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COMPLETE AND INCOMPLETE FUZZY LOGIC CONTROLLERS

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Abstract

The paper deal with the differences between a fuzzy logic controller with a complete linguistic description and one with an incomplete linguistic description. The conditions to get a complete crisp controller by using a fuzzy logic controller with incomplete description are analyzed, and an application to the control of an analog PLL circuit is described, [1].

Introduction

Usual crisp systems are <u>injective</u> mappings from a compact domain $D \subset R^n$ in R^n, i.e. $f: D \subset R^n \rightarrow R^n$, $\forall\, x \in D$, f(x) is defined. In this paper, crisp systems as above are named "complete crisp systems".

Fuzzy logic systems (FLS) are often used in conjunction with an output defuzzifier to implement a crisp system. Some questions arise:

i) Which conditions the fuzzy systems has to satisfy to achieve an implementation of a complete crisp system ? (i.e. a complete crisp control system).

ii) Can the usual FLS be simplified, still allowing the implementation of a complete crisp system ?

iii) If the second question has a positive answer, how to systematically minimize the complexity of the FLS without degrading its performance ?

The topic of this paper refers to the above questions and some related answers.

Theoretical Aspects

Let be a FLS described by a set R of rules as:

$$\mathbf{R = \{IF\ x_1\ is\ A_{1i}\ and\ x_2\ is\ A_{2j}... ...and\ x_n\ is\ A_{nm}\ THEN\ y\ is\ B_k\}} \qquad (1),$$

where x_1, x_2, ... x_n are the input variables, y is the output variable, and $A_{1i} \in \{A_{1\alpha}\}$, $A_{2j} \in \{A_{2\beta}\}, \ldots, A_{nm} \in \{A_{n\eta}\}$, $B_k \in \{B_\Theta\}$, where $\{A_{1\alpha} \mid \alpha = 1...N_1\}$; $\{A_{2\beta} \mid \alpha = 1...N_2\}$;... $\{A_{n\eta} \mid \alpha = 1...N_n\}$ are the sets of input linguistic degrees, and $\{B_\Theta \mid \Theta = 1...M\}$ is the set of output linguistic degrees.

Let $A_{\bullet\bullet}\sim$, $B^{\bullet\bullet}\sim$ be the fuzzy sets corresponding to the respective linguistic degrees $A_{\bullet\bullet}$ and $B_\bullet\sim$.

The rules system (1) maps the set of the tuples $\mathbf{A} = \{(A_{1\bullet}, A_{2\bullet}, \ldots A_{n\bullet})\}$ into the set $\mathbf{B} = \{(B_\bullet)\}$.

<u>DEFINITION:</u> The rules-based (linguistic) system is named <u>complete</u> if for any possible tuple $(A_{1\bullet}, A_{2\bullet}, \ldots A_{n\bullet})$ there exist a rule in (1), i.e. the mapping is an injection. Else, the system is named <u>incomplete</u>. So, a complete linguistic system (1) introduces $N_1 x N_2 x \ldots x N_n$

rules.

We shall suppose that the fuzzy sets $A_{\bullet\bullet}\sim$ satisfy the conditions:

i) they are defined on R, i.e. the membership functions $\mu_{A\bullet\bullet}(u_i) : R \rightarrow [0,1]$.

ii) the sets $A_{\bullet\bullet}\sim$ cover a conex domain $D \subset R^n$, in the sense that for any point $u(u_1, u_2, \ldots u_n) \in D$ there is at least one tuple $(A_{1x}\sim, \ldots, A_{nx}\sim)$ such as all corresponding membership functions do not vanish in u:

$$\forall\ u \in D\ \exists\ (A_{1x}\sim, \ldots, A_{nx}\sim) \text{ such that } \mu_{A1x}(u_1) \neq 0 \ldots \mu_{Anx}(u_n) \neq 0. \quad (2)$$

Remark 1: Under the above conditions and if the corresponding linguistic system is complete, then the fuzzy system maps the conex domain D in $\{B_\Theta\}$ and the mapping is an injection. Moreover, the mapping is not trivial, in the sense that for any $u \in D$, these is at least one rule in (1) to infer about the output with nonzero belief (non-vanishing membership functions at output).

Remark 2: The system in Remark 1, if provided with a defuzzifier, maps D in R and is an injection. Such a fuzzy system will be named complete.

So, the key point in the above discussion is that a fuzzy system described by rules (1) and by membership functions completely covering the input domain (i.e. satisfying the above described coverage conditions), when provided with a defuzzifier, maps a domain $D \subset R^n$ in R, and is an injection. Such a system can represent a crisp system $f: D \subset R^n \rightarrow R$. Let us stress that such a system allows to infer something about the output for any value u of the input, i.e. it offers a complete description of the behavior in D **(shortly: "complete description in D")**. In general a fuzzy system not allowing a complete description in this sense is not good in applications. For example, a control system should allow us to infer something about the control for any value of the input variables in the domain of interest.

