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A Multivariable Fuzzy Control System with a Coordinator

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ABSTRACT

For the design of multivariable fuzzy control systems the decomposition of control rules is preferable since it alleviates the complexity of the problem. In some systems, however, inference error of the Gupta's decomposition method is inevitable because of its approximate nature. In this paper, we propose a new multivariable fuzzy controller with a coordinator which can reduce the inference error of the decomposition method by using an index of applicability.

I. INTRODUCTION

Since the fuzzy set theory was introduced by Zadeh[1], the fuzzy logic has been used successfully in a number of control systems with single input and single output[2,3]. However, the complex industrial processes are of multivariate nature. It is difficult to infer the proper control input for these systems since the dimension of the relation matrix composed of the control rules is very large. The high dimensionality of these relation matrices accompanies computational difficulties. Therefore, recent research on the application of fuzzy set theory to the design of control systems is directed to the descriptions of the multivariable structure of these systems. Shakouri et al.[4] suggested a fuzzy control algorithm for a multivariable system which was represented by a conventional state space model. Gupta et al.[5] proposed a fuzzy control algorithm by which the multivariable fuzzy system is decomposed into a set of one-dimensional systems. However, the Gupta's decomposition algorithm in some cases results in a large inference error due to its approximate nature.

To reduce the inference error, the authors have defined an index of applicability[6] which can classify whether the Gupta's method can be applied to a multivariable fuzzy system or not. In this paper we propose a new fuzzy controller with a coordinator which can reduce the inference error of the decomposition method by using the index of applicability. An example of the closed-loop fuzzy control system is given to compare the results of two control methods.

II. OUTPUT INFERENCE FROM TWO-DIMENSIONAL RELATION MATRICES

Consider a multivariable fuzzy system with N inputs and M outputs. Suppose that K control rules of the system are given by

$$
\begin{aligned}
&IF\ X_1\ is\ X_{1(1)}\ AND \cdots X_N\ is\ X_{N(1)}\ \ THEN\ \ Y_1\ is\ Y_{1(1)}\\
&AND\ \cdots\ Y_M\ is\ Y_{M(1)}\\
&ALSO\ \cdots\\
&IF\ X_1\ is\ X_{1(k)}\ AND \cdots X_N\ is\ X_{N(k)}\ \ THEN\ \ Y_1\ is\ Y_{1(k)}\\
&AND\ \cdots\ Y_M\ is\ Y_{M(k)}\\
&ALSO\ \cdots\\
&IF\ X_1\ is\ X_{1(K)}\ AND \cdots X_N\ is\ X_{N(K)}\ \ THEN\ \ Y_1\ is\ Y_{1(K)}\\
&AND \cdots Y_M\ is\ Y_{M(K)}. \qquad (1)
\end{aligned}
$$

where the inputs and the outputs are normalized fuzzy sets, $X_{N(k)}$ is the fuzzy value of the N-th process input in the k-th control rule, and $Y_{M(k)}$ is that of the M-th process output in the k-th rule. From control rules established by experts' experience, a relation matrix which is necessary to the inference of the m-th output is defined as

$$R_m = \bigvee_{k=1}^{K} \{ X_{1(k)} \wedge X_{2(k)} \wedge \cdots \wedge X_{N(k)} \wedge Y_{m(k)} \} \qquad (2)$$

$$1 \le m \le M.$$

where $\vee$ is the max-operator and $\wedge$ is the min-operator.

Given the current inputs $X_1, X_2, \cdots, X_N$, the present m-th output Y_m is computed by the following compositional rule[7].

$$Y_m = X_1 \circ X_2 \circ X_3 \circ \cdots X_N \circ R_m \tag{3}$$

where $\circ$ is the max-min composition of fuzzy relations. When the number of system inputs is large a straight inference which is expressed by (3) accompanies computational difficulties such as computation time and storage space because of the high dimensionality of relation matrix (2).

To overcome this problem, Gupta et al.[5] proposed a decomposition method which is composed of two dimensional relation matrices. Its matrix form is represented as follows.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix} = [\, X_1 \, X_2 \cdots X_N \,] * \begin{bmatrix} R_{11} & \cdots & R_{1M} \\ R_{21} & \cdots & R_{2M} \\ \vdots & \ddots & \vdots \\ R_{N1} & \cdots & R_{NM} \end{bmatrix} \tag{4}$$

where $*$ is the $(\circ, \wedge)$ operator. From (4), the m-th output of the system is expressed by the $(\circ, \wedge)$ operator as

$$Y_m = X_1 \circ R_{1m} \wedge X_2 \circ R_{2m} \wedge \cdots \wedge X_N \circ R_{Nm} \tag{5}$$

and

$$R_{nm} = \overset{K}{\underset{k=1}{\vee}} \{ X_{n(k)} \wedge Y_{m(k)} \} \tag{6}$$

Ⅲ. AN INDEX OF APPLICABILITY FOR THE DECOMPOSITION OF CONTROL RULES

Given the current inputs X_1 and X_2, the output computed by Gupta's decomposition method has the following inequality.

