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Digital Signal Processing Based on Fuzzy Rules

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ABSTRACT

A novel digital signal processing technique based on fuzzy rules is proposed for estimating nonstationary signals, such as image signals, contaminated with additive random noises. In this filter, fuzzy rules are utilized to set the filter parameters, taking the local characteristics of the signal into consideration. The introduction of the fuzzy rules is effective, since the rules to set the filter parameters is usually expressed ambiguously. Computer simulations verify its high performance.

1.INTRODUCTION

In the field of signal processing, estimation of a signal degraded by additive random noises is one of the main concerns. For this purpose, several filtering methods have been proposed. Most of these methods are based on the assumption that the signal is represented by a stationary model and their goal is how to optimize a filtering system for the model. However, signals are often difficult to be described in this way, since they are generally nonstationary. For example, image signals are composed of two parts; one is a flat regional part and the other an abruptly changing one as edges.

In order to estimate the nonstationary signals effectively, the filter parameters should be controlled depending of the local characteristics of the signals. However, such control cannot be performed definitely, since the rules to control the parameters are usually expressed in an ambiguous style. For example, we know that the window size of the filter should be large, if the local variance of the input is small. But, we cannot definitely classify the window size or the local variance into small and large ones by a single threshold.

In this paper, a new type of digital filter utilizing fuzzy rules for parameter control is proposed for estimating the nonstationary signals, named as a *fuzzy rule-based filter* (FRB filter for short). This filter is realized as a weighted-averaging type filter in which the values of the weights are controlled. Various kinds of parameters of the input signal sequence can be considered to create the rules, but here we adopt three of them; one is the difference between the input signal values, one is the time distance between signal points, and the other the local variance in the filter window, since they have been strongly considered to influence the value of the weight and also they are easy to obtain. Usually in fuzzy rule-based systems, the membership functions for the input parameters must be given beforehand, however, in this proposal the membership functions are created automatically by a learning method. Consequently, the weights are obtained so that they minimize the mean square error of the filter output for some given training data.

2. PRINCIPLE OF FUZZY RULE-BASED FILTER

Suppose that a noisy signal x_n is obtained at time point n as $x_n=d_n+u_n$, where d_n denotes a desired signal and u_n a white gaussian noise with zero mean. The problem is how to estimate the value d_n from the input sequence $\{ x_{n-k} \}(-N\leq k\leq N)$.

Usually, d_n can be estimated by a weighted-averaging filter, the output of which is y_n expressed as follows.

$$y_n=\frac{\sum_{k=-N}^{N}\mu_{n;n-k}x_{n-k}}{\sum_{k=-N}^{N}\mu_{n;n-k}} \qquad (1)$$

Here, $\mu_{n;n-k}$ denotes the weight which represents to what extent the input x_{n-k} should be considered to get the estimation of d_n . This weight takes a value from 0 to 1. In signal processing, the following rules are known concerning how to set the weights $\mu_{n;n-k}$'s depending on the characteristics of the input signals.

[1] $\mu_{n;n-k}$ should be small for the input x_{n-k}, if the difference between the signal values x_n and x_{n-k} is large, since such x_{n-k} is considered to be in a different signal level.

[2] $\mu_{n;n-k}$ should be small for the input x_{n-k}, if the time distance $|k|$ between x_n and x_{n-k} is large, since some signal changes are considered to occur in the large time distance.

[3] If the local variance around the time point n is large, the weights should be concentrated near around the time point n on the time axis, while the weights widely spread over the time axis, if the local variance is small.

The FRB filter is the filter eq.(1), where the weights $\mu_{n;n-k}$'s are controlled on the basis of the combination of these rules. For example, if the signal difference between x_n and x_{n-k} is small and if the local variance around the time point n is large, $\mu_{n;n-k}$ is large (close to one) if $|k|$ is small, while $\mu_{n;n-k}$ is small (close to zero) if $|k|$ is large.

3. HOW TO SET THE WEIGHTS BASED ON THE RULES

In order to set the value of $\mu_{n;n-k}$ based on these rules, we propose to express $\mu_{n;n-k}$ as a function of these three parameters, the signal difference $|x_n - x_{n-k}|$, the time distance k, and the local variance σ_n, as $\mu(s_{n;n-k}, k, \sigma_n)$. Here $|x_n - x_{n-k}|$ is denoted as $s_{n;n-k}$ for short. Moreover, the function $\mu(s_{n;n-k}, k, \sigma_n)$ is approximated as a step-like function as shown in Fig.1 for the convenience of calculation. This function is expressed as μ_{jkm} at the j-th, the k-th , and the m-th step for $s_{n;n-k}$, k, and σ_n respectively.

Then, the value μ_{jkm} can be obtained for all the combination of j,k and m by the LMS algorithm (Ref.1) iteratively as follows, so that the mean square error of the filter output for some known training signals can be the minimum.

$$\mu_{jkm}(n+1) =\mu_{jkm}(n)+\alpha t_{jkm}(x_{n-k}-y_n)(d_n-y_n) / \sum_k \mu_{jkm}(n) \qquad (2)$$

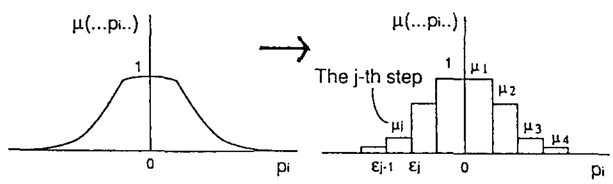
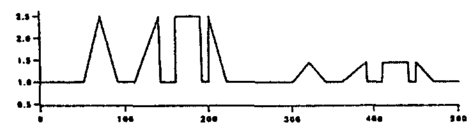


Fig.1 Approximation of the function $\mu(\cdot,\cdot,\cdot)$ as a step-like form.

