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FUZZY CHOICE IN DESIGN OF THE COMPLEX SYSTEMS.

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The base of proposed decomposing approach is multilevel process of agregation (simplificative transformation) of the description of the project structures. The new classification of fuzzy choice operators is suggested to obtain the decomposing correlations.

One of the most important tasks which occur in designs of modern complex systems is to choose a preferable projects out of a set of feasible project options which can be obtained from some physical consideration or from existing experience on similiar systems. This task, in general, cannot be solved by direct usage of conventional choice techniques for all the possible projects, due to explosion of calculation. The only way to overcome such difficulties is to use some decomposition technique. We propose a new decomposition approach, which is based on a multilevel aggregation of project descriptions; namely simplification of the detailed description of the system into the more general description.

Let X be a set of all possible variants of complex system to be designed. Each element $x \in X$ might to express, for example, a set of parametrs spesifying the system.

Some tasks in the design of complex systems can be featured by fuzzy choice. In fuzzy choice a numeric function $\mu_X : X \to [0;1]$ is generated. The values of the function $\mu_X(x)$ can be considered as a degree of preferability of element x on the set X. All information about preference on X is contained in the fuzzy set:

$\Psi(X) = \{(x, \mu_X(x)),\ x \in X,\ \mu_X : X \to [0;1]\}$, (1)

where Ψ is an operator of fuzzy choice which is determined on all possible subsets of X. Suppose the fuzzy set (1) is not empty: $\mu_X(x) > 0$ holds at least for one $x \in X$. It is considered that $\mu_{X'}(x) = 0$ for each $x \in X \setminus X'$.

Fuzzy choice is sufficient for most tasks in complex system designs.

In particular it is often possible to formalize only part of requirements for the system being designed. Therefore, the choice should be carried out in two stages. At first a set of projects must be chosen which satisfy the formalized requirements, and then the final choice is made on the basis of some empirical procedure. In order for the approach to be effective it is necessary for the set chosen at the first stage to have some optimal property. It can be provided by attempt some choice variations of the projects that satisfy the requirements with a degree greater than some fixed threshold value.

To formalize the last procedure we introduce the family of the choice functions

$$C_{\nu}(X) = \{ x \in X \mid \mu_X(x) > \nu \}.$$

At first, $C_{\nu}(X)$ with a level ν is extracted. It can be either too small (in such a case we can not get satisfactory requirements) or too lagre (in this case, the experts will not be able to use this information). In such a case further enlarging or shrinking of set $C_{\nu}(X)$ (i.e. decreasing or increasing ν respectivelly) is required. We shall refer to this procedure as *choice variation*. The range $[\alpha; \beta)$ of variatinon is determined a priori or by some empirical consideration.

Definition 1. Numerical functions μ, η: $X \to [0;1]$ are called $[\alpha;\beta)$-*equivalent* on X', if the following suffices: $\forall x \in X'$

$$\eta(x) \le \alpha, \quad \text{if} \quad \mu(x) \le \alpha;$$
$$\eta(x) = \mu(x), \quad \text{if} \quad \alpha < \mu(x) < \beta;$$
$$\eta(x) \ge \beta, \quad \text{if} \quad \mu(x) \ge \beta.$$

In a case of choice, when variation of choice is assumed, decomposition procedure has to define some more simple numeric function η_X, which is $[\alpha;\beta)$-equivalent for origin function $\mu_X = \Psi(X)$

Assertion. Two sets of level $\nu \in [\alpha; \beta)$ defined by functions μ_X and η_X : $X \to [0;1)$ are equivalent if,and only if, functions μ_X and η_X are $[\alpha; \beta)$-equivalent on X.

The main problem is how to calculate $C_{\nu}(X)$ if dimensions of set X is very large. In order to achieve this two concepts are important: decomposition and aggregation.

Decomposition of the choice task is a process in which we try to obtain such expression of set X and choice operator Ψ, that problem of calcucation $\Psi(X)$ is more easy. *Aggregation* is the process by which X mapped into another space $\bar{X}$ (a lower dimensional space which should be more tractable then X).

