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Representation of Uncertain Geometric Robot Environment Using Fuzzy Numbers

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Abstract: In this paper, we present a fuzzy-number-oriented methodology to model uncertain geometric robot environment and to manipulate geometric uncertainty between robot coordinate frames. We describe any geometric primitive of robot environment as a parameter vector in parameter space. Not only ill-known values of the parameterized geometric primitive but the uncertain quantities of coordinate transformation are represented by means of fuzzy numbers restricted to appropriate membership functions. For consistent interpretation about geometric primitives between different coordinate frames, we manipulate these uncertain quantities using fuzzy arithmetic.

1. Introduction

Probability theory originates in a frequentistic interpretation, while possibility theory which axiomatically departs from probability theory corresponds more to the evaluation of the ease of attainment or of the feasibility of events. Considering another point of view on the robot environment uncertainty, we shall present in this paper, a fuzzy-number-oriented methodology to model uncertain geometric robot environment and to manipulate geometric uncertainty between robot coordinate frames. A fuzzy number is not a measurement but a subjective valuation[8]. In other words, a fuzzy number is a fuzzy set. But we can regard it as a possibility distribution function about some universe of discourse by virtue of the possibility theory[14]. Not only ill-known values of geometric primitives of robot environment but the uncertainty of coordinate transformation is represented by means of fuzzy numbers restricted to appropriate membership functions. Then the manipulation of uncertainty measures becomes the transformation and combination of these fuzzy numbers. Calculations with fuzzy numbers are very easy to handle and thus the manipulation of uncertainty using this representation is not much more complicated than using a usual probability density function(p.d.f) representation[2,6,9].

The use of fuzzy numbers gives some advantages in uncertain geometric robot environment modeling. First, in the light of engineer's senses, fuzzy numbers are more intuitive and figurative than probabilistic representation. Second, there is no need to have precise *a priori* information and constraints about sensor observation uncertainties. Third, there are some easy and consistent computational methods for the manipulation of geometric uncertainty between robot environment coordinates using simple fuzzy arithmetic based on the α-cut representation of fuzzy numbers and interval analysis. Also we can take account of the effect of the uncertainty of coordinate frame itself. Fourth, we can easily describe the linguistic or subjective constraints of the sensor system with appropriate membership functions. Moreover, in case of reasoning about uncertain robot environment, imprecise fuzzy number representation is more flexible because a reasoning process is not usually precise.

2. Representation of Uncertain Geometric Environment Using Fuzzy Number

Representation of geometric primitives by parameterization technique has been used in the field of stereo vision and multisensory information fusion[1,6]. In this paper we describe a geometric primitive as a parameter vector p and a parameterization vector function g such that:

$$g(x,p)=0 \tag{1}$$

where g can be interpreted as a model of the physical geometric primitive that maps a set of points (region) $x \subseteq \mathbf{R}^n$ in Euclidean n-space to a point $p \in \mathbf{R}^m$ in parameter space.

For example, a straight line on a plane can be described as follows:

$$g(x,p) = r - x\cos\theta - y\sin\theta = 0,$$

$$x = [x,y]^T, \quad p = [r,\theta]^T \tag{2}$$

where r is the distance from the line to the origin, and θ is the angle of its normal with respect to the abscissa axis(Fig. 1) ($r \geq 0, 0 \leq \theta < 2\pi$). This representation is complete since it allows the description of any straight lines in the plane. (However, the representation is ambiguous: in effect, the lines $[r, \theta]$ and $[-r, \theta \pm \pi]$ are identical[1].)

Now we represent the uncertainty of geometric primitives by assigning appropriate fuzzy numbers $\tilde{p}$ to the parameter vector p. This uncertainty representation is simpler and more comprehensive and intuitive than the usual p.d.f representation because the observed real sensor data have some imprecision caused by measurement error as well as variations caused by dynamic situation of robot environment[7]. Here the term *fuzzy number* is used to indicate a normalized convex fuzzy set $\tilde{M}$ of the real line $\mathbf{R}$ such that:

a. There exists exactly one $x_0 \in \mathbf{R}$ with $\mu_{\tilde{M}}(x_0) = 1$

b. $\mu_{\tilde{M}}(x)$ is piecewise continuous

where x is the genuine value of $\tilde{M}$ and x_0 is called the mean value of $\tilde{M}$. And the meaning of the fuzzy number $\tilde{M}$ is that it has the value of *approximately* (or *about*) x_0.

