Fifth IFSA World Congress (1993), 1207-1210

Design of a Fuzzy Logic Controller for Robot Manipulators in the VSS Control Scheme

Soo Yeong Yi and Myung Jin Chung

Dept. of Electrical Engineering
Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu, Taejon, Korea

Abstract: There is an opinion of regarding a simple fuzzy logic controller as a kind of Variable Structure Controller in recent years. The opinion may provide an analytical basis which describes the robustness to uncertainty and the stability of a fuzzy logic controller. So in this paper, a fuzzy logic controller based on the Variable Structure System scheme is designed for a robot manipulator which is a class of complex, nonlinear system with uncertainty. Fuzzy control rules, membership shape of the I/O variables of the fuzzy logic controller are designed for guaranteeing the stability of an overall control system. From a computer simulation of dynamic control of a two link robot manipulator, the design procedure of the fuzzy logic controller is validated.

1. Introduction

The Fuzzy Logic Controller(FLC), which has simple computation and programming capability of human control behavior, has widen its applicability to many engineering fields in recent years. Especially, the robustness of the FLC to uncertainty makes the FLC more appropriate to an imprecise model based system control area.

A robot manipulator is a case of complex uncertain dynamic system because of uncertainties such as load variation, motor backlash, friction and external disturbances. So the simple computation and the robustness to uncertainty of the FLC may be effective in robot system control. However, there are some difficulties in the FLC design: absence of a systematic design procedure, analysis guideline, and performance evaluation; dependency on the human intuition by trial and error; its overall stability is not clear.

A simple FLC was regarded as a kind of Variable Structure System(VSS) controller by Kawaji et. al[1]. The VSS is a class of systems with discontinuous feedback control structure, which forces the system to reach a switching surface representing a desired characteristic and switches a different control structure to remain on the surface. The system is insensitive to uncertainty or disturbance on the switching surface[2][3]. For the consideration of the chattering effect, a continuous function is used for switching action. Therefore, from presumption for relation between the FLC and the VSS controller, it can be understood that the robustness of a FLC comes from the invariant switching surface of the VSS control scheme. And it may give us a systematic analysis and design techniques of a FLC.

So in this paper, a FLC will be established for tracking control of a robot manipulator with uncertainty. Based on the VSS control scheme, design specifications for the FLC such as dynamic range of the controller variables, their membership shapes, and control rules will be presented definitely.

We will describe the robot dynamics and the VSS control theory in section 2, and the FLC design procedure based on the VSS control scheme in section 3. In section 4, a computer simulation results of the proposed controller for simple two link manipulator will be given and we will conclude in section 5.

2. A Robot Dynamics and a VSS control Theory

A manipulator type robot dynamics is described as follows:

$$M(\theta)\ddot{\theta} + F(\theta,\dot{\theta}) + G(\theta) + \Delta(\theta,\dot{\theta}) = T \qquad (1)$$

where θ, $\dot{\theta}$, and $\ddot{\theta} \in R^n$ denote joint angle, angular velocity, and angular acceleration each other, $T \in R^n$ means torque input to each joint with the number of joint n. The $M(\theta)$ is an $n \times n$ inertial matrix, and the $F(\theta,\dot{\theta})$, $G(\theta)$ mean $n \times 1$ Coriolis and gravity vector. The $\Delta(\theta,\dot{\theta})$ represents lumped uncertainty with known bounded $|\Delta(\theta,\dot{\theta})| < \Delta_{max}$ containing structural and unstructural uncertainties. Let us denote a desired joint angle, angular velocity, and angular acceleration as θ_d, $\dot{\theta}_d$, and $\ddot{\theta}_d$ each other. (1) can be rewritten in state space representation as

$$\ddot{\theta} = -M^{-1}(\theta)(F(\theta,\dot{\theta})+G(\theta)) - M^{-1}(\theta)\Delta + M^{-1}(\theta)T \quad (2)$$

Let us define a surface s representing a desired error dynamics in error phase plane as

$$s = \dot{e} + \lambda e = 0 \quad (3)$$

where $e = \theta - \theta_d$, $s \in R^n$, and $\lambda = diag[\lambda_1, \cdots, \lambda_n]$, $\lambda_i > 0$ $\forall i$. Then, the VSS control law forces the system (2) to reach a surface (3) and switches a different control structure to remain on the surface. Since the surface (3) represents stable error dynamics, the e slides into equilibrium point $e = 0$ as $t \to \infty$ after reaching to (3). The sliding mode existence condition is that the Lyapunov function of the variable s

$$V = \frac{1}{2}s^t s \quad (4)$$

should decrease for all time t. Derivating (4) and inserting (3) give

$$\begin{aligned}\frac{dV}{dt} &= s^t \dot{s} \\ &= s^t(\ddot{e} + \lambda\dot{e}) \\ &= s^t\{-M^{-1}(\theta)(F(\theta,\dot{\theta}) + G(\theta)) \\ &\quad - M^{-1}(\theta)\Delta + M^{-1}(\theta)T - \ddot{\theta}_d + \lambda\dot{e}\}. \quad (5)\end{aligned}$$

