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An On-line Fuzzy Identification Method utilizing Fuzzy Model Evaluation

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Abstract

This paper proposes a new on-line fuzzy model identification(ONFID) algorithm in which the fuzzy model evaluation stage is incorporated. The fuzzy model evaluation is performed by the fuzzy equality index which is known to be a useful tool to evaluate the performance of the identified fuzzy model. Then the fuzzy model is updated according to the result of the evaluation. Proposed ONFID algorithm can sensibly identify the fuzzy model of the process, since it has adaptation capability to the system changes.

To show the usefulness of the proposed algorithm, it is applied to the fuzzy model identification problem of the gas furnace and the output prediction problem of the flexible joint manipulator which is a nonlinear system.

1. Introduction

As can be seen in literature, the fuzzy set theory has proven to be a suitable approach to the modelling of the complex and ill-defined processes. The main results until now for the fuzzy model identification are realizable in off-line fashion. The main purpose of the fuzzy model based control system (FMBCS) is to adapt the fuzzy logic controller(FLC) to the variations of process operation conditions such as setpoint changes, disturbances and parameter perturbations and enhance the performance of FLC. To achieve these purposes, in general, it is required to identify the fuzzy model of a process rapidly in on-line fashion.

Pedrycz[1] firstly proposed the notion of simultaneous fuzzy identification and control and Graham[2] applied it to the control of a liquid level rig. In these papers, the off-line fuzzy model identification(OFFID) algorithm is used iteratively as on-line fuzzy model identification (ONFID) algoritm without any other modification. It might show poor performance especially in the view point of model adaptation speed for the absence of adaptation scheme. Lu and Zu[3] proposed an interesting off-line identification algorithm which introduced an self-learning scheme and could be extended to ONFID algorithm. But this algorithm has some difficulty to be used in practical on-line applications, since there are some parameters which are to be chosen in off-line. Moreover the adaptation gain is determined at the numerical level rather than the logical level. As Pedrycz stated[4], the fuzzy model identification and control systems are formed at the logical level, therefore a way of mixing the logical level and numerical level should not be recommended.

In this paper, a new ONFID algorithm based on fuzzy relational equation approach is proposed. A fuzzy model evaluation stage in which the fuzzy equality index is used to evaluate the fuzzy model identified is incorporated in the proposed algorithm. And the fuzzy model of the precess is updated based on the evaluation result. The overall structure of ONFID scheme is constructed in the fuzzy framework.

In section II, the fuzzy model identification problem based on the fuzzy relational equation approach is stated and the method which is outlined by Pedrycz is briefly reviewd. In section III, the fuzzy equality index which is used as a fuzzy model evaluation tool is illustrated and a new ONFID algorithm incorporating the fuzzy model evaluation stage in it is proposed. In section IV, to show the usefulness of the proposed algorithm it is applied to the fuzzy modelling of a gas furnace and the output prediction problem of a flexible joint manipulator which is a nonlinear system. Finally, in section V, state the conclusions obtained.

2. Problem statement

We are concerned with the fuzzy dynamic systems of the form

$$X_{k+1} = U_{k-\tau} \circ X_k \circ R \tag{1}$$

where, $\circ$ is a composition operator(e.g. max-min, max-product composition), and X_k, $X_{k+1} \in F(\mathbb{X})$ outputs at time k, k+1, $U_{k-\tau} \in F(\mathbb{U})$ input at time $k-\tau$(τ is time delay), $R \in F(\mathbb{X}\times\mathbb{X}\times\mathbb{U})$($\times$: cartesian product) fuzzy relation which represents process. $\mathbb{X}$, $\mathbb{U}$ are the universes of discourse for the process output and input, respectively. And $F(\cdot)$ stands for the family of fuzzy sets defined on a particular universes (i.e. $X_k \in F(\mathbb{X})$, $X_k: \mathbb{X} \to [0,1]$).

Every fuzzy set in eq.(1) is defined in terms of a family of reference fuzzy sets defined on the underlying universe of discourse. For example, let the reference fuzzy sets defined on

X be $X^1, X^2, \cdots, X^r$. Then any fuzzy set $\tilde{x}_k$(a degenerated fuzzy set of which the membership function is equal to 1 in exactly one point on X, and zero otherwise) can be represented by its possibility measure of $\tilde{x}_k$ with respect to $X^1, X^2, \cdots, X^r$ as following[5]

$$X_k = [\mathrm{Poss}(\tilde{x}_k | X^1), \mathrm{Poss}(\tilde{x}_k | X^2), \cdots, \mathrm{Poss}(\tilde{x}_k | X^r)] \quad (2)$$

Now, the off-line fuzzy model identification problem can be stated as follows: given a set of input-output data, to identify a fuzzy relation matrix, R which satisfies (1) for all $k(k=1,\cdots,N)$. One of the solution is given by

$$R = \bigcup_{k=1}^{N} R_k = \bigcup_{k=1}^{N} U_{k-\tau} \times X_k \times X_{k+1} \quad (3)$$

where N is the number of input-output data, for the max-min composition × is taken to be minimum, for the max-product composition product [5].