The most crucial element in the representation of uncertain geometric primitives by fuzzy numbers is to determine the shapes of the membership functions of fuzzy numbers. An overview of the properties for constructing the membership functions and the mathematical form of the membership functions is well given in [4], especially Civanlar[3] proposed a method to obtain the fuzzy membership function from the p.d.f. For the sake of computational efficiency and ease of data acquisition, we can use an appropriate fuzzy number for representing the uncertainty of the parameterized geometric primitives. For example, on some line image, we can fit straight line to a set of edge pixels by the Hough transformation. Then the line parameter r and θ can be represented by a 2-dimensional accumulator array like Fig. 2(a). We can represent these line parameters r and θ using triangular fuzzy numbers like Fig. 2(b).

3. Uncertainty Manipulation Between Robot Coordinate Frames Using Fuzzy Numbers

In many sensor-based robotic applications, especially for integrating the information of the mobile sensor systems or the distributed sensor networks, consistent interpretation about geometric primitives between different coordinate frames is crucial. Thus we must consider how the uncertainty of geometric primitives in a coordinate frame is interpreted when it is described with respect to other coordinate frames.

Suppose a geometric primitive defined by a set of points x in Euclidean n-space R^n is expressed x_i in the i-th coordinate frame. If we want to represent the point set x_i with respect to the j-th coordinate frame, it is necessary to take account of a transformation T_i^j in order to change the description of x_i from the i-th frame to the j-th frame such that:

$$x_j = T_i^j(x_i). \tag{3}$$

Very often the usual representation of relative locations(rotations and translations) in robotics is the *homogeneous transformation*[10]:

$$T_i^j = \begin{bmatrix} rotation & translation \\ 0 & 1 \end{bmatrix} \tag{4}$$

where *rotation* is a 3×3 matrix and *translation* is a 3×1 vector. Similarly, the parameter vector p_i in the i-th coordinate frame is also changed to p_j in the j-th coordinate frame through another transformation h_i^j such as:

$$p_j = h_i^j(p_i). \tag{5}$$

Here we assume that the parameterization function g is the same in any coordinate system and the transformation T_i^j has the same form for any set x. Then h_i^j can be completely defined by the function g and fixed T_i^j[6], so that:

$$g(T_i^j(x_i), h_i^j(p_i)) = g(x_j, p_j) = 0. \tag{6}$$

For example, we assume that a transformation T_i^j is described by the following in 2-dimensional case for simplicity:

$$T_i^j = \begin{bmatrix} \cos\zeta & -sin\,\zeta & T_x \\ \sin\zeta & \cos\zeta & T_y \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

where ζ is a rotation angle about the origin and T_x and T_y are translations along x and y direction respectively. Consider a straight line on a plane in the i-th coordinate frame which is described by the following function:

$$g(x_i, p_i) = r_i - x cos\,\theta_i - y sin\,\theta_i = 0,$$

$$x_i = [x_i, y_i]^T, \quad p_i = [r_i, \theta_i]^T. \tag{8}$$

Then,

$$x_j = \begin{bmatrix} x_j \\ y_j \end{bmatrix} = T_i^j(x_i) = \begin{bmatrix} \cos\zeta & -sin\,\zeta \\ \sin\zeta & \cos\zeta \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}. \tag{9}$$

The transformation h_i^j can be found as follows[9]:

$$p_j = \begin{bmatrix} r_j \\ \theta_j \end{bmatrix} = h_i^j(p_i) = \begin{bmatrix} r_i + T_x cos(\theta_i + \zeta) + T_y sin(\theta_i + \zeta) \\ \theta_i + \zeta \end{bmatrix} \tag{10}$$

which is subject to eq. (6).

In general, parameter uncertainty transformation with an associated p.d.f between coordinate frames is not simple. For example, we assume that the parameter uncertainty is expressed by a p.d.f, $f_p(p_i)$, in the i-th coordinate frame and we consider a parameter transformation h_i^j of eq. (5). Then, in the j-th coordinate frame, the transformed p.d.f is described as follows[11]:

$$f_p(p_j) = \frac{1}{|\nabla h|} f_p(h^{-1}(p_j)). \tag{11}$$

But eq. (11) can be very complex and may introduce singularities into the Jacobian[11,12]. In addition, the geometric complexity of the transformation (5) and the difficulty of finding a p.d.f which has invariant measure over all primitives and transformations only allow us to approximate the exact transformation[6]. Moreover, in many applications of robotics, more specifically in mobile robots, the transformation T_i^j cannot be determined without uncertainties. All rotation angles and translations of T_i^j are measured using sensors or determined through calibration process. But measurements and calibration are always subject to error and such errors are often unavoidable and varying according as how the sensor is used. Thus we must take the transformation uncertainty into account for describing the propagation of parameter uncertainty between coordinate frames.

