

**A Multiple-Valued Fuzzy Approximate Analogical-Reasoning System**

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**Keywords:** fuzzy logic, multiple-valued logic, Compositional Rule of Inference (CRI), Approximate Analogical Reasoning Schema (AARS), current mode, fuzzy-inference system (FIS).

**Abstract**

We have designed a multiple-valued fuzzy Approximate Analogical-Reasoning system (AARS). The system uses a similarity measure of fuzzy sets and a threshold of similarity  $S_T$  to determine whether a rule should be fired, with a Modification Function inferred from the Similarity Measure to deduce a consequent. Multiple-valued basic fuzzy blocks are used to construct the system. A description of the system is presented to illustrate the operation of the schema. The results of simulations show that the system can perform about  $3.5 \times 10^6$  inferences per second. Finally, we compare the system with Yamakawa's chip which is based on the Compositional Rule of Inference (CRI) with Mamdani's implication.

**Introduction**

Fuzzy inference systems have been widely used in automatic control and pattern recognition, particularly in home appliances. The inference methods used in these applications are more or less the same, namely a special case of the Compositional Rule of Inference (CRI). However, there are many other inference methods available. One of them, named Approximate Analogical-Reasoning (AAR), has been proposed by Turksen and Zhong [6,7], where a modification function, based on a similarity measure, is used to deduce a consequent. It has been shown to be an effective method for fuzzy reasoning which bypasses the matrix operations required for obtaining the implication relations involved in

CRI. Thus, potentially, AAR can greatly increase inference speed and reduce chip area. Its computational effectiveness is especially suitable to the real-time control of multi-variable fuzzy-inference systems. Furthermore, if more complex modification schemes are used in AAR, the results will fall fully within the bounds proved by Turksen and Tian [8].

This paper presents a design of AAR fuzzy-inference system using multiple-valued (MV) blocks. These blocks base on multiple-valued logic and current-mirror technologies. We intend to construct an AAR fuzzy-inference system module using MV blocks. The module handles a three-antecedent single-consequent case. Since multiple-valued logic and current-mirror technologies are used, circuit configurations are simple, and explicit block functions such as summation can be omitted. In this way, the chip area is reduced, and the speed of fuzzy inference is increased. Simulations of the blocks have been done. It is estimated that the time taken for an inference is less than 20ns.

**The AAR Method and System**

For a particular version of the point-valued AAR inference method, based on distance [6], the main idea is to compute the Hamming distance between the observed system state  $A^*$  and the left-hand side of each rule  $A_k$  in the rule base:

$$D_{Hi} = \sum_{i=1}^5 |a^*_i - a_{ki}|, k=1,2,3; i=1,\dots,5 \quad (1)$$

where the similarity measure is as follows:

$$S_k = (1 + D_{Hi})^{-1} \quad (2)$$

Then, the rule is fired that has the minimum distance (maximum similarity) to the observed system state. To obtain the decision  $B^*_k$ , we use the following "membership-function-expansion" and "membership-function reduction" forms:

$$1) \text{ The "membership-function-expansion" : } \\ \mu_{B_k U^*}(y_j) = \text{Min} (1, \mu_{B_k}(y_j)/S_k), \quad (3)$$

identifying an upper-bound decision  $B_k U^*$ , which equals the Min of  $(1, \mu_{B_k}(y_j)/S_k)$ ;

$$2) \text{ The "membership-function-reduction" : } \\ \mu_{B_k L^*}(y_j) = \mu_{B_k}(y_j) \cdot S_k, \quad (4)$$

identifying a lower-bound decision  $B_k L^*$ , which simply equals  $\mu_{B_k}(y_j)$  times  $S_k$ .

The two equations together yield a decision as follows:

$$\mu_{B^*_k}(y_j) = 1/2 [\text{Min} (1, \mu_{B_k}(y_j)/S_k) + \mu_{B_k}(y_j) \cdot S_k] \quad (5)$$

This means that the decision  $\mu_{B^*_k}(y_j)$  is the average of the upper-and lower-bound decisions.

