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Optimization of Fuzzy Relational Models

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Abstract: The problem of the optimization of fuzzy relational models for dealing with (non-fuzzy) numerical data is investigated. In this context, interfaces optimization assumes particular importance, becoming a determinant factor in what concerns the overall model performance. Considering this, several scenarios for building fuzzy relational models are presented. These are: (i) optimizing I/O interfaces in advance (independently from the linguistic part of the model); (ii) optimizing I/O interfaces in advance and allowing that their optimized parameters may change during the learning of the linguistic part of the model; (iii) build simultaneously both interfaces and the linguistic subsystem; and (iv) build simultaneously both linguistic subsystem and interfaces, now subject to semantic integrity constraints. As linguistic subsystems, both a basic type and an extended versions of fuzzy relation equations are exploited in each one of these scenarios. A comparative analysis of the different approaches is summarized.

1 Introduction

In constructing fuzzy relational models, the problem of building I/O interfaces becomes a critical issue. On one hand the meaning of the linguistic terms of these interfaces should be semantically valid, on the other hand the performance of I/O interfaces determines the overall system performance. This is of particular interest when the models have to be applied in closed-loop structures. With this respect one can look at fuzzy controllers as a particular class of fuzzy models heavily utilizing the above mentioned interfaces. If no interface optimization ever takes place some drawbacks are supposed to happen. For instance, it could happen that the model simply will not be able to match effectively the observed data. Even if the linguistic subsystem (partially) compensates the conversion errors, it could be "pushed" in such way (e.g. over trained) that it may loose some of its desirable characteristics such as its generalization capabilities. Thus some type of optimization becomes inevitable. In what concerns the linguistic subsystem of the fuzzy model, a single level relational structure under t-s composition is first exploited. Then a structure representing an extended version of fuzzy relation equation is discussed.

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2 Fuzzy relational models

Fuzzy relational models for handling (non-fuzzy) numerical data are viewed here as three block systems, consisting of (i) an input or numerical/linguistic (N/L) interface, that translates numerical data into linguistic data (fuzzy sets); (ii) a linguistic (relational) subsystem, whose both input and output are linguistic terms, and where the linguistic relationships are captured; and (iii) an output or linguistic/numerical (L/N) interface, that converts linguistic information back to numerical data.

2.1 I/O interfaces

A N/L interface can be built based on the fuzzy discretization concept. This concept enables one to address the problem the N/L interface construction as the problem of generating c reference fuzzy sets [1]. In this work, reference fuzzy sets are defined by two-parameters normal, and convex membership functions. As L/N interface, the center of gravity method is adopted.

2.2 Linguistic system

Several fuzzy relational structures have been proposed lately [3, 4, 5, 7]. For the sake of simplicity we will start with single level structures. A single level relational structure can be viewed as a collection of units processing linguistic terms (fuzzy sets) defined in respective input and output spaces – see Fig. 1. The structure encapsulates relationships between inputs and outputs at a level of reference fuzzy sets. Let X and Y be collections of activations of the reference fuzzy sets produced by the N/L interface, such as $X = [x_1 \ldots x_n]$ and $Y = [y_1 \ldots y_m]$. The output of an generic unit, say y_j, can be given by:

$$y_j = \mathrm{T}_{i=1}^{n}(x_i \; s \; r_{ij}) \tag{1}$$

where T and s stand for t-norm and s-norm respectively. The matrix formed by all r_{ij} can also be understood as a fuzzy relation. Extended versions of such structures as those proposed in [7] are also considered.

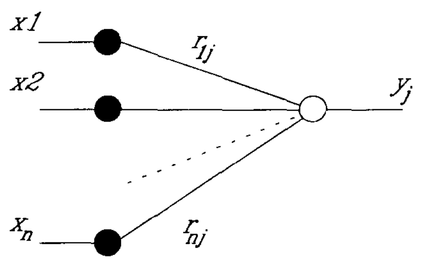

Figure 1: Unit of single level relational structure.

3 Optimization scenarios

Four different scenarios for optimizing fuzzy relational models are analyzed:

A: Optimizing I/O interfaces in advance and separately from the linguistic subsystem. That is regarding the interfaces as standalone subsystems providing a (nearly) zero conversion error;

B: Optimizing I/O interfaces allowing however that their optimized parameters may change during the learning process of the linguistic system;

C: Optimizing simultaneously both the I/O interfaces and the linguistic subsystem. This scenario correspond to the usual optimization process of fuzzy systems reported in the literature;

D: Optimizing simultaneously both the linguistic subsystem and the interfaces subject to some integrity constraints.

The required interface optimization for scenarios A and B is provided by the PAFIO algorithm [8]. This algorithm optimizes the parameters of membership functions such as an equivalence of information processed by a series of N/L and L/N interfaces is achieved. More generally, the construction of the model is based on an adaptive version of supervised learning where both the fuzzy relations as well as the reference fuzzy sets are optimized. In general, the learning process is driven by a standard performance index, the MSE criterion:

$$Q = \sum_{j=1}^{N}(t_j - y_j)^2$$

where t_j is the j-th target output of the model and y_j is its actual value.

