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Design technique of fuzzy controller using pole assignment method and the stability analysis of the system

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abstract

In this paper, the design technique of fuzzy controller using pole placement method and the stability analysis of the system are discussed. The consequent parts of the fuzzy model representing the fuzzy control system are descrived by linear state equations. It cannot be guaranteed that the total fuzzy system is stable even if all subsystems are stable. The range of the consequent parameters of fuzzy feedback controller which is stable for each fuzzy subspace of the input space are derived, using a rather simplified stability criterion. Then, the consequent parameters of fuzzy controller is determined with the sufficient condition that the fuzzy feedback controller maintain robust stability for the model of other subspace.

1. Introduction

Fuzzy control has been applied to many practical industrial applications. However, we have depended upon expert's knowledge and experience to design a fuzzy controller because of the lack of analytic tool for fuzzy system. Recently, the studies on the design technique such as self-tuning, neural network, genetic algorithm are being progressed to solve this difficulty.[2][3][4] But, it remains the probrem that there is no consideration about the properties of the system such as stability and controlibility.

It come to be possible to analyze the properties of fuzzy control system as the fuzzy model of process is introduced. Takagi and Sugeno model the process with fuzzy implications that descrive the relation of input and output of the process by linear equation for each devided fuzzy subspace.[8] Tanaka and Sugeno proposed a design technique using the stability criterion of model based fuzzy controller.[9] It remains also as a problem that the stability is checked heuristically and repeatedly.

So, instead of checking the existence of common positive definite matrix P, we derive the stable range of the consequent parameters of fuzzy feedback controller which is stable for each fuzzy subspace of the input space, using a rather simplified stability criterion. Then, the consequent parameters of fuzzy controller is determined with the sufficient condition that the fuzzy feedback controller maintain robust stability for the model of other subspace.

2. Takagi and Sugeno's fuzzy model

In system analysis, the modeling algorithm for the mathematical representation of a system is an important probrem. In this paper, Takagi and Sugeno's fuzzy model is used.

$$L^i: \text{If } x(k) \text{ is } A_1^i \text{ and } \cdots \text{ and } x(k-n+1) \text{ is } A_n^i \text{ and } u(k) \text{ is } B_1^i \text{ and } \cdots \text{ and } u(k-m+1) \text{ is } B_m^i$$
$$\text{then, } x^i(k+1) = a_0^i + a_1^i x(k) + \cdots + a_n^i x(k-n+1) + b_1^i u(k) + \cdots + b_m^i u(k-m+1) \tag{2.1}$$

where, $L^i(i=1, 2, \cdot\cdot, l)$denotes i-implication, l is the number of fuzzy implications, $x^i(k+1)$ is the output from i-thimplication, $a_p^i(p=0, 1, \cdots, n)$ and $b_q^i(q=1, 2, \cdots, m)$ are consequent parameters, $x(k), \cdot\cdot\cdot, x(k-n+1)$ are state variables, $u(k), \cdots, u(k-m+1)$ are input variables, A_i and B_i are fuzzy sets whose membership functions are continuos piecewise polynomial functions.

Given an input $(x(k), x(k-1), \cdots, x(k-n+1), u(k), u(k-1), \cdots, u(k-m+1))$, the output of a fuzzy model is inferred by taking the average of the $x^i(k+1)$'s :

$$X(k+1) = \frac{\sum_{i=1}^{l} w_i\{A_i X(k) + B_i U(k)\}}{\sum_{i=1}^{l} w_i} \tag{2.2}$$

$$w_i = \prod_{p=1}^{n} A_p^i(\ x(k-p+1)\ ) \times \prod_{q=1}^{m} B_q^i(\ u(k-q+1)\ ) \tag{2.3}$$

where, w_i is the true value of the premise of the i-th implication which is calculated by Eq.(2.1.3). A_i and B_i are the matrix representations of a_p^i and b_q^i, and $X(k)$ and $U(k)$ are the input vectors. A fuzzy controller is represented as following fuzzy model:

$$U(k)=\frac{\sum_{j=1}^{r} w_j H_j X(k)}{\sum_{j=1}^{r} w_j} \qquad (2.4)$$

Combining Eq.(2.2) and Eq.(2.4), the resulting fuzzy feedback system becomes Eq. (2.5)

$$X(k+1)=\frac{\sum_{ij=1}^{lr} w_{ij}(A_i-B_iH_j)X(k)}{\sum_{ij=1}^{lr} w_{ij}} \qquad (2.5)$$

