Attenuation of High-Frequency Wave Energy Due to Opposing Currents

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1 Introduction

In coastal waters, more often than not, waves propagate on currents driven by tidal forces, earth's gravity, or wind. There have been a number of studies for dealing with the change of wave spectrum due to the presence of current.

Based on the conservation of wave action, Hedges et al. (1985) have proposed an equation which describes the influence of current on the change of wave spectrum in water of finite depth. The equation of Hedges et al. (1985) is useful for predicting the change of a wave spectrum due to the presence of current unless the wave spectrum reaches an equilibrium (or saturation). When waves propagate onto an opposing current, their growth may be limited by the breaking of waves, especially of the waves in the range of high frequency. By the use of Phillips' (1958) equilibrium range constraint, Hedges (1981) has derived an equation for the equilibrium range of a deep-water wave spectrum when the waves encounter an opposing current (also can be found in Hedges et al. (1985)). On the other hand, again by reexamining and extending the Phillips' equilibrium range concept in deep water, Kitaigordskii et al. (1975) have proposed two separate equations for the equilibrium range of wave spectra each describing the effect of finite water depth and the effect of current, which were later combined into one equation by Gadzhiyev et al. (1978) to include together the effects of finite depth and presence of current.

In the present study, we extend the Hedges' (1981) approach to obtain an equation for the equilibrium range of wave spectra in water of finite depth and in the presence of current. In order to examine the predictability of the theoretical equation, laboratory experiments were made for the change of TMA shallow-water spectra of various water depths and spectral parameters propagating onto following or opposing currents of various speeds. Since the cases for following currents could be predicted reasonably well by the equation of Hedges et al. (1985) and are of less interest, only the cases for opposing currents are reported here. The results show that the theory is in reasonable agreement with observations.

2 Theory

Based on the conservation of wave action, Hedges et al. (1985) have shown that the wave spectrum, $S(\omega_a, d, U)$, under the influence of current, is related to $S_o(\omega_a, d)$, the spectrum in quiescent water without current, as

$$\frac{S(\omega_a, d, U)}{S_o(\omega_a, d)} = \frac{\omega_r \left[1 + \left(\frac{2k_o d}{\sinh 2k_o d}\right)\right]}{2k_o \left\{U + \left[1 + \left(\frac{2kd}{\sinh 2kd}\right)\right] \frac{\omega_r}{2k}\right\}} \tag{1}$$

in which the subscript 'o' refers to the quantities in the zero-current area, U = vertically uniform current velocity in the direction of wave propagation (i.e., positive for following current), d = water depth, k_o and k are the wave numbers in the zero-current area and current area, respectively, and ω_a is the absolute angular frequency in the stationary frame of reference which is related to the relative wave frequency, ω_r, in the frame of reference moving with the current by

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$$\omega_a = \omega_r + kU \tag{2}$$

The wave numbers, k_o and k, are calculated from the dispersion relationships, $\omega_a^2 = gk_o \tanh k_o d$ and $\omega_r^2 = (\omega_a - kU)^2 = gk \tanh kd$, respectively, in which g is the gravitational acceleration.

The growth of waves under the influence of current cannot continue indefinitely. Their growth may be limited by the process of wave breaking, especially for the wave components in the high-frequency range. Phillips (1958) has suggested that there should be an equilibrium range of the spectrum of wind-generated deep-water gravity waves in which the spectral density has an upper bound. The high-frequency portion of the more recently developed wave spectra such as Pierson-Moskowitz, JONSWAP, or TMA spectrum also contains an equilibrium range constraint.

