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Optimum number of Fuzzy Labeling and Control Performance for Fuzzy Control.

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Abstract: We consider a fuzzy controller corresponding to PI controller. This controller is applied to a controlled object which is a first order lag system with dead time. An antecedent part is divided into 3,5, and 7 parts (membership function of triangle shape), and a consequent part into 3,5, and 7 parts (membership function of singleton).
In each combination of an antecedent part and a conseqent one . We compare control efficiency under the performance criteria such that the overshoot is kept 20% and the ITAE index is minimized.

1. Introduction

In constructing fuzzy control systems, the control performance is influenced considerably by control rules, membership functions, scaling factors, and so on, whose are structural elements of fuzzy inference.

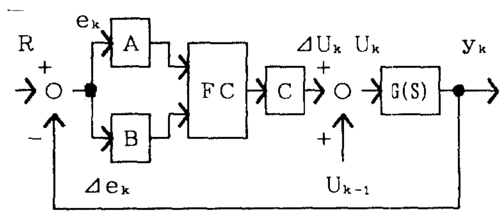
It has been an unresolved problem how the optimum control system using fuzzy control depends on number of divisions of an antecedent part and a consequent part, and also their combinations, as well as scaling factors. The purpose of the present paper is to determine the optimun scaling factors and the optimum combination of number of divisions.

In this paper, we consider a fuzzy controller as the one corresponding to PI controller. This controller has been applied to a controlled object which has a first order lag system with dead time. There are 3, 5, and 7 divisions for both an antecedent part and a consequent one. Membership functions are a triangular type for the former part and a singleton for the latter one.

Our performance criterion is that, keeping the overshoot at 20%, the ITAE index is minimized for a step response. Under the criterion, we search for an optimum combination of scaling factors so that the performance criterion with respect to step response is minimized for each combination of divisions, and then determine the optimum combination of divisions by comparing these results.

2. Fuzzy control system

The fuzzy control system used in this study is shown in Fig.1.



FC: Fuzzy Controller G(S):Plant

Fig.1. Block diagram of a fuzzy control system.

Notation

R : Reference.

U_k: Manipulated variable at k-th sampling time.

y_k: Controlled variable at k-th sampling time

$e_k(=R-y_k)$: Error at k-th sampling time.

$\varDelta e_k(=e_k-e_{k-1})$: First order difference of error at k-th sampling time.

$\varDelta U_k(=U_k-U_{k-1})$: First order difference of manipulated variable at k-th sampling time.

A: Scaling factor for e_k.

B: Scaling factor for $\varDelta e_k$.

C: Scaling factor for $\varDelta U_k$.

e_k and $\varDelta e_k$ are variables for an antecedent part. $\varDelta U_k$ for a consequent one.

Control rules are described as follows,

R_1: If e_k is P and $\varDelta e_k$ is P then $\varDelta U_k$ is P.

R_2: If e_k is N and Δe_k is N then ΔU_k is N

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:

The numbers of divisions for an antecedent part and a consequent one,and the number of those combinations are shown in Table 1. The control rules for each combination are shown in Table 2.

The membership functions are shown in Figs.2 and 3.

Table 1. Numbers of divisions and those conbination names

	Consequent parts		
Antecedent parts	3	5	7
3	3*3	3*5	☆
5	5*3	5*5	5*7
7	7*3	7*5	7*7

Table.2 Control rules.

3*3

e \ Δe	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P

3*5

e \ Δe	N	Z	P
N	NB	NS	Z
Z	NS	Z	PS
P	Z	PS	PB

5*3

e \ Δe	NB	NS	Z	PS	PB
NB	N	N	N	N	Z
NS	N	N	N	Z	P
Z	N	N	Z	P	P
PS	N	Z	P	P	P
PB	Z	P	P	P	P

5*5

e \ Δe	NB	NS	Z	PS	PB
NB	NB	NB	NB	NS	Z
NS	NB	NB	NS	Z	PS
Z	NB	NS	Z	PS	PB
PS	NS	Z	PS	PB	PB
PB	Z	PS	PB	PB	PB

5*7

e \ Δe	NB	NS	Z	PS	PB
NB	NB	NB	NM	NS	Z
NS	NB	NM	NS	Z	PS
Z	NM	NS	Z	PS	PM
PS	NS	Z	PS	PM	PB
PB	Z	PS	PM	PB	PB

7*3

e \ Δe	NB	NM	NS	Z	PS	PM	PB
NB	N	N	N	N	N	N	Z
NM	N	N	N	N	N	Z	P
NS	N	N	N	N	Z	P	P
Z	N	N	N	Z	P	P	P
PS	N	N	Z	P	P	P	P
PM	N	Z	P	P	P	P	P
PB	Z	P	P	P	P	P	P

