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AUTOMATIC TUNING OF FUZZY OPTIMAL CONTROL SYSTEMS

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ABSTRACT

We investigate a systematic design procedure of **automated rule generation** of fuzzy logic based controllers for uncertain dynamic systems such as an engine dynamic model. "Automated Tuning" means autonomous clustering or collection of such meaningful **transitional relations** in the state-space. **Optimal control strategies** are included in the design procedures, such as minimum squared error, minimum time, minimum energy or combined performance criteria. Fuzzy feedback control systems designed by the **cell-state transition method** have the properties of closed-loop **stability, robustness** under parameter variabtions, and a certain degree of **optimality**. Most of all, the main advantage of the proposed approach is that **reliability** can be potentially increased even if a large grain of uncertainty is involved within the control system under consideration. A numerical example is shown in which we apply our strategic fuzzy controller design to a highly nonlinear model of engine idle speed control.

1. Introduction

Fuzzy logic/linguistic control can be categorized as a knowledge-based system or an expert control paradigm, the reason for which is that every control action derived by the fuzzy inference engine is based on some a priori knowledge source [1]. However, the difficulties in constructing a rule base have prevented the fuzzy engineers from approaching to a generalized methodology for fuzzy rule based control systems [2] as shown in Figure 1.

We propose a systematic design procedure of automated rule generation for uncertain dynamic processes. Fuzzy logic based feedback control is suitable for our physical target of automated rule generation. Membership functions stored in a fuzzy logic controller can be easily modified and updated without repetitive tedious re-evaluation of different dynamic models. This procedure of generating the rules required in a fuzzy logic controller should guarantee stability of the closed-loop system and robustness under parameter variations. We utilize the cell-to-cell mapping theory originally introduced by Hsu [3] and later applied to fuzzy dynamic systems by Chen et al.[4] The key point in making a stable control rule base is that every stabilizable feedback system has a chain of state transitions from one cell-state to another. Consequential elements of such transitions are anticipated according to applied control action of each rule. Data required for this transitional set of rules can be collected via [5]

* *A priori* information such as experimental results
* Numerical simulation runs based on dynamic models
* Expertise and heuristics

Specifications, accuracy and precision, of the system tolerances can be arbitrarily adjusted and are a function of resolution of design parameter[6]. The next section deals with the step-by-step procedure as to the synthesis of a fuzzy logic control rule base based on the given input-state data pairs of a particular nonlinear dynamic model. These are the training data for approximate learning.

2. Automatic Tuning for Fuzzy Logic Controllers

2.1 Fuzzy Logic Control Based On Cell-State Transitions

General fuzzy controllers have four components: fuzzifier, rule base, fuzzy inference engine, and defuzzifier. The control rules can be determined by using the cell-to-cell mapping theory[3] and the cell-state transitions. Comparing with the point-to-point mapping theory, this concept make use of the intervals and a finite number of cells in the cell-state. The dynamical characteristics are preserved as far as the resolution allows.

DEFINITION 1: A "Cell" z is defined as an n-tuple of integers in the cell-state space Z such that

$$\mathbf{z} = (z_1, z_2, \ldots, z_n) = \sum_{i=1 \ldots n} z_i \, e_i \qquad (1)$$

where e_i is the unit vector in the direction of z_i and the corresponding x_i is represented by an interval X_i.

$$(z_i - 0.5)\, h_i < x_i < (z_i + 0.5)\, h_i \qquad (2)$$

(h_i ... interval size, z_i ... integer representing x_i) ■

DEFINITION 2: A "Cell-to-Cell Mapping" F is a relation between cells in the cell-state space, F: Z —> Z, and the function values have one-to-one correspondence with the point-to-point mapping f such that

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k) \in \mathbf{z}^* \longleftrightarrow \mathbf{z}^* = F(\mathbf{z}_k) \qquad (3)$$ ■

DEFINITION 3: An "Equilibrium Cell" $\mathbf{z}^*$ (Invariant Cell) of F satisfies

$$\mathbf{z}^* = F(\mathbf{z}^*) \qquad (4)$$ ■

DEFINITION 4: A k-Period Motion Cell is the distinct k cells z(1),...,z(k) that satisfy

$$z(1) = F^k(z(1)) \text{ and } z(m+1) = F^m(z(1)).$$ ■

2.2 Systematic Procedure for Automated Rule Generation

Step 1: Consider a two-state dynamic model given by

$$dx_1/dt = f_1(x_1, x_2, \delta, \theta) \quad (5a)$$
$$dx_2/dt = f_2(x_1, x_2, \delta, \theta) \quad (5b)$$