The solution (3) can be used to predict the process output in FMBC only when input-output data which represent the process completely are gathered and/or unexpected system changes(e.g. disturbances, parameter perturbations) do not exist. Almost all the processes are not the case. The available input-output data are finite and unexpected system changes do always exist. Therefore ONFID algorithm which can adapt the fuzzy model to the system changes is needed. In [1][2], they used OFFID algorithm (3) iteratively as ONFID without any modification. This algorithm adequately identify a fuzzy model from the consistent process data. However, it will not sensibly adapt the fuzzy model to the variations of process operation conditions.

Xu and Lu[3] proposed a self-learning algorithm which can improve the performance of fuzzy model identified in off-line fashion and asserted that it can be used for the on-line applications. But, it requires more or less labor to determine the parameters related to the adaptation scheme such as the threshold which is used to select the elements of fuzzy relation needed to be modified, and the adaptation gain which is dertermined at the numerical level and used to determine the imporatance weights for the fuzzy model identified at the previous time and the subrelation at the fuzzy model modification time. The performance of the identified fuzzy model is dependent on the parameters and likely to be a problem in the practical applications[6].

3. On-line fuzzy model identification(ONFID)

In this section, a new ONFID algorithm incorporating a fuzzy model evaluation stage in it is proposed. In this algorithm, the fuzzy model evaluation is performed by use of fuzzy equality index which is known to be a useful evaluation tool. And the result of it is utilized to update the fuzzy model to adapt to the variations of the process operation conditions.

3.1 Fuzzy equality index

The fuzzy equality index is a fuzzy measure which compares two fuzzy sets(represent linguistic quantities) defined on the same universe of discourse. Its properties(e.g. symmetricity) are determined by the definitions of logical connectives and implication used. The comparison results of several types of fuzzy equality index according to them are well described in [7], and the usefulness of the fuzzy equality index for the evaluation of the identified fuzzy model is well presented in [4][7]. The definition of fuzzy equality index used in this paper is as follows.

Let A, B be two fuzzy sets which are interested in comparing (A, B : X → [0,1]). The set-theoretic definition of the fuzzy equality index for one point, x_0 on X is as following

$$[A \equiv B](x_0) = \frac{1}{2}[(A \subset B) \wedge (A \supset B) + (\overline{A} \subset \overline{B}) \wedge (\overline{A} \supset \overline{B})](x_0)$$

where '⊂' and '∧' can be modeled by the φ-operator(pseudocomplement) and min-operator, respectively. In this paper, following form of φ-operator is used

$$a \,\varphi\, b = \begin{cases} 1 & \text{if } a \le b \\ b/a & \text{if } a > b \end{cases} \quad (4)$$

Then the fuzzy equality index for one point x_0 on X can be computed as follows

$$[A \equiv B](x_0) = \begin{cases} \frac{1}{2}\left[\frac{b}{a} + \frac{(1-a)}{(1-b)}\right] & \text{if } a > b \\ 1 & \text{if } a = b \\ \frac{1}{2}\left[\frac{a}{b} + \frac{(1-b)}{(1-a)}\right] & \text{if } a < b \end{cases} \quad (5)$$

where $a = A(x_0)$, $b = B(x_0)$.

In the finite set case, the fuzzy equality index δ between two fuzzy sets A, B can be computed as follows

$$\delta = [A \equiv B] = \min_{x_i} [A \equiv B](x_i), \quad (i = 1, 2, \ldots, M) \quad (6)$$

where M is the number of data point. This form of fuzzy equality index is called as a pessimistic one.

3.2 On-line fuzzy model identification(ONFID)

In this subsection, is proposed a new ONFID algorithm in which a fuzzy model evaluation stage is incorporated. The fuzzy model evaluation is performed by computing the fuzzy equality index between X_{k+1} (measured output at time k+1) and $\hat{X}_{k+1}$ (predicted output from the fuzzy model R_k identified at time k). The index represents the relevancy of R_k to the process operation condition at time k+1. The fuzzy model is updated based on the evaluation result. A detailed description of the algorithm is as follows.