To overcome the difficulty of the p.d.f formalism and to take the transformation uncertainty into consideration, in this paper, we present an explicit method for manipulating the uncertainties represented by fuzzy numbers. Similarly, we describe the transformation uncertainty by assigning appropriate fuzzy numbers to the rotation angles and translations such as:

$$\tilde{T}_i^j = \begin{bmatrix} \tilde{rotation} & \tilde{translation} \\ 0 & 1 \end{bmatrix}. \tag{12}$$

For example, in eq. (7) the rotation angle ζ is represented by fuzzy number $\tilde{\zeta}$ and the translations T_x and T_y are also represented by fuzzy numbers $\tilde{T}_x$ and $\tilde{T}_y$ respectively as follows:

$$\tilde{T}_i^j = \begin{bmatrix} \cos\tilde{\zeta} & -\sin\tilde{\zeta} & \tilde{T}_x \\ \sin\tilde{\zeta} & \cos\tilde{\zeta} & \tilde{T}_y \\ 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Because the parameter vector p of a geometric primitive in one coordinate frame and the transformation T_i^j are described by fuzzy numbers $\tilde{p}$ and $\tilde{T}_i^j$ respectively, we can manipulate these uncertain quantities between coordinate frames in a consistent manner using fuzzy arithmetic[8]. Here we assume that the parameter uncertainties are noninteractive with the transformation uncertainties.

For example, 2-dimensional straight line case, the parameter vector p_i in the i-th coordinate frame described by the fuzzy number $\tilde{p}_i$ can be propagated to the j-th coordinate frame through the transformation $\tilde{T}_i^j$ using eq. (10) as follows:

$$\tilde{p}_j = \begin{bmatrix} \tilde{r}_j \\ \tilde{\theta}_j \end{bmatrix} = \begin{bmatrix} \tilde{r}_i + \tilde{T}_x \cos(\tilde{\theta}_i + \tilde{\zeta}) + \tilde{T}_y \sin(\tilde{\theta}_i + \tilde{\zeta}) \\ \tilde{\theta}_i + \tilde{\zeta} \end{bmatrix} \tag{14}$$

The manipulation of fuzzy number operations, sometimes called the extended operations, can be processed by means of the *extension principle* of Zadeh[13]. This principle is one of the most basic concepts of fuzzy set theory which can be used to generalize crisp mathematical concepts to fuzzy sets. Although the calculation can be done using the extension principle, the procedure for obtaining a solution is not trivial because nonlinear programming problem must be solved. However, fortunately, there is an efficient and simple calculation method based on the α-cut representation of fuzzy numbers and interval analysis[5,8]. We can process the extended additions, extended multiplications, and fuzzy trigonometric functions in the following steps:

(1) Select a particular α value, where $0 \le \alpha \le 1$.

(2) Find the α-cuts about $\tilde{p}_i$ and $\tilde{T}_i^j$ (the intervals which correspond to α : α level set).

(3) Find the range of a trigonometric function about the α-cuts. This range is another interval which consists of the maximum and minimum values of the trigonometric function about the α-cuts.

(4) Using the intervals of (2) and (3), construct the α-cuts of $\tilde{p}_j$ according to the usual interval analysis.

(5) Repeat these steps as many different values of α as needed to refine the solution.

This method provides a discrete but sufficiently exact solution to the propagation of parameter uncertainty between coordinate frames. And there is no need to perform the combinatorial interval analysis by virtue of the simple occurrence of variables.

4. Conclusions

In this paper we have proposed a representation method of uncertain robot environment by assigning appropriate fuzzy numbers to the parameters of geometric primitives and coordinate transformation. We also have dealt with the manipulation of geometric uncertainties between coordinate frames. We think that the proposed method might be useful to many application areas of sensor-based robotics in which conventional probabilistic approach is insufficient or complex. In practice, since we cannot easily obtain the exact distribution of uncertainties of sensor observations, the uncertainty representation using fuzzy numbers is more flexible and intuitive in the light of engineer's senses for many application areas.