where x_1, x_2 are the states (or the errors); f_1, f_2 are the nonlinear functions; and δ, θ are the control inputs. From the admissible controls, we select finite representative constant values of δ's and θ's and we call them δ_i's and θ_j's. These crisp numbers will be fuzzified later after the performance is satisfied. Moreover, we choose finite representative points in the state space to anticipate the trajectories from one subspace to another. These countless trajectories are called a 'manifold' and one set of data is collected by setting δ_i and θ_j constant. Repeated collections of such information are used next in order to obtain a rule base for feedback regulation of x_1 and x_2, i.e., x_1, $x_2 \longrightarrow 0$ as t increases. We denote X_{1m}, X_{2n} the m-th and the n-th interval sets of x_1 and x_2, respectively. Linguistically, we define

$$L_{mn} = X_{1m} \times X_{2n} \quad (6)$$

and then L_{mn} is a finite region in the state space. By applying fixed controls, δ_i and θ_j, one set of transitional relations is obtained for example,

Rules of Dynamical Behavior
(Cell-State Transitions by (δ_1, θ_1)):

$$\begin{aligned}
&(\delta_1, \theta_1): \ z\{L_{12}\} \longrightarrow z\{L_{34}\} \\
&(\delta_1, \theta_1): \ z\{L_{23}\} \longrightarrow z\{L_{33}\} \\
&\quad \cdots \qquad\qquad \cdots \\
&(\delta_1, \theta_1): \ z\{L_{55}\} \longrightarrow z\{L_{73}\}
\end{aligned} \quad (7)$$

Controls	Prev.State	Next State	Time t	Perf.Index
δ1 θ1	{1,2}	{3,4}	0.12sec	5.2
	{2,3}	{3,3}	0.23sec	7.7
	{4,3}*	{4,3}*	∞ sec	0.0
δ1 θ2	{1,3}	{1,2}	0.41sec	4.3
	{2,1}	{4,3}	0.37sec	11.2
	...	...	...	...

Table 1. Cell-State Transition Table in the 2-dim. Cell-State Space

The above transitions are valid if the average dynamic behavior of the system shows the above rules. We continue to change (δ_i, θ_j) to get other sets of transitional rules and exhaustively gather finite number of transitional relations. When we store the above information, we add time required during transitions, and other optimal performance indices such as energy, squared errors, etc. These transition relations are stored in the table which we call a "cell-state transition table". An example of cell-state transition table in the 3 dimensional cell-state space is shown in Table 1. Changing (δ_i, θ_j), we may have the same L_{mn} for the source and the destination and this is called an 'invariant cell' or an 'invariant manifold'. For an invariant cell, there should be a design limit in the transition time since it is an indefinite stay in that L_{mn}. It is emphasized that the target L_{mn} (the specified goal) must have an invariant manifold for some fixed controls (δ_i, θ_j) for convergence and asymptotic stability. This is equivalent to the 'reachability condition' in the classical control theory. The target is denoted as L^*.

Step 2: From the collected data, we generate an N-ary tree that connects from one node L_{mn} to another. The root of the tree is L^* and we avoid any looping structures. We proceed with a backward chaining search technique in artificial intelligence (AI) and the search procedure is initiated by finding all possible dynamic transitions to the target region L^*. There may be multiple paths from one region to another and we eliminate multiplicity and extract only one transition by considering the following concepts with priority from P1 to P3:

P1. Minimum Euclidean Distance
P2. Optimal Strategies
Minimum Energy, Minimum Time,
Minimum Squared Errors, or Combinations
P3. Redundancy in Controls, Transitions

The elements of the finalized tree constitute a set of control rule base. These rules are automatically generated on the basis of optimal performance criteria such as minimum time or minimum energy concepts. Simply, the transitional relations that force the trajectories from any points in the state space to the desired goal within the prescribed tolerances are themselves the control rules for feedback regulation. We store the membership functions for the transition relations in matrices P_i and C_j, in which rows the numerical values in [0,1] are the chosen membership functions.

Step 3: The fuzzification procedure undertakes the rule generation so that the crisp transitions between the regions in the state space can be smoothed out. To each L_{mn} is assigned as many elements as accuracy and precision can allow. In a practical sense, five to seven elements are suitable for fuzzification of L_{mn} in the state space. Membership functions may be triangular or of simple functional type. The trade-off's between the number of quantization in Lmn and the transition smoothness, the total numbers of X_{1m}, X_{2n} and the performance are important and these issues are related to heuristics. For each rule, numbers between 0 and 1 are stored for each vector array of one membership function. In our example, a two-input two-output fuzzy controller has two vector arrays for the conditional parts and two vector arrays for the action parts for each rule. The fuzzy sets for control inputs δ_i and θ_j are denoted as Δ_i and Θ_j, respectively while χ_1 and χ_2 are the fuzzy sets for x_1 and x_2, respectively.