The fuzzy model evaluation is performed by comparing X_{k+1} and $\hat{X}_{k+1}$ using the fuzzy equality index.

$$\delta_{k+1} = [X_{k+1} \equiv \hat{X}_{k+1}] \quad (7)$$

Since the fuzzy equality index is a fuzzy measure of which the value is bounded in [0,1], the fuzzy model evaluation based on it is a logical one which is quite different from the methods which is based on the conventional numerical index such as root mean square error in crisp. Although the ultimate performance of the identified fuzzy model could be evaluated in numerical level, the evaluation should be performed in logical level in order to avoid mixing the numerical concept and logical one and construct the fuzzy model identification algorithm in the fuzzy framework.

Based on the evaluation result, the fuzzy model is updated for the elements of fuzzy relation which contributed to the prediction of output at time k+1 as following

$$r^{ijp}_{k+1} = \begin{cases} \delta_{k+1} r^{ijp}_{k} + (1-\delta_{k+1})(u^{i}_{k-\tau} \times x^{j}_{k} \times \varepsilon^{p}_{k+1}) \\ \qquad , \text{if } u^{i}_{k-\tau} > 0,\ x^{j}_{k} > 0,\ \varepsilon^{p}_{k+1} > 0 \\ r^{ijp}_{k} \qquad , \text{otherwise} \end{cases} \tag{8}$$

where $i = 1,..,c_u$, $j,p = 1,..,c_x$ (c_u, c_x are the number of reference fuzzy sets for the process input and output, respectively), $\varepsilon_k = X_k \ominus \hat{X}_k$ ($\ominus$ represents bounded difference of fuzzy sets) and the initial fuzzy model is $R_0 = 0$. In the proposed fuzzy model adaptaion scheme, the fuzzy equality index δ_{k+1} plays the role of importance weight for the aggregation of r^{ijp}_{k+1}, $u^{i}_{k-\tau} \times x^{j}_{k} \times \varepsilon^{p}_{k+1}$ which is known as an exponential smoothing type aggregation of two linguistic quantity[8]. Updation law (8) can be illustrated like that the contribution of R_k to the updated fuzzy model R_{k+1} is constrained by δ_{k+1} and the rest portion of R_{k+1} is complemented by the subrelation $U_{k-\tau} \times X_k \times \varepsilon_{k+1}$.

For the practical purpose, to prevent significant changes of the fuzzy model by the transient abnormal operations, filtered fuzzy equality index α_k is used as the adaptation gain. The filter is taken as the following form

$$\alpha_k = \gamma\left(\frac{k}{1+k}\right)\alpha_{k-1} + (1-\gamma)\left(\frac{k}{1+k}\right)\delta_k \tag{9}$$

where γ ($0 < \gamma < 1$) is the forgetting factor or memory decreasing rate. Then the history of fuzzy equality index can be memoried. The bigger γ is, the longer the memory duration is and the less significant current equality index at time k is.

4. Numerical Examples

In this section, the usefulness of the proposed fuzzy model identification algorithm is shown by applying it to the modelling problem of a gas furnace and the output prediction problem of flexible joint manipiulator which is a nonliner system.

4.1 Fuzzy modelling of Gas Furnace

The modelling is performed based on the gas furnace data of Box and Jenkins[9]. This data is well known and frequently used as a standard test for the identification algorithms. The data set consists of 296 pairs of input-output observations where the input is the gas flow rate into the gas furnace and the output is the concentration of CO_2 in the exhaust gas. The sampling interval is equal to 9 seconds. The reference fuzzy sets defined on the input space **U** and output space **X** are of triangular form and depicted in Fig.1 for the case of c_u, c_x= 5. The simulation is carried out for c_u, c_x = 5, 7, 9, respectively. For the comparison purpose, Pedrycz's ONFID[1], OFFID[2] schemes are also simulated with the same environment. The performances are checked by root mean-square prediction error of the form (12) and the simulation results are listed in Table.1.

$$Q = \sum_{k=11}^{290} (x_k - \hat{x}_k)^2 \tag{12}$$

It can be seen that the result of the proposed algorithm is somewhat better than those of Pedrycz's scheme.

4.2 Output prediction of flexible joint manipulator

In this subsection, the proposed fuzzy model identification algorithm is applied to the on-line output prediction problem of flexible joint manipulator. The mathematical model of the flexible joint manipulator used in this example is as follows.