If the control input is

$$T = T_{eq} + M(\theta)\delta T \quad (6)$$

where $T_{eq} = F(\theta,\dot{\theta}) + G(\theta)$, $\delta T = -k \cdot sgn(s) \triangleq [-k_i \cdot sgn(s_i)]_{n\times 1}$, and

$$\begin{aligned}k &= |-\ddot{\theta}_d + \lambda\dot{e}| + |M^{-1}(\theta)\Delta_{max}| \triangleq k_{min} \quad (7) \\ &> |-\ddot{\theta}_d + \lambda\dot{e}| + |M^{-1}(\theta)\Delta|,\end{aligned}$$

then (5) becomes

$$\begin{aligned}&s^t\{-M^{-1}(\theta)\Delta - |M^{-1}(\theta)|\Delta_{max} \cdot sgn(s) \\ &\quad -\ddot{\theta}_d + \lambda\dot{e} + |-\ddot{\theta}_d + \lambda\dot{e}| \cdot sgn(s)\} < 0, \ \forall s, \forall t. \quad (8)\end{aligned}$$

(5), (7), and (8) are the well-known sliding mode existence condition in the VSS theory.

Compromise between a tracking accuracy and chattering problem due to a time delay or numerical error of control hardware make substitute the $sgn(s)$ function with $sat(\frac{s}{\phi})$ where

$$sat(\frac{s_i}{\phi}) = \begin{cases} 1 & \text{if } s_i > \phi \\ \frac{s_i}{\phi} & \text{if } |s_i| \le \phi \\ -1 & \text{if } s_i < -\phi, \end{cases} \quad (9)$$

then, the switching part of the control input (6) becomes

$$\delta T = -k_{min} \cdot sat(\frac{s}{\phi}). \quad (10)$$

Note that (6), (8), (9) and (10) mean that if the switching part δT of the control input (6) satisfies

$$\begin{aligned}&k > k_{min} \quad \text{in } s < -\phi \text{ and} \\ &k < -k_{min} \quad \text{in } s > \phi \quad (11)\end{aligned}$$

then, the error dynamics becomes

$$|s| = |\dot{e} + \lambda e| \le \phi \quad \text{as } t \to \infty. \quad (12)$$

It implies that the error dynamics is ultimately bounded with ϕ.

3. A FLC Design based on the VSS control scheme

For a FLC design, we should determine the controller input, output fuzzy sets with membership shapes, control rules, reasoning method, and defuzzification method. Let us define a surface $s(e, \dot{e}) = \dot{e} + \lambda e = 0$ in the error phase plane.

3.1. Controller input fuzzy sets; We take some real numbers C_{ZRE}, C_{PSE}, and C_{PBE}, which are $C_{ZRE} = 0 < C_{PSE} < C_{PBE}$ as the means of fuzzy sets ZeRo Error, Positive Small Error, and Positive Big Error in the universe of discourse of tracking error. Then, let us take the followings.

$$\begin{aligned}&C_{NSE} = -C_{PSE}, \ C_{NBE} = -C_{PBE} \text{ for symmetry and} \\ &C_{ZRDE} = 0, \ C_{PSDE} = \lambda C_{PSE} = -C_{NSDE} \\ &\qquad C_{PBDE} = \lambda C_{PBE} = -C_{NBDE} \quad (13)\end{aligned}$$

for consistency of e and $\dot{e}$ on the s. In the sequel, NSE(Negative Small Error), PBE(Positive Big Error), $NSDE$(Negative Small Derivative of Error), and $ZRDE$(ZeRo Derivative of Error), etc., are used as linguistic notation of the fuzzy sets and C_{NSE}, C_{PBE}, C_{NSDE}, and C_{ZRDE}, etc., denote means of the corresponding fuzzy sets.

Then, the membership shape of each fuzzy set is constructed so that only two membership shape are crossed between the means of two near fuzzy sets as shown in fig. 1.

3.2. Output fuzzy sets and defuzzification method; As the means of output fuzzy sets $\tilde{k}$, some real numbers

$$\begin{aligned}&C_{ZRK} = 0, \\ &C_{PSK} = -C_{NSK}, \ C_{PBK} = -C_{NBK}\end{aligned}$$

are chosen where

$$\frac{\int_0^{C_{PBK}} k \cdot \mu_{PSK}(k)dk}{\int_0^{C_{PBK}} \mu_{PSK}(k)dk} = k_{min}, \qquad \frac{\int_{C_{PSK}}^{C_{PBK}} k \cdot \mu_{PBK}(k)dk}{\int_{C_{PSK}}^{C_{PBK}} \mu_{PBK}(k)dk} = k_{max}, \quad (14)$$

the k_{min} is as defined in (7) and $k_{max} \gg k_{min}$ as shown in fig. 2.