2.3 Fuzzy Inference Using Decomposition of Fuzzy Hypercubes

For practical purpose, a discrete version of fuzzy controllers is needed and it is convenient if we utilize a decomposed fuzzy hypercube [7,8] which is suitable for implementing a fuzzy logic controller with the cell-state mapping concept. Each rule numerically stored in a fuzzy hypercube corresponds with each cell in the cell-state. For each rule, one membership function in X_{1m} is stored in the premise matrix no.1, P_1, and so is another in the premise matrix no.2, P_2, and so on. Each row in P_1 or P_2 is the membership function obtained in the stepwise procedure stated earlier. The same is true for the consequence matrices, C_1 and C_2, representing Δ_i and Θ_j for each rule. Let the max-min product be denoted as "$\circ$", then for given fuzzy sets χ_1 in X_{1m} and χ_2 in X_{2n}, the control input fuzzy sets, Δ and Θ, are obtained as

$$\Delta = C_1^T \circ \{ (P_1 \circ \chi_1) \otimes (P_2 \circ \chi_2) \} \quad (8a)$$
$$\Theta = C_2^T \circ \{ (P_1 \circ \chi_1) \otimes (P_2 \circ \chi_2) \} \quad (8b)$$

where C_i^T is the transpose of the matrix C_i and "$\otimes$" is the element-wise minimum operator. The crisp results of Δ and Θ are

$$\delta = \text{DEFUZZIFIER}(\Delta) \quad (9a)$$
$$\theta = \text{DEFUZZIFIER}(\Theta) \quad (9b)$$

where DEFUZZIFIER(.) is a defuzzification operator chosen among the maximum criterion method, the mean of maxima procedure, and the centroid algorithm.

3. Application: Design of An Engine Idling Speed Fuzzy Controller

Simulation Model: The well-known model for engine idling speed control has been rigorously studied in [9,10]. For simulation input-state training pairs, we collect the data exhaustively for the given fixed control values from the following nonlinear model with uncertainty: Let x_1=N (Engine Rotor Speed [rpm]), x_2=P (Manifold Pressure [kPascal]).

Rotating Dynamics:

$$dx_1/dt = K_n\,(T_i(x_1,\ \delta,\ m_{ao}(t-\tau)) - T_L(x_1, Td)) \quad (10a)$$

Manifold Dynamics:

$$dx_2/dt = K_p\,(m_{ai}(x_2,\ \theta) - m_{ao}(x_1, x_2)) \quad (10b)$$

Equations (10) are highly nonlinear two state engine model for idle speed control. We will obtain cell-state transitions from the above model in order to derive fuzzy logic control rules that stabilize and regulate the state trajectories toward the goal state, and in our case, the goal is N = x_1 = 750.0 (rpm), P = x_2 = 34.0 (kPascal). It is noted that $L^* = L_{43}$ in the state trajectories where the interval of X_{14} is 750.0±166.67/2, and that of X_{23} is 34.0±15.0/2.

Cell-State Transitions: As in Step 1, we gathered 9 kinds of the complete state trajectories by using 9 fixed controls from (δ_1, θ_1) to (δ_3, θ_3). Every initial state starting from the representative positions in the cell-state space is collected for the cell-state transition table. We can find an equilibrium cell with fixed $(\delta_2, \theta_2) = U_{22}$ in $L^* = L_{43}$. The next step (Step 2) is to find a chain of connections among the cells with the assigned controls according to the chosen optimal strategy. This procedure is the most important one in the design of fuzzy logic controllers.

Results: The finalized 30 control rules are determined by using the backward chaining algorithm in Figure 2 where only 10 of them are shown. L_{43} is the root of N-ary tree in the backward tracking search algorithm. In our example, 5 elements are assigned to each cell, and the support of each membership function has 7 elements, thus making 2 overlapping elements (Step 3). In Figures 3-a and 3-b, the responses of the minimum squared error (MSE) and the minimum control effort (MCE) control results are shown with the inferred control actions together. In Figure 3-b, (a) and (b) are the idle speed control results for MSE and MCE, respectively, while (c) and (d) represents the throttle angles for MSE and MCE.

4. Conclusions

With a two-input two-output multivariable fuzzy logic control scheme for the automated design of a fuzzy controller rule base, we can easily generalize the systematic procedure for a m-input n-output multivariable fuzzy control system. The automated production design of fuzzy logic control rule bases for different optimal control strategies and the associated simulation results ensure **versatility** and **flexibility** of the proposed **cell-state transition method**. Emphasis is placed upon the fact that, for given arbitrary systems, we can make fuzzy logic based control rule bases that **stabilize** the closed-loop feedback control systems, and that the design procedure is totally **automated**. Furthermore, the rules are determined according to the chosen **optimal strategy**. Numerical simulation results strongly suggest the automated rule design of fuzzy logic controllers for uncertain dynamic systems as a promising controller design paradigm for intelligent control.