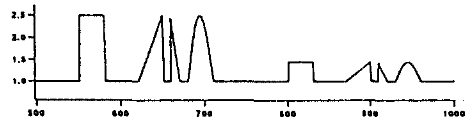
Here, $\mu_{jkm}(n)$ is the value of μ_{jkm} at time point n, α a factor for the stability and convergence, and t_{jkm} is a value as follows for the input x_{n-k} at time point n.

$$t_{jkm} = \begin{cases} 1 & ; \text{if } \varepsilon_{j-1} \leq s_{n;n-k} < \varepsilon_j \text{ and } \varepsilon_{m-1} \leq \sigma_n < \varepsilon_m \\ 0 & ; \text{otherwise.} \end{cases} \qquad (3)$$

ε_j and ε_m are the parameters to divide the values of $s_{n;n-k}$ and σ_n into pieces. The procedure eq.(2) is repeated till $\mu_{jkm}(n)$ converges.



(a) The signal for training.



(b) The signal to be processing

Fig.2 Waveforms of the signals used in computer simulations.

4. COMPUTER SIMULATIONS

The original signal waveforms of the training signal and the signal to be processed are shown in Fig.2(a)(b). White gaussian noises with zero mean and the variance 0.01 are added to both of them. The weights μ_{jkm}'s are obtained for the training signal and then they are applied to the noisy signal to be processed. Table 1 shows the mean square error of the filter output for the signal to be processed when various combination of the rules [1][2][3] are applied. For example, when only the rule [2] is used, μ_{jkm} is represented as μ_k and this FRB filter corresponds to a linear Wiener filter. Since the rule [3] must be accompanied by the parameter k, the rule [3] is applied with the rule [2].

We can see that the performance is the best, when all the rules [1][2][3] are used. Fig.3 shows the waveforms of the input and the output of the FRB filters. We can see that by adding the rule [3] to the rules [1][2], the flat part of the signal is restored more satisfactorily, that is to say, the low-frequency noise is more clearly filtered out. Fig.4 shows examples of the weights obtained for small amplitude signal difference; (a) is when only the rules [1][2] are applied, and (b) is when all the rules [1][2][3] are applied. When the rules [1][2][3] are applied, μ_{jkm} takes a large value for large $|k|$ if the signal is almost flat (the input variance is small), but small if the signal is changing (the input variance is large). On the other hand, when only the rules [1][2] are applied, an intermediate result between the two cases is obtained. We can see the weights μ_{jkm} can be more effectively controlled by adding the rule [3].

5. CONCLUSIONS

A novel digital filter based on fuzzy rules is proposed for noise reduction of nonstationary signals. This filter is realized as a weighted averaging filter in which the weights are controlled based on the rules for signal estimation. Moreover, the values of the weights are determined by a learning method for some given training data so that the mean square error of the output can be the minimum. Computer simulations verify the high performance of this filter.

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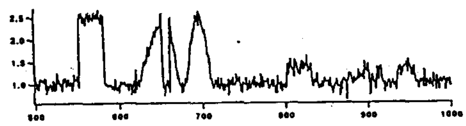
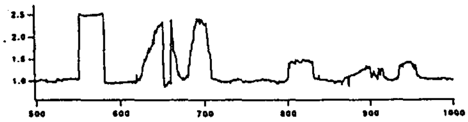
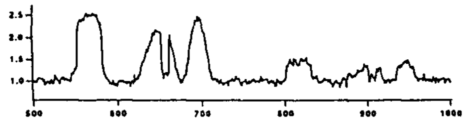


(a) The input signal.

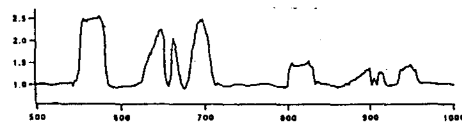


(b) When the rule [1] is used.

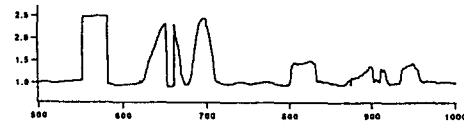


(c) When the rule [2] is used.

(d) When the rules [1][2] are used.

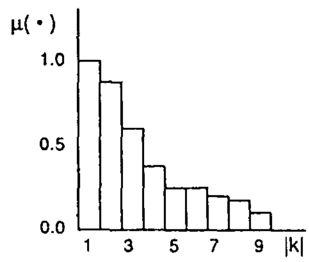


(e) When the rules [2][3] are used.

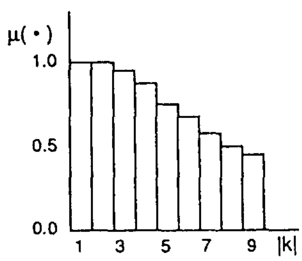


(f) When the rules [1][2][3] are used.

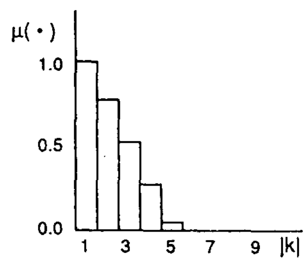
Fig.3 Waveforms of the noisy input and output of the FRB filters.



(a) When the rules [1][2] are applied.



(b-i) When the input variance is small.



(b-ii) When the input variance is large.

(b) When the rules [1][2][3] are applied.

Fig.4 Examples of the weights versus |k| obtained by training for small signal difference.

Used Rules	[1]	[2]	[1][2]	[2][3]	[1][2][3]
MSE	0.0039	0.0082	0.0023	0.0064	0.0019

Table 1 The mean square error of the output of the FRB filters using various rules.