That is to say, ij-th implication of combined fuzzy feedback system is as follows.

$$S^{ij}: \text{If } X(k) \text{ is } (A^i \text{ and } C^j) \text{ and } U(k) \text{ is } (B^i \text{ and } D^j)$$
$$\text{then } X^{ij}(k+1) = (A_i - B_iH_j)X(k) \qquad (2.6)$$

3. The range of parameter ensuring stability

3-1. stability condition

Stability is one of the most important considerations related to the property of control system. Among various researches about the stability of fuzzy control system[5][6], Tanaka and Sugeno proposed a criterion deciding the stability of the system. But, It has a difficulty to check the existence of common positive matrix P, the solution of Lyapunov equation. So, the stability criterion can check the stability from each implication of fuzzy model is used in this paper. Assuming that a process is represented as follows :

$$X(k+1)=\sum_{i=1}^{l}\alpha_i A_iX(k), \quad \alpha_i = \frac{\omega_i}{\sum_{i=1}^{l}\omega_i} \qquad (3.1.1)$$

then Eq.(3.1.1) is divided into constant part and variable part.

$$X(k+1)=\sum_{i=1}^{l}\alpha_i (A_i + \Delta A_i)X(k) \qquad (3.1.2)$$

where, $\sum_{i=1}^{l}\alpha_i = 1$ therefore,

$$X(k+1)=(A + \sum_{i=1}^{l}\alpha_i \Delta A_i)X(k) \qquad (3.1.3)$$

It is known that the fuzzy system (3.1.1) is stable if system A is stable, and $\varepsilon > 0$ satisfying (3.1.4) exists

$$\sigma_{\max}(G^TPG) < [1-(\Lambda_{\max} \mid A \mid + \varepsilon)^2]\sigma_{\min}(Q) \qquad (3.1.4)$$

where, $G = \sum_{i=1}^{l} \alpha_i \Delta A_i$, $\sigma_{\max}(M)$, $\sigma_{\min}(M)$ represent maximum and minimum singular value of the matrix M and $\Lambda_{\max} \mid A \mid$ is maximum value of $\mid \Lambda_i(A) \mid$, and P, Q are positive definite matrix satisfying Lyapunov equation.

$$A^TPA-P = -Q \qquad (3.1.5)$$

Considering the stability condition represented by eq(3.1.4) as second order inequality about ε, it becomes (3.1.6)

$$\sigma_{\max}(G^TPG) < [1-(\Lambda_{\max} \mid A \mid)^2]\sigma_{\min}(Q) \qquad (3.1.6)$$

and its necessary and sufficient condition is that

$$\sigma_{\max}(\Delta A_i^TP\Delta A_i) < [1-(\Lambda_{\max} \mid A \mid)^2]\sigma_{\min}(Q) \qquad (3.1.7)$$

By eqation (3.1.7), the stability of the system expressed by eq.(3.1.3) is examineed from each rule (that is, ΔA_i) without obtaing the common positive definite matrix P of Lyapnov equation. Therefore, in the case that the consequent part of ij_{th} rule describing total fuzzy feedback system is expressed as eq(2.6), the feedback system matrix $G_{ij} = A_i + B_iH_j$ is divided into A_i and B_iH_j, and we can obtain the range of feedback matrix H_j $(j = 1, 2, \cdots, r)$ ensuring the stability with regard to i_{th} system matrix $A_i(i = 1,2,\cdots,l)$ of spatial-partitioned plant. Regarding i_{th} system matrix A_i,

$$\sigma_{\max}[(B_iH_j)^TP(B_iH_j)] < [1-(\Lambda_{\max} \mid A_i \mid)^2]\sigma_{\min}(Q_i)$$
$$j = 1, 2, \cdots, r \qquad (3.1.8)$$

where, P_i, Q_i are the solutions of $A_i^TP_iA_i - P_i = -Q_i$. Although the common region of parameter of $H_j(j = 1, 2, \cdots, r)$ obtained from eq(3.1.8) guarantee the stability for individual A_i, the feedback controller H_j must have robust property in order to maintain the stability cope with variation of system matrix A_i.

