacement γ is the same for both types of motor.

6. References

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