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= -\frac{MgL}{I}\sin q_1 - \frac{K_s}{I}(q_1 - q_3) \\ \dot{q}_3 &= q_4 \\ \dot{q}_4 &= \frac{K_s}{J_r}(q_1 - q_3) + \frac{1}{J_r}u \end{aligned} \tag{10}$$

where the parameters are listed in Table.2, q_1 is the link position to be predicted and u is the input. In this application, nine reference fuzzy sets of triangular form is defined for q_1, u (i.e. c_u, c_x = 9) and the universes of discourse are [-2.8, 2.8] for input, [-4.5 , 4.5] for output. Fig. 2 shows the simulation results of proposed ONFID scheme and Fig.3 that of Pedrycz's ONFID scheme when $u = 0.3\sin(1.5t)$ is applied and the disturbance which increases the initia of the link is applied at time 4[sec]. Proposed ONFID scheme shows good prediction results being compared with the Pedrycz's algorithm.

5. Conclusions

A new on-line fuzzy model identification algorithm incorporating the fuzzy model evaluation stage in it is proposed. The evaluation for the identified fuzzy model is performed by use of fuzzy equality index. The result of evaluation is utilized to adapt the fuzzy model to the variations of process operation conditions. Through the simulation study it can be seen that the proposed ONFID shows better performance than Pedrycz's ONFID scheme especialy in the case of on-line implimentation. We think that a FMBCS based on it can be designed for the practical purpose. And the study on the fuzzy model identification scheme for MIMO(multi input-multi output) system is expected.

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Table1 Comparison of proposed FID and Pedrycz's FID

	r	Max-prod Comp. On-line	Max-prod Comp. Off-line	Max-min Comp. On-line	Max-min Comp. Off-line
Proposed FID	5	0.533	0.508	0.736	0.698
	7	0.347	0.289	0.426	0.379
	9	0.295	0.207	0.369	0.295
Pedrycz's FID	5	0.544	0.526	0.963	1.001
	7	0.364	0.311	0.563	0.424
	9	0.299	0.207	0.395	0.304

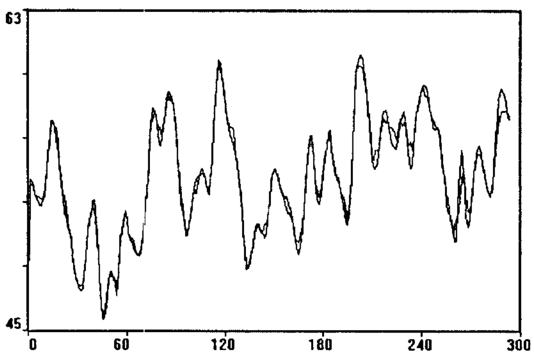

Fig.1 Output prediction result with the fuzzy model identified by the proposed ONFID scheme (c_u = c_x = 9, max-product composition)

Table2 Parameters of flexible joint manipulator

PARAMETER		Value [Unit]
Link Mass	M	0.4 [Kg]
Link Length	L	0.2 [m]
Joint Stiffness	K_s	5 [N-m/rad]
Link Inertia	I	0.031 [Kg-m^2]
Rotor Inertia	J_r	0.004 [Kg-m^2]
Gravity	g	9.8 [Kg-m/m^2]

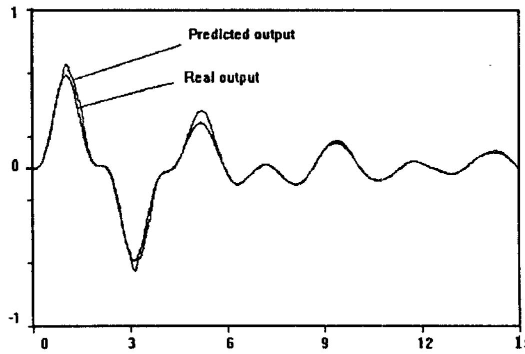

Fig.2 Output prediction result of flexible joint manipulator with the fuzzy model identified by the proposed ONFID scheme(c_u = c_x = 9, max-product composition)

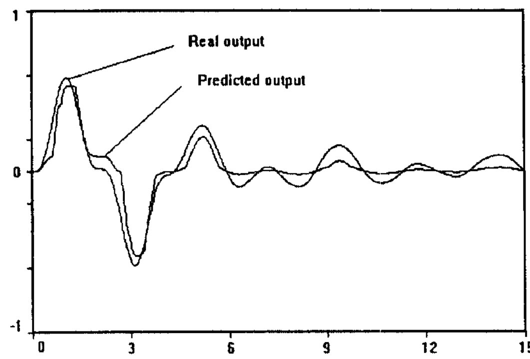

Fig.3 Output prediction result of flexible joint manipulator with the fuzzy model identified by Pedrycz's ONFID scheme(c_u = c_x = 9, max-product composition)