The weighted-sum-average defuzzification method is

$$k(e,\dot{e}) = defuzzify(\tilde{k})$$

$$= \frac{\int k \cdot \mu_{\tilde{k}} dk}{\int \mu_{\tilde{k}} dk}. \tag{15}$$

3.3. Control rules; We assign a control output fuzzy set to each cell of rule tables as shown in fig. 3 and table 1. The assign law is as follows: Assign ZRK to the cells which are laid on $s=0$, and draw a parallel region $|s| = |\dot{e} + \lambda e| = 2\lambda C_{PSE}$ with $s=0$ in the error phase plane. Then, assign NSK to the inner cells of the region above $s=0$ and PSK to the belows. Among the outer cells of the region, assign NBK to the cells above $s=0$, and PBK to the belows. It becomes so called the linear rules which is generally referred in the other fuzzy control literatures.

3.4. Reasoning method; We take the max-min operation, which is frequently used in a FLC design.

According to the rule table 1., there are some characteristic of the control output $k(e,\dot{e})$: its sign is changed near $s=0$ as $k(e,\dot{e}) > 0$ below the $s=0$ and $k(e,\dot{e}) < 0$ above the $s=0$ and its magnitude has trend to increase in accordance with distance to $s=0$. Therefore, since the membership shapes of e and $\dot{e}$, the max-min operation of reasoning and the defuzzification for the controller output are continuous, there exists a ϕ such that (11) is satisfied. It implies that if the interesting region of the FLC output, that is, $C_{NBE} < e < C_{PBE}$, $C_{NBDE} < \dot{e} < C_{PBDE}$ is large enough, (12) can be satisfied.

4. Simulation Results

Consider the dynamics of two link manipulator with known bounded uncertainty as shown in fig. 4.

$$\begin{bmatrix} m_2 l^2_2 + 2m_2 l_1 l_2 C_2 + (m_1+m_2) l^2_1 + J_1 & m_2 l^2_2 + m_2 l_1 l_2 C_2 \\ m_2 l_1 l_2 C_2 + m_2 l^2_2 & m_2 l^2_2 + J_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -m_2 l_1 l_2 S_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 S_2 \dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} m_2 l_2 C_{12} G + (m_1+m_2) l_1 G C_1 \\ m_2 l_2 G C_{12} \end{bmatrix}$$

$$+ \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{16}$$

Assume that the nominal parameters m_1, m_2, l_1, l_2, J_1, J_2 of (16) are known and the desired trajectory is a circle in Cartesian space as

$$x = 0.7 + 0.5 sin(\alpha)$$

$$y = 0.7 + 0.5 cos(\alpha), \quad -\pi \le \alpha(t) \le \pi, \quad 0 \le t \le P,$$

where $\alpha(t) = -\pi\cos(\frac{\pi}{P}t)$, and $P = 5$.

The control input has form of (6), where the δT is the FLC as designed in section 3. The T_{eq} is nonlinear compensation term for nominal dynamics of the robot. There are two pages of the control rule table 3. for each joint torque input T_j, $j = 1, 2$, having same form and different parameters of means of membership shapes of I/O fuzzy sets. The simulation result is shown in fig. 5.

5. Conclusion

There are advantageous properties in the FLC, which are robustness to uncertainty, and arbitrary nonlinear mapping capability, etc. But since its design method depends only on the human intuition by trial and error, there is not a systematic design and analysis procedure, so the controller stability is not clear. In this paper, based on the well-proved VSS control scheme, we develop the design guideline for guaranteeing the asymptotic attractivity and apply the controller design to a simple two link robot dynamics with uncertainty. Since the controller has not discontinuous switching but continuous switching and its magnitude increases according to the distance to the switching region with minimum magnitude near the switching region, it improves the reaching time and chattering effect, which have been serious problem of the standard VSS control scheme.