$$\begin{aligned} Y &= X_1 \circ R_{11} \wedge X_2 \circ R_{21} \\ &= X_1 \circ A_2 \circ \{ \overset{K}{\underset{k=1}{\vee}} X_{1(k)} \wedge A_{2(k)} \wedge Y_{(k)} \} \\ &\quad \wedge A_1 \circ X_2 \circ \{ \overset{K}{\underset{k=1}{\vee}} A_{1(k)} \wedge X_{2(k)} \wedge Y_{(k)} \} \\ &\geq X_1 \circ X_2 \circ \{ \overset{K}{\underset{k=1}{\vee}} X_{1(k)} \wedge X_{2(k)} \wedge Y_{(k)} \} \end{aligned} \tag{7}$$

where A_1 and A_2 are fuzzy sets whose membership function is identical to one over the universe of discourse, and the last term of (7) is the inference output from the original relation matrix. This equation shows the possible inference error generated by the decomposition method.

In order to quantify the inference error generated by the decomposition method, an index of applicability[6], C is employed to judge whether the decomposition method of control rules can be applied to the multivariable fuzzy system or not. The index has been defined as

$$C = \frac{1}{M} \sum_{m=1}^{M} C_m, \qquad 0 \leq C_m < 1 \tag{8}$$

and

$$C_m = \frac{\sum_{i_1, \cdots, i_N, j_m} \{ \overset{N}{\underset{n=1}{\wedge}} P_{nm} - R_m(i_1, i_2, \cdots, i_N, j_m) \}}{q_1 \times \cdots \cdots \times q_N \times p_m} \tag{9}$$

where

$$\overset{N}{\underset{n=1}{\wedge}} P_{nm} = P_{1m} \wedge P_{2m} \wedge \cdots \wedge P_{Nm}$$

$$\begin{aligned} P_{nm} &= \mathrm{Proj}_{X_n = i_n} R_m \\ &= \sup_{\substack{x_1 \in X_1 \\ x_2 \in X_2 \\ \vdots \\ x_n \in X_N}} R_m(x_1, \cdots, X_n = i_n, \cdots, x_N, Y_m = j_m), \end{aligned}$$

$$m = 1, 2, \cdots, M, \qquad n = 1, 2, \cdots, N$$

$$\sum_{i_1, \cdots, i_N, j_m} = \sum_{j_m=1}^{p_m} \sum_{i_N=1}^{q_N} \cdots \sum_{i_2=1}^{q_2} \sum_{i_1=1}^{q_1}$$

where, P_{nm} is the projection of R_m through all input variables except that x_n is fixed at a point i_n. The index C_m represents percentage rate of the normalized average area difference between the m-th inference output membership functions from the decomposition method and those from the original relation matrix. Thus the decomposition of the original relation matrix is successful if and only if the index is close to zero.

Ⅳ. MULTIVARIABLE CLOSED-LOOP FUZZY CONTROL SYSTEMS

In implementing a practical system, much inference error of fuzzy controller could be a main source of degradation of the performance. To overcome this problem, we propose a structure of a new fuzzy controller with a coordinator in the upper level as shown in Fig. 1.

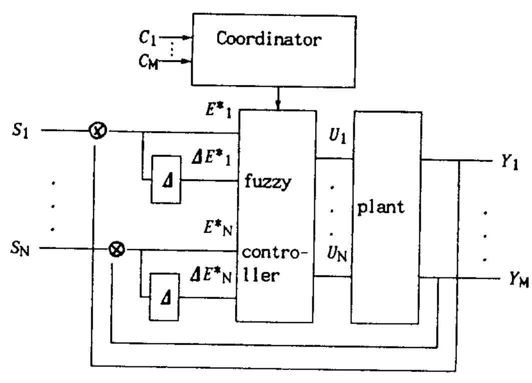

Fig. 1. Structure of a new fuzzy controller with a coordinator

In this figure, S_i denotes a desired value of the i-th output of the system and E^*_i and ΔE^*_i represent the i-th actual error and error change, respectively.

The design procedure of the fuzzy controller by the proposed method can be summarized as follows:

Step 1, Assign an upper bound(ζ) of the indices of applicability C_m in the upper level.

Step 2, Compute the indices C_1 , C_2 , $\cdots$, C_M corresponding to each controller output from the relation matrix of the fuzzy controller.

Step 3, Check whether the indices are within the pre-assigned upper bound. If not, increase the dimension of the corresponding relation matrices to three dimension. Otherwise, set the relation matrices to two dimensional ones.

As an example, we consider a fuzzy controller with four inputs and two outputs. Suppose that the index C_1 is greater than the upper bound and C_2 is less than that. Then Fig. 2 shows the block diagram of the fuzzy controller by the proposed method.