In the development of the above wave spectra the presence of currents was ignored, thus for the direct use of the proposed form of the spectra in a current region the relative frequency in the moving frame of reference should be used. It may be more convenient to express the wave spectrum in terms of the absolute frequency in the stationary frame of reference. For this, we extend the Hedges' (1981) formula for deep water to the water of finite depth. This can be readily done by noting that the wave energy itself does not change whether it is expressed in terms of the absolute frequency or in terms of the relative frequency, i.e.,

$$S(\omega_a)\, d\omega_a = S(\omega_r)\, d\omega_r \tag{3}$$

Eckart (1951) has proposed the following approximate dispersion relationship which can be solved directly for the wave number, k, and generally is in error by only a few percent:

$$\omega_r^2 = gk\sqrt{\tanh\left(\frac{\omega_r^2}{g}d\right)} \tag{4}$$

Substitution of the above equation into Eq. (2) for k and differentiation of ω_a with respect to ω_r gives

$$\frac{d\omega_a}{d\omega_r}(\equiv D) = 1 + \frac{\omega_r U}{g}\left\{\frac{2}{\sqrt{\tanh\left(\frac{\omega_r^2}{g}d\right)}} - \frac{\frac{\omega_r^2 d}{g}}{\cosh^2\left(\frac{\omega_r^2}{g}d\right)\left[\tanh\left(\frac{\omega_r^2}{g}d\right)\right]^{3/2}}\right\} \tag{5}$$

Now the spectral density in the equilibrium range can be expressed in terms of the absolute frequency as follows:

$$S^e(\omega_a, d, U) = \frac{1}{D}S^e(\omega_r, d, U) \tag{6}$$

in which the superscript 'e' denotes an equilibrium range spectrum. In deep water, D reduces to $(1 + 2\omega_r U/g)$ as in Hedges (1981).

In the following sections, Eq. (6) will be checked against the experimental data in the equilibrium range. If the spectral density calculated by Eq. (1) for a certain frequency exceeds that calculated by Eq. (6), then all of the excess wave energy may be assumed to be dissipated and Eq. (6) may be used. Otherwise, Eq. (1) may be assumed to apply.

3 Experiment and Data Analysis

3.1 Experimental Apparatus

Experiments were carried out in the wave-current flume in the Ocean Engineering Division of the Korea Ocean Research and Development Institute. The flume is 53 m long, 1.25 m high,

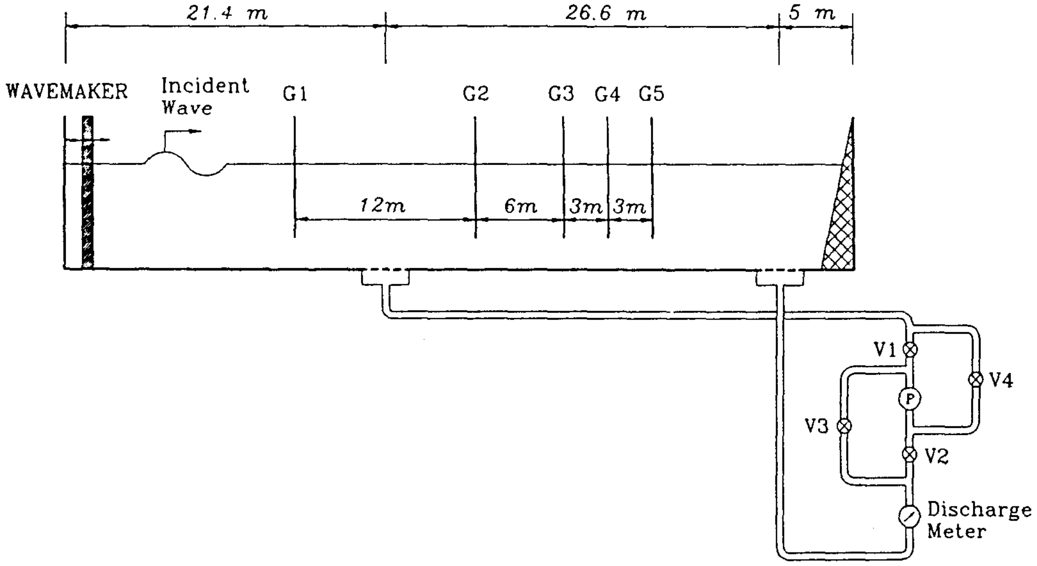

Figure 1: Illustration of Experimental Setup

and 1 m wide, and is equipped with a piston-type random wave generator. Currents can be generated by pumping the water through the pipe underneath the flume as shown in Fig. 1. The bottom of the flume at the entrance and exit of the current is replaced by perforated steel plates with many holes through which the current may pass. Water surface displacement was measured with resistance-type wave gauges, and current velocity was measured using a WVM bi-directional impeller liquid-velocity meter manufactured in the Delft Hydraulics Laboratory.