3-2 Robust stability

In the case that a process is modeled by fuzzy implications with consequent part expressed by linear state equation, feedback controller H_j must be able to maintain stability robustly, because the system matrix represent the process vary as state vector vary. Theorem (3.1) propose a sufficient condition that the total fuzzy control system maintains stability robustly. By this theorem, the robust stability region of parameter of feedback controller is able to obtained. Consider the consequent part descriving fuzzy model varies as follows.

$$X(k+1) = (G + \Delta G)X(k) \qquad (3.2.1)$$

Assuming that ΔG is bounded under U, that is $\mid \Delta G \mid \le U$ then following is the sufficient condition that $G + \Delta G$ is stable.

Therem 3.1

The system $G + \Delta G$ is stable if $\rho(\mid G \mid + U) < 1$ where, $\rho(A) = \max_i \mid \lambda_i(A) \mid$, $\mid A \mid$ is matrix with absolute values of each components of A

proof

The system $G+\Delta G$ is stable $M = I - |G+\Delta G|$ is M matrix. It is then sufficient that $\lambda(|G+\Delta G|) < 1$ or $\rho(|G+\Delta G|) < 1$. From $|\Delta G| \le U$, we have $|G+\Delta G| \le |G| + U$. Therefore, if $\rho(|G| + U) < 1$ is satisfied , system $G+\Delta G$ is stable.

Q. E. D.

If fuzzy model of process and fuzzy controller is expressed as eq.(2.2) and eq.(2.4) respectively, the total fuzzy feedback system to design in this paper is expressed by fuzzy model with consequent structure, $G_{ij} = A_i - B_i H_j$ as following eq. (3.2).

$$X(k+1) = \frac{\sum_{ij=1}^{k} w_{ij} G_{ij} X(k)}{\sum_{ij=1}^{k} w_{ij}}$$

$$\text{where,} \quad G_{ij} = A_i - B_i H_j \qquad (3.2)$$

Consequent structures of the closed loop fuzzy feedback system for i_{th} implication of process (A_i, B_i) are represented as following equations.

$$G_1 \triangleq G_{i1} = A_i - G_i H_1$$
$$G_2 \triangleq G_{i2} = A_i - G_i H_2$$
$$\vdots$$
$$G_1 \triangleq G_{i1} = A_i - G_i H_1$$

and those for k_{th} implication are

$$G_{k1} = A_i - B_i H_1 + (A_k - A_i) + (B_k - B_i)H_1 \triangleq G_1 + \Delta G_{k1}$$
$$G_{k2} = A_i - B_i H_2 + (A_k - A_i) + (B_k - B_i)H_2 \triangleq G_2 + \Delta G_{k2}$$
$$\vdots$$
$$G_{kr} = A_i - B_i H_r + (A_k - A_i) + (B_k - B_i)H_r \triangleq G_r + \Delta G_{kr}$$

then, H_j has its parameter in the region satisfying the theorem 4.2 regarding to U_j in order to control the process maintaining stability robustly. Therefore, we can design parameters of fuzzy controller H_j which has desired property in the common region that is obtained using stability condition and robust stability condition.

4. Controller Design and Simulation

4-1. Design Algorithm

By the stability condition in Sec. 3.1 and the robust stability condition showed in Sec. 3.2, we can design the fuzzy controller so as that the total fuzzy feedback system has a desired charicteristic and satisfies the stability as follows ;

Step1 : Design the controller with a unknown parameter, where the premise of the controller was constructed as that of a plant fuzzy model.

Step2 : Make the model S_{ij} that describes fuzzy feedback system by combining model of a fuzzy plant and the fuzzy controller designed in step1 .

Step3 : Calculate the region of the parameters of the controller, $H_i\ (i = 1, 2, \cdots, r)$, satisfying the stability in Sec. 3.1 .

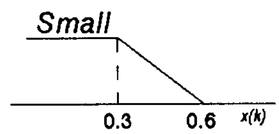
Step4 : Calculate the region of the parameters of the controller, $H_i\ (i = 1, 2, \cdots, r)$, satisfying the robust stability for each plant model by Theorem 3.2.1 .