The fuzzy choice scheme which defines the operator Ψ is very difficult to implement in real tasks. In practice, the decomposition approach discussed above is useful especially in the case that a task is simplified by aggregating the projects and by organizing the fuzzy choice task on the aggregated level.

The decomposition should be organized such that the main part of the options from X will be adopted or rejected by the relativelly simpler choice at the aggregated level. To carry out the decission for other options an addition choice should be applied.

The aggregation process is described by the following equation:

$$\bar{x} = F(x), \quad x \in X, \quad \bar{x} \in \bar{X}, \qquad (2)$$

where F is some operator of aggregation, and $\bar{X}$ is a new set of projections, on which a simpler fuzzy choice operator $\bar{\Psi}$ is defined. Here, let us examine only the case where the decomposition includes only one stage of aggregation (2), without loss of generality.

As for the aggregating operator F, it should be noted that F must have reverse operator F^{-1} and satisfy the some conditions, which will be formulate later.

This simplification enables to determine the fuzzy set of preferable project options at hhe aggregated level $\bar{\Psi}(\bar{X})$. It's obvious that for restoring set $\Psi(X)$ from set $\bar{\Psi}(\bar{X})$, the numerical functions Ψ and $\bar{\Psi}$ should be co-ordinated with the aggregating operator F. We shall give the definition of some classes of aggregation, which corresponds to the situation where projects features are changed only in some restricted limits. We consider that operator F defines one-to-one reflection.

Definition 2. Fuzzy choice operators $\bar{\Psi}$, Ψ and the aggregating operator F are in the *limited incoordination (δ-incoordination)* condition if the following inequalities hold for any $\bar{x}\in\bar{X}$, $x\in F^{-1}(\bar{x})$:

$$\bar{\mu}_{\bar{X}}(\bar{x}) - \delta_1 \le \mu_X(x) \le \bar{\mu}_{\bar{X}}(\bar{x}) + \delta_2.$$

Let us formulate characteristical conditions for operators of fuzzy choice which would be of use to obtain decomposing relations. Consider that $C_\nu(X)\neq\emptyset$ suffices for any $\nu\in[\alpha,\beta)$ and set X.

$$\text{Let} \quad \gamma(X') = \begin{cases} \sup_{y\in X'} \mu_X(y), & \text{if } X'\neq\emptyset; \\ 0, & \text{if } X'=\emptyset. \end{cases}$$

$\gamma(X')$ determines such a threshold value of level for each unempty $X'\subseteq X$, that $C_\nu(X) \cap X'\neq\emptyset$ is true for any $\nu \in [0; \gamma(X'))$. Besides that, value $\gamma(X\setminus X')$ determines such threshold valueof level, that for any $\nu \in [\gamma(X\setminus X'); 1)$ sets of levels $C_\nu(X)\subseteq X'$ or $C_\nu(X)=\emptyset$ is true.

Definition 3. The fuzzy choice operator Ψ is said to satisfy on an interval $[\alpha;\beta)$ the condition:

(a) of *summing* (denotement: $\Psi \in \mathbb{S}_{[\alpha;\beta)}$), if for any $X'\subseteq X$ functions μ_X and $\mu_{X'}$ are $[\alpha;\beta)$-equivalent on X';

(b) of *constant* (denotement $\Psi \in \mathbb{K}_{[\alpha;\beta)}$), if for any $X' \subset X$, so that $\gamma(X') > \alpha$, functions μ_X and $\mu_{X'}$ are $[\alpha, \min(\gamma(X'),\beta))$-equivalent on X'; and also for any $x \in X$, from $\gamma(X) < \beta$ the $\mu_{X'}(x) < \max(\alpha; \gamma(X))$ follows.

(c) of *independence of the rejected options* (denotement: $\Psi \in \mathbb{I}_{[\alpha;\beta)}$), if for any $X'\subset X$ so that $\gamma(X')>\gamma(X\setminus X')$, $\gamma(X') > \alpha$, $\gamma(X\setminus X') \le \beta$, functions μ_X and $\mu_{X'}$ are $[\max(\alpha; \gamma(X\setminus X')); \min(\gamma(X'); \beta))$-equivalent on X.