3.2 Experimental Procedure

The wave spectrum used for the experiment was the TMA shallow-water spectrum proposed by Bouws et al. (1985). A preliminary test performed without generation of current has shown that the wave energy dissipation due to the bottom and side-wall friction is negligible but somewhat significant wave attenuation occurs when waves pass the porous bottom at the current exit. Thus the wave spectrum measured at G4 in Fig. 1, while generating waves but not currents, was taken as the wave spectrum in quiescent water. The wave spectrum measured at the zero-current region (G1 in Fig. 1) was used only for a reference. The TMA spectral parameters for the wave spectra measured at G4, while generating only waves, are given in Table 1.

Currents were generated, while the wavemaker was not in operation, and current velocities were measured at nine elevations dividing the total water depth by ten equidistant intervals at G4. At each elevation, measurement was made for 60 seconds at a samling rate of 20 Hz to obtain the time-averaged velocity. After finishing the current measurement, waves were generated and wave measurements were made at the five locations shown in Fig. 1, one (G1) at the zero-current region and others (G2 to G5) at the current region. A total of 12,000 data points were collected at the sampling rate of 20 Hz for each of the five wave gauges.

The previous theories for wave-current interaction were developed based on the assumption of vertically uniform current, but the actual current generated in the experiment was a depth-varying shear current. Recently Hedges and Lee (1992) has introduced the so-called equivalent

Table 1: Test Conditions and Indices of Agreement

Test No.	Depth (cm)	ω_{a_p} (rad/s)	α	γ	σ_a	σ_b	U_e (cm/s)	Index of agreement	
								Present theory	Gadzhiyev et al. (1978)
1	30	4.109	0.00864	1.870	0.6557	0.0613	13.9	0.980	0.989
2	30	4.109	0.00864	1.870	0.6557	0.0613	22.7	0.979	0.987
3	30	4.109	0.00864	1.870	0.6557	0.0613	33.7	0.972	0.975
4	40	4.172	0.00741	2.597	0.6391	0.1146	13.3	0.988	0.982
5	40	4.172	0.00741	2.597	0.6391	0.1146	21.7	0.968	0.959
6	40	4.172	0.00741	2.597	0.6391	0.1146	31.4	0.961	0.959
7	50	4.172	0.00796	2.471	0.6951	0.0839	11.2	0.992	0.990
8	50	4.172	0.00796	2.471	0.6951	0.0839	20.3	0.977	0.970
9	50	4.172	0.00796	2.471	0.6951	0.0839	33.0	0.975	0.976

uniform current, U_e, defined as the uniform current which produces the same wavelength, L, as the actual depth-varying current for a given observed wave period, wave height and water depth. The equivalent uniform current of Hedges and Lee (1992) was developed for regular waves. In this study, we used the equivalent uniform current corresponding to the wave component at the spectral peak. The equivalent uniform currents thus calculated are given in Table 1.

3.3 Data Analysis

In the spectral analysis of the data, the smoothing techniques presented in Otnes and Enochson (1978) were used. Of 12,000 data points in each test record, the first 10,240 data points were processed in 9 segments of 2048 points per segment. These segments overlap by 50 percent for smoother and more statistically significant spectral estimates. The time series of each segment was corrected by applying a 10 % cosine taper on both ends and was subjected to spectral analysis. The raw spectra were then ensemble-averaged. Further smoothing was made by band-averaging over five neighboring frequency bands, the total number of degrees of freedom being 60. Analysis of the data has shown that the spectral density near the peak frequency at G2 is somewhat greater than those at G3 to G5 but the wave spectrum does not change significantly from G3 to G5, thus the data at G4 are used in the following analysis.