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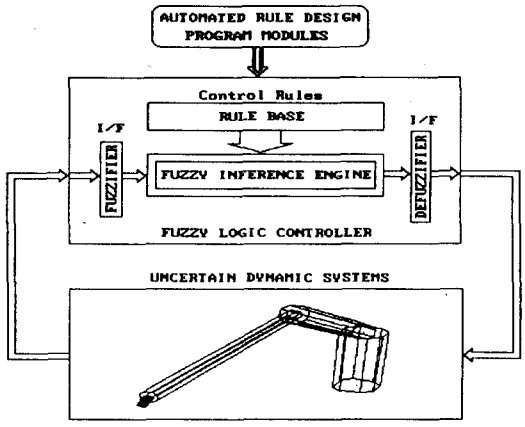


Figure 1. Block Diagram of Fuzzy Control System

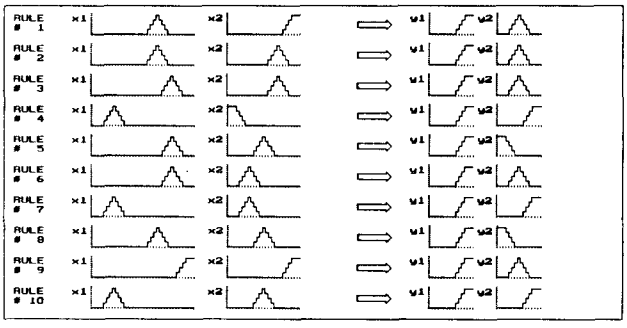


Figure 2. 10/30 Rules Derived MCE Rules by Automated Design

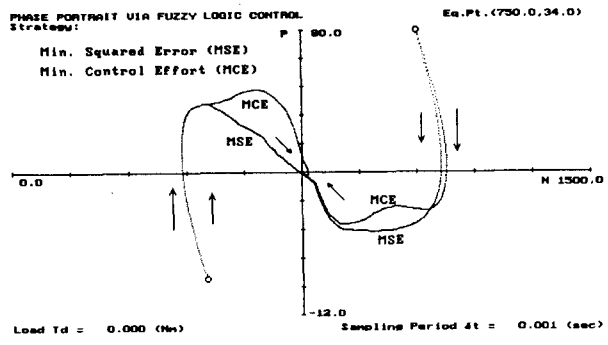


Figure 3-a. Simulation Results MCE vs MSE: Phase Plot

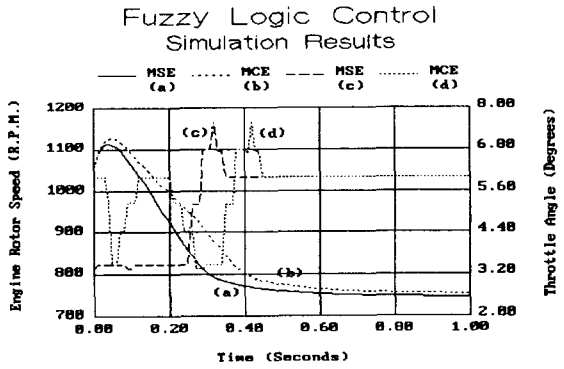
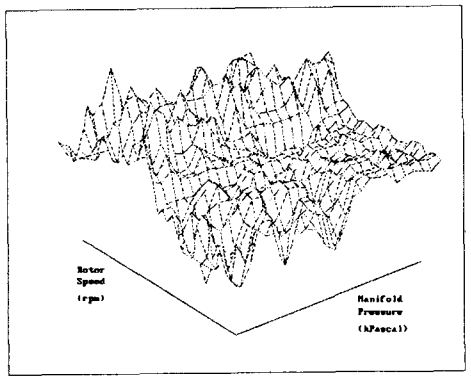
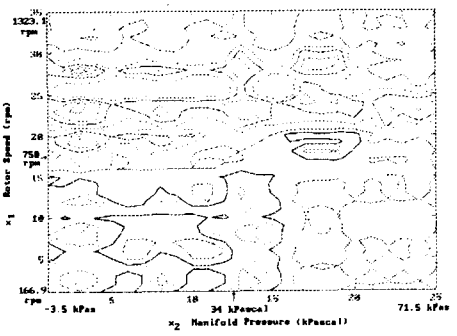


Figure 3-b. Simulation Results MCE vs MSE: Time Responses



(a) Three Dimensional Graphic Representation of Function - Throttle vs. Rotor Speed & Manifold Pressure



(b) Contour Map of Throttle Angle vs. Rotor Speed & Manifold Pressure

Figure 4. Three Dimensional Graphics of Action (θ vs. x_1 & x_2)