4 Numerical considerations

Let us discuss the optimization of a fuzzy model using learning data coming from the following static non-linearity $t(x) = \frac{1}{1+e^{-x}}$, where x is a non-uniformly distributed sequence with 50 points [6]. Furthermore, it is assumed that both the input and the output linguistic terms are of interest. Notice that in [6] only the input linguistic terms were assumed as interesting. In the following, three linguistic terms at both interfaces modelled by Gaussian membership functions are used. Refer to [6] for a discussion on the impact of different shapes of membership functions. Here we are concerned with both the performance of the optimized models, and the semantic meaning of their interfaces. The optimization efficiency is also to be considered. For assessing these features, a model is built according to each scenario, and then validated by a testing data set with 121 points. From Fig. 2 one can visualize a perfect cross validation performance for all models but for the model derived with scenario A.

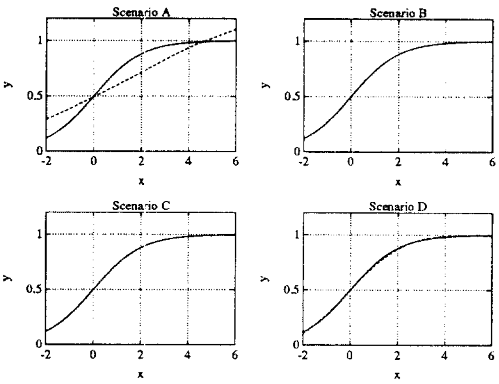

Figure 2: Cross validation for systems obtained from different scenarios: solid line – target; dashed line – model output

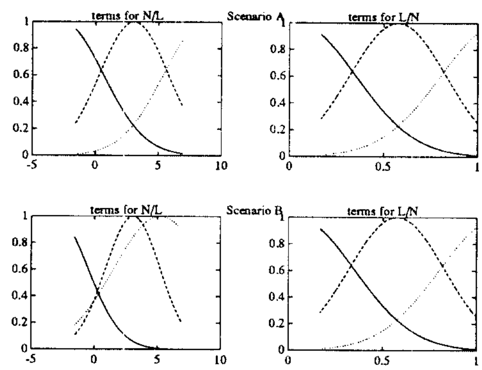

Figure 3: Resulting linguistic terms: scenarios A and B.

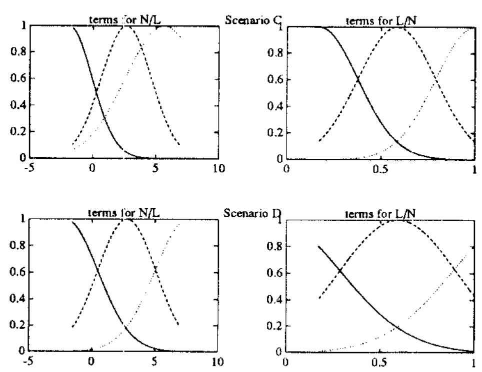

Figure 4: Resulting linguistic terms: scenarios C and D.

Looking for the resulting interfaces (Figs. 3 and 4) one can see that scenarios A and D produce linguistic terms semanticly meaningful, that can be easily termed *small*, *medium*, and *large*, for instance. By the opposite, those input linguistic terms generated by scenarios B and C (they produce nearly the same results) hold a meaning that is difficult to figure out. This has also its implications from a information processing point of view: some points of the UoD (e.g. $x = 2$) have a higher value of possibility (they are more "important") than others (e.g. $x = -2$) without any other reason than to compensate the lack of "capacity" of the linguistic subsystem.

This lack of "capacity" is also the reason for the poor performance of the model derived from A. To overcome this drawback we should select from the family of optimal interfaces one that also optimizes the overall system performance, or what is the same, optimize the model subject to the constraint of optimal interfaces. This is what scenario D is aiming at. Notice that the interfaces optimized by scenarios A and D are both optimal. Obviously, if we use as linguistic subsystem an extended relational structure such as those proposed in [7], where there is more degrees of freedom, a fuzzy model derived from scenario A can perform successfully – see Fig. 5. In such extended structures the output of a generic unit, say y_j, is given by:

$$y_j = \mathrm{T}^{n_1}_{j_1=1} \dots \mathrm{T}^{n_n}_{j_n=1}(x1_{j_1} s \dots s xn_{j_n} s r_{j_1,\dots,j_n,j}) \tag{2}$$

From an optimization efficiency perspective, scenario B is the most computationally demanding, while scenario A is the most efficient. Scenarios C and D require nearly the same computa-

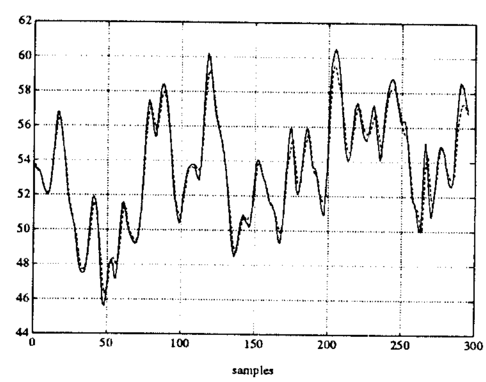

Figure 5: Performance of scenario A for an extended fuzzy relational structure: solid line – target; dashed line – model output

tion effort.

5 Conclusions

The optimization of fuzzy relation models for dealing with numerical data, where the meaning of its linguistic terms is relevant, was summarized. Four different optimization strategies, including the conventional one (scenario C), were considered. It was argued that the optimization simultaneous of both I/O interfaces and the linguistic subsystem with no integrity constraints can generate meaningless linguistic terms. Therefore, for small "capacity" linguistic subsystems, scenario D should be used while for larger linguistic subsystems scenario A can be advantageous over scenario D in terms of efficiency.

Finally it should be stressed that the use of fuzzy relational models allow us to generalize this study to those fuzzy systems embedded in that class.

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