4 Results

Fig. 2 shows the comparison between measurement and theories for Test 3 as an example. The calculation was stopped when $\omega_a = -g/4U$, because at this point Eq. (1) blows up. The results show that Eq. (1) agrees well with the experimental data up to the frequency somewhat greater than the peak frequency while the present theory and the theory of Gadzhiyev et al. (1978) provide reasonable agreement in the higher frequency range where Eq. (1) overpredicts values.

For a quantitative comparison of the theoretical values with the experimental data, we use the index of agreement proposed by Willmott (1981). The values of the index of agreement vary between 0 and 1.0, where 1.0 indicates perfect agreement between observation and prediction, and 0 connotes complete disagreement. The indices of agreement computed using the data between

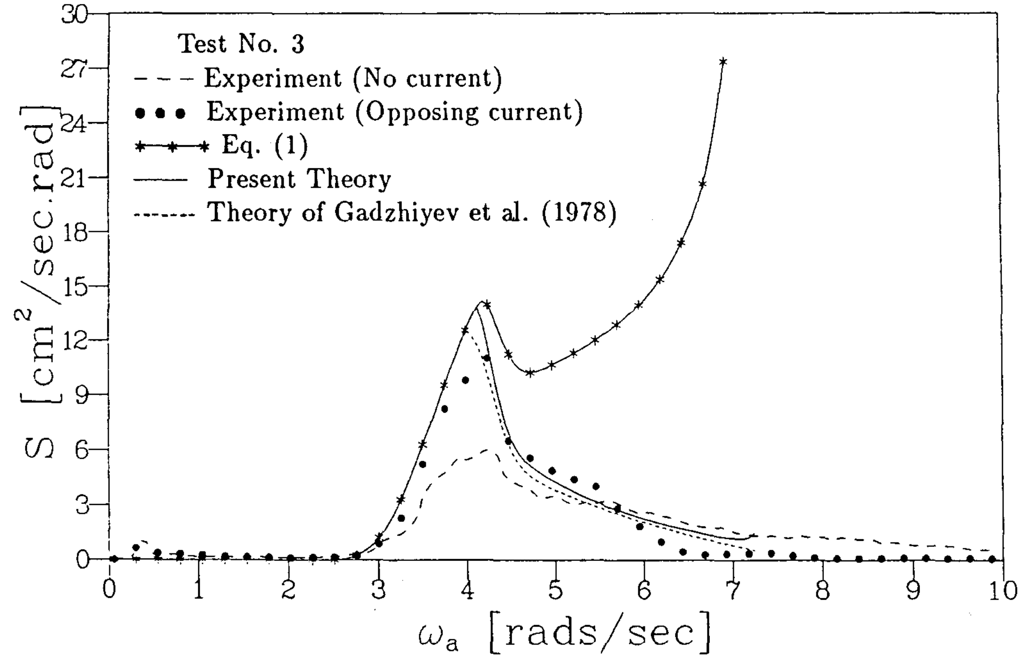

Figure 2: Comparison between theory and experiment

$0.7\omega_{a_p}$ and $2.0\omega_{a_p}$ for each test are given in Table 1. The indices of agreement do not show big differences between the theoretical models and are close to 1.0, indicating that both of the theoretical models work quite well. On the whole, the agreement between observation and prediction becomes better as the current velocity becomes smaller.

5 Conclusion

The present study has proposed an equation for the equilibrium range spectrum of waves propagating on an opposing current in finite depth water. This is an extension of the Hedges' (1981) work for deep-water spectrum to a finite depth water. In order to examine the predictability of the proposed equation, laboratory experiments have been made for the change of TMA shallow-water spectra in water of various depths and current velocities. Comparison with the experimental data has shown that the proposed equation predicts reasonably well the attenuation of high-frequency energy of wave spectra due to opposing currents.

Comparison has also been made with the theoretical expression developed by Gadzhiyev et al. (1978). A statistical analysis has shown that the present equation and the equation of Gadzhiyev et al. have almost the same degree of predictability, though the equation developed in the present study is simpler than that of Gadzhiyev et al. in its derivation and final form.

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