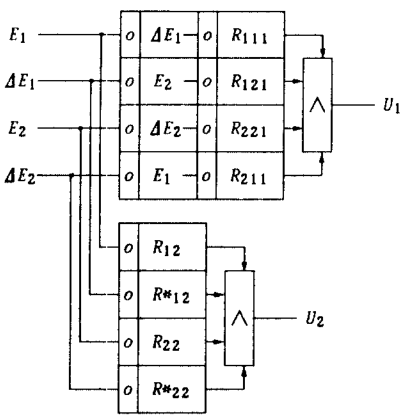

Fig. 2. Block diagram of the fuzzy controller by the proposed method

V. AN NUMERICAL EXAMPLE

Consider a multivariable closed-loop fuzzy control system with transfer function described as follows.

$$G(s) = \begin{bmatrix} \dfrac{1}{s(s+3.6)} & \dfrac{0.15}{s(s+2.8)} \\ \\ \dfrac{0.29}{s(s+3.0)} & \dfrac{1.28}{s(s+4.0)} \end{bmatrix}$$

Table 1. shows the membership functions of inputs and outputs of the fuzzy controller.

Table 1. Membership function of inputs and outputs for the controller

Ele-ments	Linguistic fuzzy sets									
	LN	*BSL*	*SN*	*BZS*	*ZE*	*BSZ*	*SP*	*BLS*	*LP*	*ANY*
-4	1	0.8	0.5	0.2	0	0	0	0	0	1
-3	0.8	1.0	0.8	0.5	0.2	0	0	0	0	0.5
-2	0.5	0.8	1	0.8	0.5	0.2	0	0	0	1
-1	0.2	0.5	0.8	1	0.8	0.5	0.2	0	0	0.5
0	0	0.2	0.5	0.8	1.0	0.8	0.5	0.2	0	1
1	0	0	0.2	0.5	0.8	1	0.8	0.5	0.2	0.5
2	0	0	0	0.2	0.5	0.8	1.0	0.8	0.5	1
3	0	0	0	0	0.2	0.5	0.8	1.0	0.8	0.5
4	0	0	0	0	0	0	0.5	0.8	1.0	1

The linguistic rules of the fuzzy controller with four inputs(E_1, ΔE_1,E_2, ΔE_2) and two outputs(U_1,U_2) are given in appropriate way. Suppose that the upper-bounds of the indices corresponding to each controller output is set to 0.10. And, the indices C_1 and C_2 are calculated by (8) as C_1=0.081, C_2=0.125. Now, in order to infer U_1 two dimensional relation matrix is sufficent because the C_1 is within the upper-bound. On the contrary, since the index C_2 is greater than the upper-bound three dimensional relational matrix is employed in the inference of U_2.

The unit step response Y_2 of the system by three different methods is shown in the following figures. The simulation gives similar results on the step response Y_1, but figures are not shown here for simplicity.

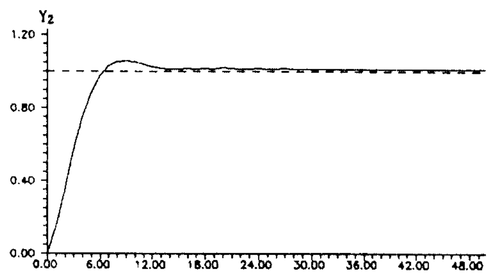

Fig. 3. Unit step response Y_2 by original relation matrix

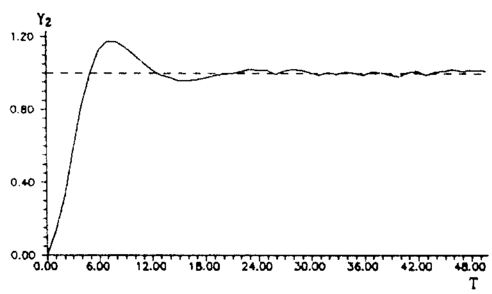

Fig. 4. Unit step response Y_2 by the Gupta's method

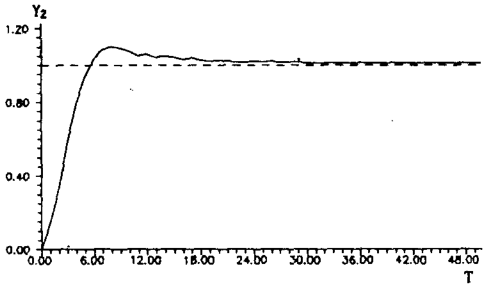

Fig. 5. Unit step response Y_2 by the proposed method

Table 2. shows the comparison of performances of output Y_1 and output Y_2 by three different methods. In the table, ISE means integral squre error.

Table 2. Comparison of performances of outputs

		original relation matrix	Gupta's method	proposed method
maximum overshoot (%)	Y_1	1.21	11.52	7.20
	Y_2	6.03	17.32	10.19
settling time (sec)	Y_1	11	18	15
	Y_2	12	19	15
ISE	Y_1	4.3926	5.9107	4.6425
	Y_2	4.0058	4.4485	4.2851

From these results we know that performance of the proposed method is better than that of Gupta's method in all categories.

VI. CONCLUSION

We proposed a feedback control method of multivariable fuzzy systems by means of decomposition of the control rules. A coordinator in the upper level calculate the inference error of the decomposition by the index of applicability and decide whether it is appropriate or not. A numerical example showed that the coordinator could prevent a large amount of inference error induced by the Gupta's decomposition method.

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