Step5 : Determine the parameters of the controller by using the pole placement method in order for the parameters, which was constituted with the poles of characteristic equation, to be in the stability region of controller obtained from step4 and 5

4-2. Simulation

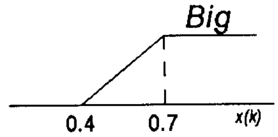
For the object process modeled as fig 4.1, the fuzzy controller designed using the method showed in Sec. 4.1 and the simulation result is represented in fig. 4.2 - fig. 4.5

If $x(k)$ *is*



$$\text{then } X(k+1) = \begin{bmatrix} 0.7 & -0.12 \\ 1 & 0 \end{bmatrix} X(k) + \begin{bmatrix} 0.3 \\ 0 \end{bmatrix} U(k)$$

If $x(k)$ *is*



$$\text{then } X(k+1) = \begin{bmatrix} 1.0 & -0.25 \\ 1 & 0 \end{bmatrix} X(k) + \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} U(k)$$

Fig 4.1 The fuzzy model of a object system

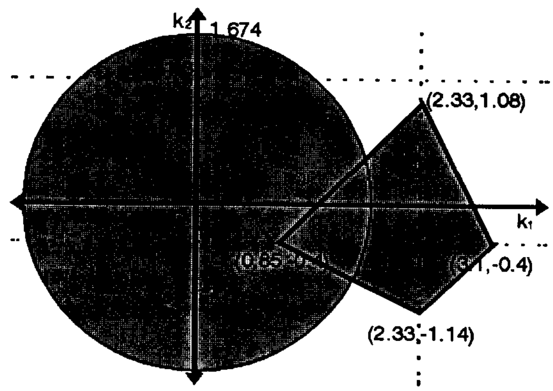
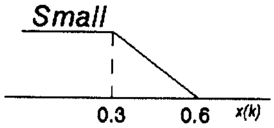


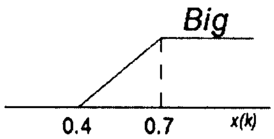
Fig. 4.2 Calulated stable region

If $x(k)$ *is*



then $u(k) = -1.3\,x(k) + 0.28x(k-1)$

If $x(k)$ *is*



then $u(k) = -1.1\,x(k) + 0.01x(k-1)$

Fig. 4.3 Designed rule of fuzzy controller

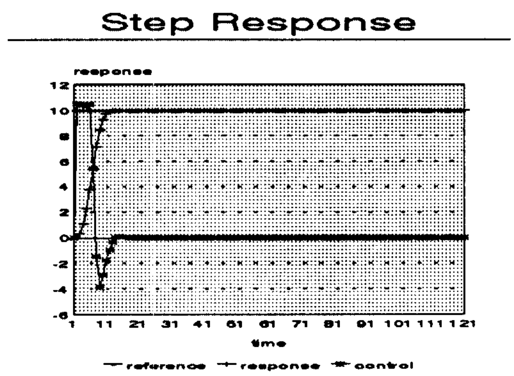


Fig. 4.4 The step response of the controller

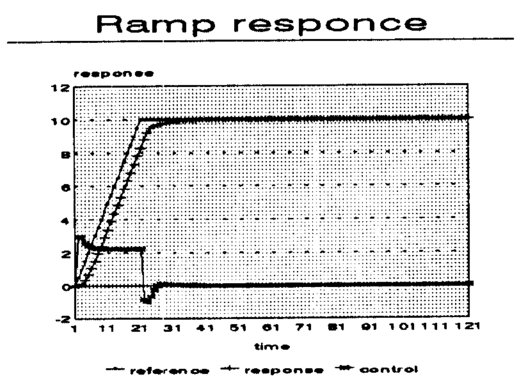


Fig 4.5 The ramp response of a controller

6. Conclusion

In the case that a process is modeled with fuzzy implications whose consequent parts are represented by linear state equations ablut input variables, it is difficult that we test the stability of the system by examine the existence of common positive definite P of the Lyapunov equation. So, using the stability criterion that can be applied to each rule of fuzzy model representing the process to examine the stability of a fuzzy system, the controller is designed so as to ensure the stability for the fuzzy model of each fuzzy subspace of the input space. Then the consequent parameters of fuzzy controller is determined with the sufficient condition that the fuzzy feedback system can maintain the robust stability for models of other spaces. Since the stable consequent parameters is given by certain region, the fuzzy controller is designed by determining parameters of feedback matrix in order that total fuzzy system has a desired characteristic. The controller designed with the method considering the robust stability is able to control the process of fixed model, the stability can't be guranteed when the model of the object process varies from disturbance.

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