7*5

e \ Δe	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NB	NS	NS	Z
NM	NB	NB	NB	NS	NS	Z	PS
NS	NB	NB	NS	NS	Z	PS	PS
Z	NB	NS	NS	Z	PS	PS	PB
PS	NS	NS	Z	PS	PS	PB	PB
PM	NS	Z	PS	PS	PB	PB	PB
PB	Z	PS	PS	PB	PB	PB	PB

7*7

e \ Δe	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	Z
NM	NB	NB	NM	NM	NS	Z	PS
NS	NB	NM	NM	NS	Z	PS	PM
Z	NM	NM	NS	Z	PS	PM	PM
PS	NM	NS	Z	PS	PM	PM	PB
PM	NS	Z	PS	PM	PM	PB	PB
PB	Z	PS	PM	PM	PB	PB	PB

NB:Negative Big.
NM:Negative Medium.
NS:Negative Small.
N :Negative.
Z :Zero.
P :Positive.
PS:Positive Small.
PM:Positive Medium.
PB:Positive Big.

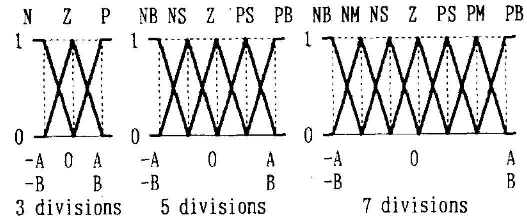

Fig.2. Membership functions $\mu_{Ai}(e_k)$ and $\mu_{Bi}(\Delta e_k)$ of antecedent parts.

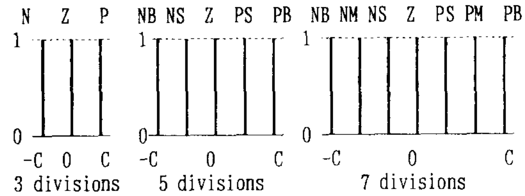

Fig.3 Membership function of consequent parts.

If non-fuzzy values e_k and Δe_k are applied to the fuzzy control device, an output non-fuzzy value ΔU_k is inferred as follows.

First, the matching grade ω_i of an antecedent part for i-th rule are calculated as eq.(1),

$$\omega_i = \min\{\mu_{Ai}(e_k), \mu_{Bi}(\Delta e_k)\} \tag{1}$$

were A_i and B_i are fuzzy label of i-th rule for e_k and Δe_k, respectively.

If the maximum value of grades $\omega(C_j)$ in each label for a consequent part is determined,then ΔU_k is inferred as the next equation byusing the maximum average method with weight.Were C_j is value of singleton for a consequent part.

$$\Delta U_k = \frac{\sum_{j=1}^{n} \omega(C_j) \cdot C_j}{\sum_{j=1}^{n} \omega(C_j)} \tag{2}$$

n:Number of divisions for a consequent part

3. Optimization of scaling factors B and C.

We describe the searching method of an optimum scaling factors. An above-mentioned performance criterion DE is defined as in the next equation.

$$DE = \int |e(t)| \cdot t \cdot dt + K \cdot |PO-20| \tag{3}$$

where PO is the percent overshoot for a step response, and K is weight.

In eq.(3), the first term of right side is the ITAE index. This is designated the integral of time multiplied by an absolute value of an error. There are some other performance indexes for a step response. We choose the ITAE index in order to increase a response rate.

Scaling factors to be tuned are A,B and C. However, A is always assumed to be coincident with a reference, because the maximum values of an error never exceeds a reference. Scaling factors B and C are tuned so that DE for a response is minimized. Our tuning technique is the simplex method. This method is applied for minimizing the wide range of functions.

Using this method, first, appropriate values are fed as initial values of B and C. Second, we search for values near optimums B and C by simulation. Finally, based on these values, optimums B and C are to be determined by repetition of simulation.