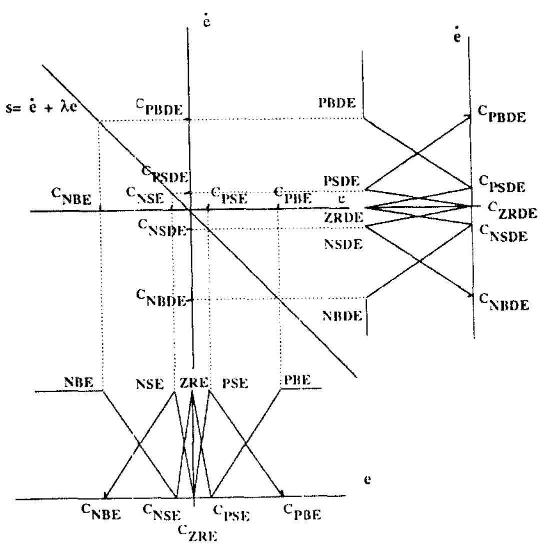

Fig. 1. Design of the Controller Input Fuzzy Sets

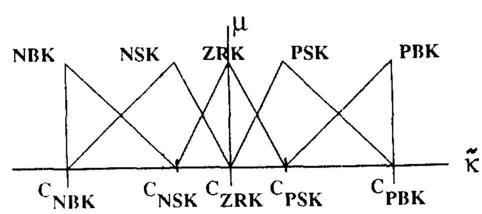

Fig. 2. The Output Fuzzy Sets of Controller

It may be possible to estimate the ϕ, which satisfies (11), since the FLC output is continuous on its argument $e, \dot{e}$. It is under study.

References

[1] S. Kawaji, N. Matsunaga, "Fuzzy Control of VSS type and Its Robustness", *Proceedings of IFSA - Engineering, Brussels*, 1991.

[2] Jean-Jacques E. Slotine, Weiping Li, *Applied Nonlinear Control*, Prentice-Hall International, 1991.

[3] Kar-Keung Young, "Controller Design for a Manipulator Using Theory of Variable Structure Systems", *IEEE Trans. on SMC*, Vol. SMC-8, No. 2, 1978.

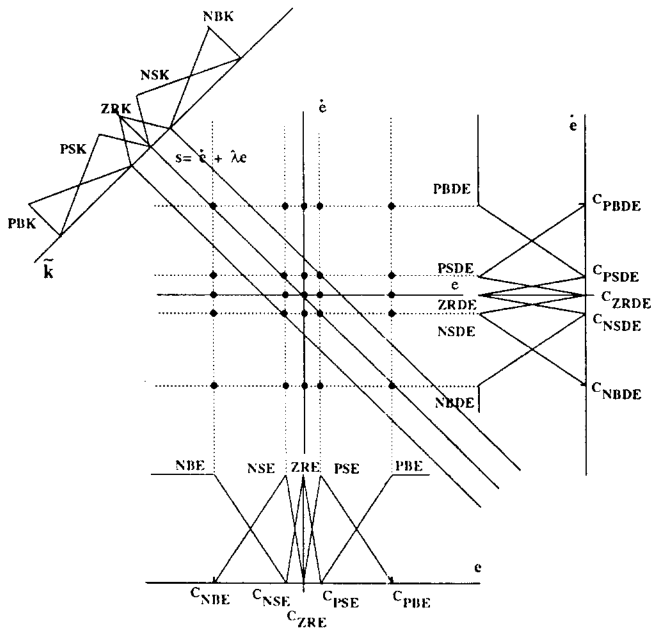

Fig. 3. Assign Law of The Fuzzy Control Rules

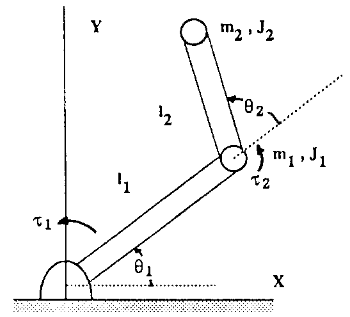

Fig. 4. A Two Link Manipulator

$\dot{e} \backslash e$	*NBE*	*NSE*	*ZRE*	*PSE*	*PBE*
PBDE	**ZRK**	**NBK**	**NBK**	**NBK**	**NBK**
PSDE	**PBK**	**ZRK**	**NSK**	**NSK**	**NBK**
ZRDE	**PBK**	**PSK**	**ZRK**	**NSK**	**NBK**
NSDE	**PBK**	**PSK**	**PSK**	**ZRK**	**NBK**
NBDE	**PBK**	**PBK**	**PBK**	**PBK**	**ZRK**

Table 1. The Fuzzy Control Rules

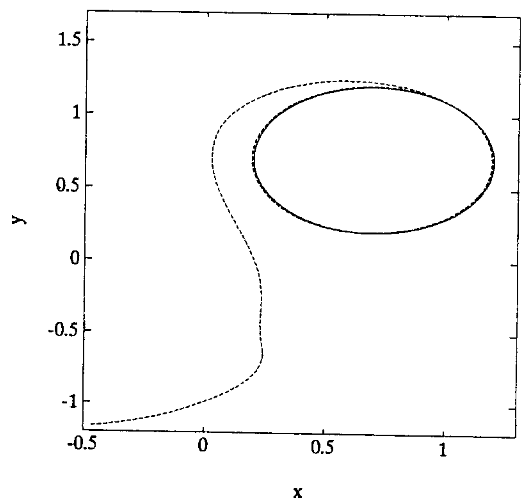

Fig. 5. The Simulation Results in Cartesian Space