We think that further researches are needed on the modeling of robot environment using fuzzy numbers especially in: (1) the selection of parameters which represent the environment more comprehensive; (2) the determination of the shapes of fuzzy numbers according to the characteristics of sensor systems; (3) the consistent modeling method of 3-dimensional complex objects; (4) the representation of the relation between objects using their topological properties.

References

[1] N. Ayache, *Artificial Vision for Mobile Robots: Stereo Vision and Multisensory Perception* (MIT Press, Cambridge, 1991).

[2] N. Ayache and O. D. Faugeras, Maintaining representations of the environment of a mobile robot, *IEEE trans. on Robotics and Automat.* 5(6) (1989) 804-819.

[3] M. R. Civanlar and H. J. Trussel, Constructing membership functions using statistical data, *Fuzzy Sets and Systems* **18** (1986) 1-13.

[4] J. Dombi, Membership function as an evaluation, *Fuzzy Sets and Systems* **35** (1990) 1-21.

[5] W. M. Dong and F. S. Wong, Fuzzy weighted averages and implementation of the extension principle, *Fuzzy Sets and Systems* **21** (1987) 183-199.

[6] H. F. Durrant-Whyte, *Integration, Coordination, and Control of Multi-Sensor Robot Systems* (Kluwer Academic Publ., Boston, 1987).

[7] H. Farreny and H. Prade, Tackling uncertainty and imprecision in robotics, in O. D. Faugeras and G. Giralt(ed.), *Robotics Research: The Third Int. Symposium* (MIT Press, Cambridge, 1986).

[8] A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic* (Van Nostrand Reinhold, New York, 1985).

[9] M. G. Kendall and P. A. P. Moran, *Geometrical Probability* (Charles Griffin, London, 1963).

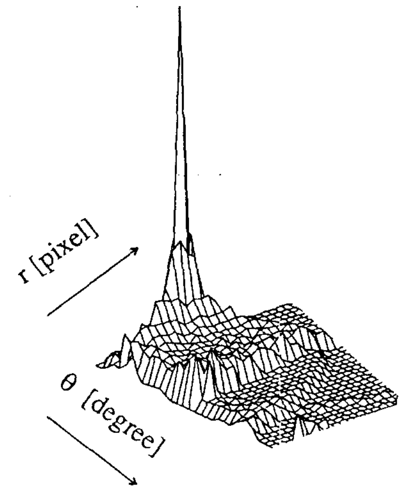
[10] R. P. Paul, *Robot Manipulators: Mathematics, Programming, and Control* (MIT Press, Cambridge, 1981).

[11] A. Papoulis, *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, Tokyo, 1984).

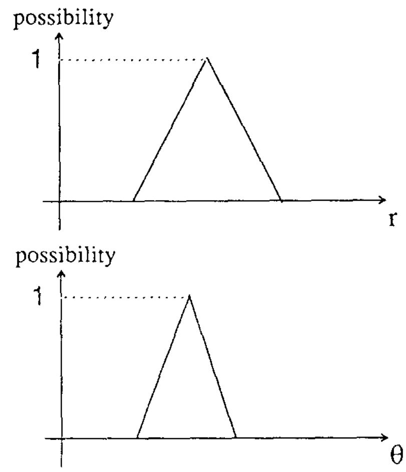
[12] R. C. Smith and P. Cheeseman, On the representation and estimation of spatial uncertainty, *Int. J. of Robotics Research* 5(4) (1986) 56-68.

[13] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, *Information Science* **8** (1975) 199-249.

[14] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems* **1** (1978) 3-28.



(a)



(b)

Fig. 2. Uncertainty representation using triangular fuzzy number
(a) Line parameter representation using Hough transformation
(b) Triangular fuzzy number representation

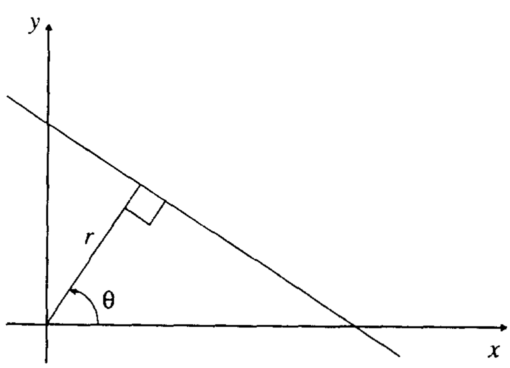


Fig. 1. Parameterization of a straight line on a plane