#### MV-AAR-FIS

We have designed an architecture for the AAR inference method. The AAR system design (Fig.1) is based on the AAR method. Before processing data and completing an inference, the observed data and the rule data are stored in the RAM. Then, all these data are sent to HAMM to obtain Hamming distances which are sent into the MIN-COMP module. Finally, the data concerning the smallest  $d_k$  is sent into RAM to fire (select) the rule and obtain the decision  $B_k$ .  $B_k$  can be modified by membership-function expansion and reduction. Therefore,  $B_k$  and  $S_k$  are sent to MODIF to obtain the final decision  $B_k^*$ .

#### Yamakawa's Chip with Mamdani Implication

Yamakawa implemented a chip for the CRI inference method. The chip can perform one million inferences per second. The operation of the chip is shown in Fig.2. The CRI method, also known as the Generalized Modus Ponens, can be stated as :

$$B^* = A^* \circ (A \rightarrow B) \quad (6)$$

which, more explicitly, means that  $B^*$  is equal to the Max of all  $\text{Min}\{A^* \text{ and } A \text{ implies } B\}$  where  $A \rightarrow B$  is the rule,  $\circ$  is the MaxMin composition operator,  $\rightarrow$  is the

implication,  $A^*$  is the observed system state, and  $B^*$  is the consequence.

The CRI with Mamdani implication can also be called the interpolation method. Its inference procedure is shown in the following formula:

$$\mu_{B^*}(y_j) = S_k (T(\gamma_k, \mu_{B_k}(y_j))) \quad \text{for } k=1,2,\dots,m \\ \text{where } \gamma_k = S_j (T(\mu_{A_k}(x_i), \mu_{A^*}(x_i))) \quad \text{for } j=1,2,\dots,n \\ \quad \quad \quad x_i \quad \quad \quad \text{for } i=1,2,\dots,p \quad (7)$$

where S means Max and T means Min.

#### Comparison Between MV-AAR and Yamakawa's Chip

Here, we compare the MV-AAR system and Yamakawa's fuzzy-inference system chip: The speed of MV-AAR is much faster than that of Yamakawa's chip and the chip area of MV-AAR is much smaller than Yamakawa's. The reasons for this can be seen as follows: Firstly, the circuits being used are relatively simple CMOS multiple-valued circuits based on current-mode CMOS topologies. Similar circuits have already been implemented in Current's converter [1] where they show a delay time of 55ns. We have completed a simulation of a converter very similar to Current's. The results of this simulation show a delay time of approximately 5ns (but at a higher current rate), a ten-fold decrease in time over Current's converter. Correspondingly, the overall delay time of the AAR system based on simulated blocks is approximately 20ns. Even using Current's ten-fold increase in delay time as a worst-case scenario (giving a time of 200ns (10 x 20ns)), and taking only 30% of the simulation's performance in consideration of the complexity of the system, a performance of about  $3.5 \times 10^6$  inferences per second may be expected. Secondly, the circuits have fewer CMOS transistors, and correspondingly a small chip area. As a result, the MV-AAR system is both faster and more compact, and more efficient overall.

#### Conclusion

There are many inference methods proposed in the literature. However, in the published literature only one special case of the CRI method

has been applied to fuzzy-inference hardware systems. A more effective approach is needed for many real situations. Results of simulation show that the AAR fuzzy inference can be employed as an effective fuzzy-inference system by using multiple-valued logic and current-mirror technologies. It is estimated that it can perform more than  $3.5 \times 10^6$  inferences per second.

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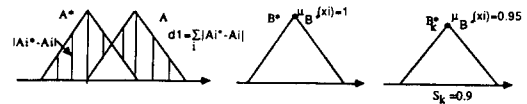


Fig.1 AAR Architecture

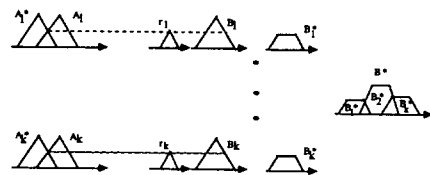


Fig. 2 The Operation of Yamakawa's Chip