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FUZZY CONTROL AS INTERPOLATION

B. Kovalerchuk

H. Yusupov

Uzinform, 8 Druzhby narodov St. Tashkent, 700043,

Uzbekistan, CIS E-mail boris@apb.tashkent.su

Abstract.

The purpose of the paper is to explain some heuristic, common sense suppositions of fuzzy control.

It is shown that Fuzzy Control is a kind of quasilinear interpolation of prototypes. Control function can be sufficiently exact represented as piecewise-linear function. The best interpolation is connected with normalized intersected fuzzy sets.

1. Introduction.

Spectacular success is reached in fuzzy control applications now. But we have no any guarantee that it will be stable in new applications.

At the first glance it looks strange. But the facts are as follows. P. Cheeseman in his deep paper [1,p.101] concludes :"The main reason for the successful use of the fuzzy approach, particularly fuzzy set theory, seems to come from a careful choice of application domain and the use of a great deal of common sense in its interpretations. Unfortunately for AI, this common sense is not well formalized, so naive attempts to use the fuzzy approach in AI programs could lead to spectacular failures".

Main reason here is very substantial influence of skills of designers on the result. We can refer to the history how people tried to build bridges. Some ancient bridges we can use till now, but many of bridges had failures (in direct sense) during the building period. Principal improving began with progress in Newton theoretical mechanics and study of strength of materials, i.e. after many centuries.

So we suppose that we have two ways:

(w1) to give to newcomers informalized skills of fuzzy control;

(w2) to develop the analog of theoretical mechanics and study of strength of materials.

Second way was impossible in ancient time. Now as we suppose there is a base to develop its analog for fuzzy control. Unfortunately attention to this matter is very insufficient.

The purpose of the paper is to EXPLICATE SOME HEURISTIC , COMMON SENSE SUPPOSITIONS of fuzzy control as a preliminary step to the second way.

To realize our purpose we formulate and prove the general statement (G).

(G) General statement. FUZZY CONTROL IS A KIND OF QUASILINEAR INTERPOLATION OF KNOWN POINTS. Control function v=f(a) can be sufficiently exact represented as PIECEWISE-LINEAR FUNCTION.

This statement explicates one of the main heuristic ideas of fuzzy control.

We call observed points as KNOWN POINTS, if it is known control for them. For example, for the point a=1 (the angle of deviation of pendulum) is given control v=1 (the velocity of pivot) (a=1 6 v=1). Here a is a known point. In pattern recognition terms this point is called a PROTOTYPE. For fuzzy set "normal temperature of body", the temperature 36,6 C is a prototype. Different correspondences are possible between fuzzy sets included in fuzzy control system.
They define a kind of interpolation, its exactness and completeness. Characteristics of INTERSECTION of fuzzy sets are most important for interpolation.

In the paper we consider following characteristics:
1. empty/ non-empty intersection;
2. normalized/ non-normalized intersection.

Normalized intersection is most popular in fuzzy control (see fig. 3).

One of the main conclusion of the paper is that NORMALIZED INTERSECTION gives the BEST INTERPOLATION. It explicates and explains popularity of normalized intersection.

2. Description of problem

We show correctness of general statement (G) on the example below. This result can be generalized for more complicated "if- then" rules.

Let us consider the example:
a is deviation angle of inverted pendulum, installed on a vehicle, A={a};
v is a velocity of a vehicle, V={v}.

There are given linguistical "if-then" rules:

If a is PL then v is PL (1)
If a is PM then v is PM (2)
If a is PS then v is PS (3)
If a is ZR then v is ZR (4)
If a is NM then v is NM (5)
If a is NS then v is NS (6)
If a is NL then v is NL (7)

Here, NL is negative large, NM is negative medium, NS is negative small, ZR is approximately zero, PS is positive small and PM is positive medium.

In usual natural language rule (2) means :"If the angle ais positive medium then the velocity of vehicle v must be positive medium".

Let us introduce membership function m(a/A,PS) for angles A and linguistical label PS (positive small). Analogously we introduce m functions for other pairs as:

m(v/,V,NL), m(a/A,PL).

Let us define known points through membership functions. We consider point a under condition (8)

m(a/A,PS)=1. (8)

Let us denote it as a(PS). Analogously we define v(PS). From (3) we have a(PS)=>v(PS). So a(PS) is a prototype (known point).

Let us suppose that scales for A and V are normalized, i.e. points under condition (8) have numerical values from fig 3. Here NL=-3; NM=-2; NS=-1; ZR=0; PS=1; PM=2; PL=3.

FUZZY CONTROL PROBLEM is to find control v . Information for that is: (1)-(7) , membership functions as

m(v/V,PM) , m(a/A,PS)

for arguments a, which are not a prototype. For prototype a=1 it is very simple to find v (see (2)).

In the paper we use standard fuzzy control scheme to obtain v (max min strategy of intersection and union of fuzzy sets and defuzzification by center of gravity).