(d) of level's *heredity* conditions (denotement: $\Psi \in \mathbb{H}_\alpha$), if for any $X' \subseteq X$, $x \in X'$, so that $\mu_X(x) > \nu$ follows

$\mu_{X'}(x) > \nu$.

The following lemmas use the conditions are formulated above to simplify the choice task.

Lemma 1. Let $\Psi \in \mathbb{D}_{[\alpha,\beta)}$ and for set X' $X' \supseteq \{ x' \in X | \mu_X(x') > \alpha \}$, $X' \neq \emptyset$ is true. Then numerical functions μ_X and $\mu_{X'} = \Psi(X')$ are $[\alpha; \min(\gamma(X);\beta))$ -equivalent on X.

Lemma 2. Let $\Psi \in \mathbb{H}_\alpha \cap \mathbb{D}_{[\alpha,\beta)}$ and for some sets $X^i \subset X$, $i=\overline{1,n}$, $\bigcup_{i=1}^{n} X^i = X$, $\gamma(X) > \alpha$, set $\{x' \in X | \max_{i=1,n} \mu_{X^i}(x') > \alpha \} \neq \emptyset$ is defined. Then numerical functions μ_X and $\mu_{X'} = \Psi(X')$ are $[\alpha; \min(\gamma(X);\beta))$ -equivalent on X.

Lemmas 1, 2 allow to replace the choice perfomed on the whole permissible set X, with the choices on its special subsets. Obviously these lemmas generalizes the known properties for choice functions which are satisfied by the condition of independence of rejected options and Plott's condition in the case of fuzzy choice operators.

Let us introduce notation $C'_\nu = F^{-1}(\bar{C}_\nu(\bar{X}))$.

Then, the following theorem holds.

<u>Theorem</u>. If the conditions of δ-incoordination are satisfied for Ψ, $\bar{\Psi}$ and F, then the numerical function η_X beaing $[\alpha; \beta)$-equivalent to initial function $\mu_X = \Psi(X)$ is given as

$$\forall x \in X \quad \eta_X(x) = \begin{cases} \alpha, & \text{if } x \in X \setminus C'_{\alpha-\delta_2}; \\ 1, & \text{if } x \in C'_{\beta+\delta_1}; \\ \mu_Y(x), & \text{if } x \in C'_{\alpha-\delta_2} \setminus C'_{\beta+\delta_1}; \end{cases}$$

where

$$Y = \begin{cases} \{x\}, & \text{if } \Psi \in \mathbb{S}_{[\alpha,\beta)}; \\ \{x;\ y\}, \text{ where } y \in C'_{\beta+\delta_1} \text{ -arbitrary option}, & \text{if } \Psi \in \mathbb{K}_{[\alpha,\beta)}; \\ C'_{\alpha+\delta_1} \cup C_\alpha(C'_{\alpha-\delta_2} \setminus C'_{\alpha+\delta_1}), & \text{if } \Psi \in \mathbb{H}_\alpha \cap \mathbb{D}_{[\alpha,\beta)}; \\ C'_{\alpha-\delta_2}, & \text{if } \Psi \in \mathbb{D}_{[\alpha,\beta)}; \\ X, & \text{in other cases.} \end{cases}$$

The proposed method of decomposition in connection with fuzzy approach allows us to replace a difficult choice task with a more simple choice task on the aggregated level. This approach is based on aggregation (or simplification) the project options described on a permissible set. Classification of fuzzy choice operators allowed us to formulate correlations which are the base of decomposition methods to solve the choice tasks. Naturally it is not always possible to specify the aggregate expressions (2) to given design task, but there are many cases in which the proposed aggregation technique is effective. The aggregation of such classes must satisfy the coordination condition for choice at the initial and aggregated levels. It should also be stressed that checkup of coordination conditions must be based on the analisys of technical specifications of the sphere of the designing.