4. A control performance due to number of divisions and those combinations.

We assumed the plant in our simulation as;

$$G(S) = \frac{K \cdot e^{-LS}}{TS+1} \quad (4)$$

where K,T and L denote the gain, the time constant and the dead time,respectively.

We determine the conditions for the simulation as:

(1)Both reference R and gain K are assumed to be unity.

(2)$1s \leqq L \leqq 16s$, $0.1s \leqq T \leqq 40s$.

(3)sampling time is 0.1s.

(4)scaling factor A for an antecedent part is assumed to be unity.

(5)time interval for simulation is 300s.

Under these conditions, we examined scaling factors B,C giving an optimum response. Results are shown in Table 3. In Table 3, DE is a performance criterion, RT is reaching time; the duration of the increasing controlled variable up to reference. From Table 3, it is found that 3*3 combination gives the best control performance within above-mentioned conditions. Some examples of time responses are shown in Figs. 4 and 5. As well as these figures, it is found that 3*3 combination gives the best control performance.

Table 3. Scaling factors and performance criterion for combinations.

L= 1	R=K=1						
		T= 0.1	T= 1	T= 5	T= 10	T= 20	T= 40
3*3	B	0.2242	0.098938	0.049847	0.038994	0.036856	0.031339
	C	0.094524	0.093988	0.199066	0.360215	0.607049	1.24613
	DE	23.03322	47.38628	77.29352	71.80504	82.45598	113.0883
	RT	2.2	3.1	3.8	3.9	4.1	4.1
3*5	B	0.373903	0.135161	0.050221	0.043172	0.038766	0.034511
	C	0.167075	0.154807	0.291446	0.581213	1.093344	2.233372
	DE	27.5418	64.364	109.106	108.5821	93.90521	115.4359
	RT	2.2	3.3	4.4	4.2	4.2	4.2
5*3	B	0.649067	0.129243	0.053195	0.045444	0.042243	0.039942
	C	0.055628	0.072348	0.122222	0.236367	0.464420	0.921287
	DE	61.05125	92.41287	105.0442	100.2793	100.0559	128.6138
	RT	3.1	3.5	4.5	4.4	4.3	4.3
5*5	B	0.339718	0.122556	0.052006	0.043217	0.038056	0.034641
	C	0.082648	0.076287	0.015466	0.300533	0.573642	1.129039
	DE	28.50607	63.08721	110.6286	113.9794	96.75591	119.463
	RT	2.4	3.5	4.3	4.2	4.2	4.3
5*7	B	0.345551	0.121376	0.052036	0.042953	0.038128	0.034493
	C	0.122954	0.115129	0.230567	0.376832	0.854919	1.694714
	DE	27.50053	61.61873	108.7870	121.3502	94.29982	117.1445
	RT	2.3	3.4	4.3	4.5	4.2	4.2
7*3	B	∞	0.121031	0.058466	0.050240	0.043413	0.035112
	C	0.03323	0.048333	0.090167	0.162916	0.214255	0.249317
	DE	93.5879	101.1867	131.9559	129.5014	159.9222	288.8443
	RT	4.1	4.1	5.0	5.1	6.0	7.6
7*5	B	0.253969	0.116785	0.053131	0.043701	0.039783	0.036488
	C	0.083978	0.081328	0.166917	0.288183	0.543810	1.125829
	DE	31.97574	63.43896	94.63531	96.78925	96.49487	126.4946
	RT	2.4	3.3	4.2	4.4	4.4	4.3
7*7	B	0.467003	0.151117	0.054962	0.045221	0.040972	0.039460
	C	0.097259	0.084026	0.164191	0.271211	0.569360	1.208956
	DE	40.03960	81.68269	129.2098	142.6454	122.8151	145.2941
	RT	2.7	3.9	4.7	5.0	4.8	4.6