3. Results

Main results are represented in fig.1,2,3 for given example. We consider three kinds of representation of set linguistical terms {NL,NM,NS,ZR,PS,PM,PL}:

(R1) non-intersected intervals (fig.1). Linguistical terms are represented by usual intervals without fuzzy sets.

(R2) partly-intersected fuzzy sets (fig.2)

(R3) normalized-intersected fuzzy sets (fig.3). Maximum of m of one fuzzy set is a minimum of the next.

For (R1) we obtained rough interpolation for intermediate points, which are not known points. Known points are {-3;-2;-1,0; 1;2;3}.

In the case (R2) interpolation for intermediate points is better. In the case (R3) interpolation for intermediate points is best. In the table 1 there are given deviations of centers of gravity from linear interpolation of known points a1=1 and a2=2. For them v1=1 and v2=2, correspondently.

Table 1.

Argument	m1 value	Center of gravity	Linear interp.	Deviation
a	m1	Cq	La	Cg-La
1.10	0,90	1,133	1,10	0,033
1.20	0,80	1,241	1,20	0,041
1.30	0,70	1,335	1,30	0,035
1.40	0,60	1,419	1,40	0,019
1.50	0,50	1,500	1,50	0,033
1.60	0,40	1,581	1,60	-0,019
1.70	0,30	1,665	1,70	-0,035
1.80	0,20	1,759	1,80	-0,041
1.90	0,10	1,867	1,90	-0,033
2.00	0.00	2.000	2.00	0,000

As we see from table 1 deviation is no more than 5% of half of support of considered fuzzy set PS for v. The length of support is equal 2. If we consider deviation as a share of support it will be no more than 2,5% of it.

It proves our statement about quasi-linear interpolation for each nearest known points. It means the existence of piecewise-linear function v=f(a) with good interpolation V on all interval [-3; +3] of A. This function is represented in fig. 3.

Analogously it is possible to show

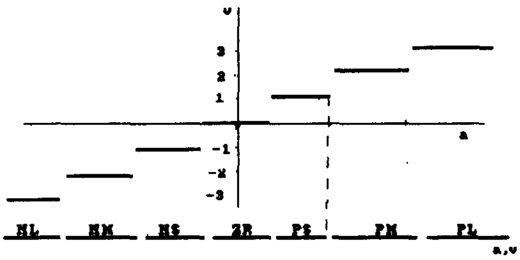

FIG. 1. INTERPOLATION FOR NON-INTERSECTED INTERVALS.

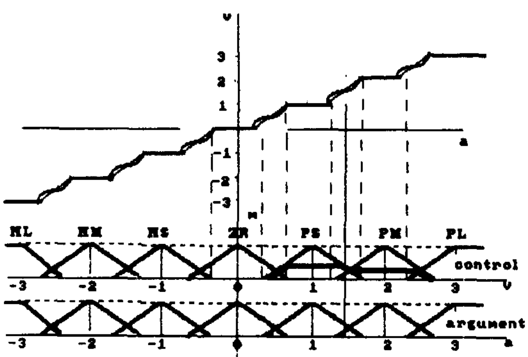

FIG. 2. INTERPOLATION FOR PARTY-INTERSECTED FUZZY SETS.

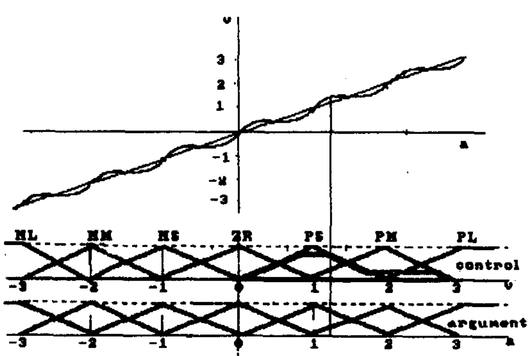

FIG. 3. INTERPOLATION FOR NORMALIZED INTERSECTED FUZZY SETS.

the same for "if-then" rules with two, tree and more antecedents. It will be multi-dimensional piecewise-linear interpolation. After this explanation one of the main common sense ideas of fuzzy control we have possibility to test and to improve control function f(v)=a. For this purpose here it is possible to use achievements of interpolation and pattern recognition theories to construct more strict fuzzy control theory with less part of non-formalized skills.

The next problems of interpolation approach are to develop methods:

1) to construct piecewise-linear interpolation in one- and multi-dimensional spaces;

2) to realize piecewise-linear interpolation in hardware effectively;

3) to test rules and control function f(v)=a;

4) to find contradictions among rules;

5) to improve control function f(v)=a;

6) to tune of membership functions;

7) to supply and preserve normalized intersection;

8) to minimize the number of rules.

In the report we develop some ideas to decide these problems. Detailed study is the topic of the further investigation.

References.

[1] P. Cheeseman. Probabilistic versus fuzzy reasoning. In L. N. Kanal and J. F. Lemner, eds. Uncertainty in Artificial Intelligence, pp 85-102, Elsevir Sci. Publ. B. V. North Holland, 1986.