L=4	R=K=1						
		T= 0.1	T= 1	T= 5	T= 10	T= 20	T= 40
3*3	B	0.057708	0.046760	0.021343	0.015537	0.012542	0.009824
	C	0.025930	0.022952	0.025951	0.034036	0.052602	0.094691
	DE	317.7924	444.2150	812.4327	1018.072	1199.603	1110.071
	RT	8.0	9.5	12.7	14.0	14.9	15.3
3*5	B	0.097521	0.072256	0.027183	0.017181	0.012635	0.010529
	C	0.046076	0.040985	0.042245	0.052835	0.079301	0.175804
	DE	397.9723	579.0431	1138.579	1443.604	1693.024	1484.611
	RT	8.4	10.1	14.1	15.9	17.2	15.6
5*3	B	0.157366	0.083778	0.027575	0.017730	0.013355	0.011634
	C	0.015460	0.015799	0.018871	0.022020	0.032500	0.065724
	DE	836.2134	1142.815	1380.828	1465.153	1609.596	1489.348
	RT	11.2	12.0	14.4	16.4	17.6	16.8
5*5	B	0.0750	0.062786	0.026074	0.017614	0.012996	0.010470
	C	0.0211	0.020094	0.020821	0.027289	0.041866	0.088212
	DE	388.4767	559.1998	1087.420	1413.608	1663.046	1520.256
	RT	8.7	10.3	14.1	15.5	16.6	15.7
5*7	B	0.084350	0.062615	0.026052	0.017614	0.013004	0.010462
	C	0.034009	0.030153	0.031277	0.041076	0.062815	0.132456
	DE	384.6803	554.6907	1081.384	1406.899	1657.491	1498.315
	RT	8.6	10.3	14.1	15.5	16.6	15.7
7*3	B	∞	∞	0.026823	0.018771	0.014730	0.012470
	C	0.008928	0.007497	0.012338	0.015857	0.022686	0.033508
	DE	1376.75	1965.258	1562.205	1793.612	2038.248	2378.453
	RT	15.3	18.3	16.9	18.4	20.1	22.0
7*5	B	0.068964	0.055155	0.025527	0.017871	0.013246	0.011088
	C	0.022988	0.020031	0.022613	0.031310	0.044428	0.085034
	DE	434.8604	616.4244	1078.893	1318.993	1450.042	1410.461
	RT	8.7	10.4	13.6	14.7	16.4	16.2
7*7	B	0.121826	0.085975	0.030029	0.018540	0.013749	0.011510
	C	0.026403	0.023115	0.023565	0.029738	0.045518	0.096457
	DE	556.7992	778.2609	1380.254	1674.520	1935.881	1870.233
	RT	10.1	11.9	15.7	17.2	18.1	17.1

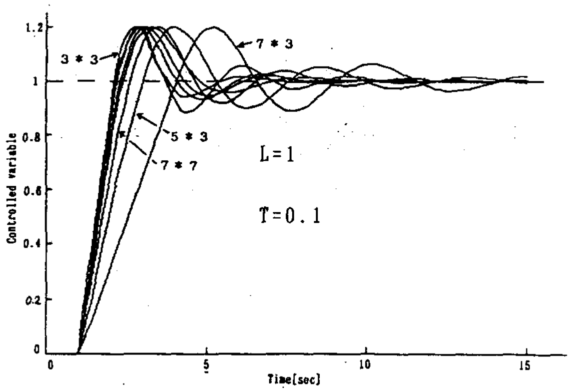

Fig.4. Time response

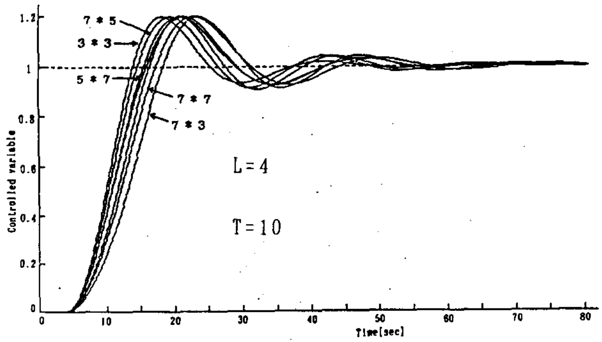

Fig.5. Time response

5. Conclusion

The purpose is to determine the optimun scaling factors and the optimum combination of number of divisions. We considered a fuzzy controller which has been applied to a controlled object having a first order lag system with dead time.

Our performance criterion is that, keeping the overshoot at 20%, the ITAE index is minimized for a step response.

Under the criterion, we searched for an optimum combination of scaling factors so that the performance criterion with respect to step response is minimized for each combination of divisions, and then determined the optimum combination of divisions by the simulation.

We have determined the value of the optimum scaling factors and it has found that 3*3 combination gives the best control performance.

The simular simulation with respect to disturbance may be carried out, and relation between a group of parameters T,L,k,R and scaling factors B,C may be formulated due to the simulation for the next step.