The problem dealt with in this paper is if a fuzzy system has to satisfy all the conditions given above to realize a complete description in D. The main results in this subsequent section are:

i) the above conditions are not necessary for a complete description in D;

ii) almost the same characteristic $f: R^n \rightarrow R$ can be achieved with a much simplified fuzzy system with defuzzifier, if the coverage of D by the membership functions is suitable choused.

Main Results

P1: If there is a subset of tuples $\boldsymbol{A_o} \subset \boldsymbol{A}$ such that condition (2) is satisfied, then let us select for the set of rules (1) a subset $\boldsymbol{R_o} \subset \boldsymbol{R}$ including only those rules having in antecedent the linguistic degrees from $\boldsymbol{A_o}$. Then, the system $(\boldsymbol{A_o}, \boldsymbol{R_o})$ allows a complete description in D.

This prof is obvious by applying Remarks 1 and 2.

P2: Consider the system in the above proposition satisfies the conditions:

i) all $A_{\bullet\bullet}\sim$ and $B_\bullet\sim$ have "symmetrical shape" membership functions;

ii) a center of gravity defuzzifier is used.

Then, the characteristic functions of the system $(\boldsymbol{R_o})$ and of the system $(\boldsymbol{R})$ have at least as many common points as many rules are in $(\boldsymbol{R_o})$, i.e. $\exists\ u_1, \ldots, u_p : f_{(Ro;Ao\rightarrow B)}(u_i) = f_{(R;A\rightarrow B)}(u_i)$.

Prof: For u_i such as $\mu_{A1\sim}(u_1) = \ldots = \mu_{An\sim}(u_n) = 1$, the output is the center of gravity of the corresponding overlapping $B_k\sim$ for both systems, or, for the complete linguistic system, it is included between two subsequent output values of the incomplete linguistic system. As the center of gravity defuzzification provides a continuous mapping, the two characteristics have a common point.

The above results allow to simplify the fuzzy systems (e.g. the fuzzy control systems), as regarded as universal approximators.

Example

Consider a simple example: a FLC with only one input and one output, with three triangle membership functions for input (A_i, i=1..3), and three triangle membership functions for output, (B_k, k=1..3).

The fuzzy controller is considered to have a complete linguistic description, the rules describing it being:

IF ***input*** **is A_1, THEN** ***output*** **is B_3,**

IF ***input*** **is A_2, THEN** ***output*** **is B_2,** (3)

IF ***input*** **is A_3, THEN** ***output*** **is B_1,**

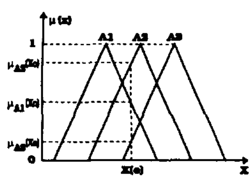

Fig.1 Fuzzified Values for Complete Description

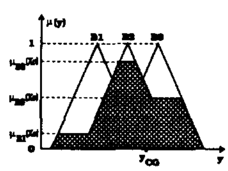

Fig.2 Defuzzified Output for Complete Description

Figure 1 shows the input fuzzification process (values $\mu_{A1}(X_o)$, $\mu_{A2}(X_o)$ for a given crisp value X_o).

According to the set of rules (3), the output membership function and the defuzzified output value y_{CG}, for input crisp value X_o, looks like in Figure 2.

The defuzzified output is:

$$y_{CG} = \frac{\int_{-\infty}^{\infty} y H(y)\, dy}{\int_{-\infty}^{\infty} H(y)\, dy} \quad (4)$$

where H(y) is implicitly function of X_o.

The defuzzified output value y_{CG} is determined by the center of gravity method. *H(y)* represents the value of the output function set by the MAX operator and *y* represents the current value.

When X_o takes all possible values, the y_{CG}(x) value is:

$$y_{CG}(x) = \frac{a_3x^3 + a_2x^2 + a_1x + a_o}{b_3x^2 + b_1x + b_o} \quad (5)$$

Further details can be found in [3].

Compared to a complete description, an incomplete description of a FLC means that there exist same possible rules which are missing. For example, compare the description (6):

IF ***input*** **is A_1, THEN** ***output*** **is B_3,**

IF ***input*** **is A_3, THEN** ***output*** **is B_1, (6)**

where a rule is missing (IF *input* is A_2, THEN *output* is B_2) with the rules set (3).

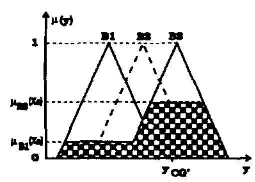

Fig.3 Defuzzified Output for Incomplete Description

According to the set of rules (6), the output membership function is plotted in Figure 3.

Remark: An incompletely described fuzzy logic system covers the Euclidian input domain (i.e. it is a complete system) only if the bases of two successive input membership functions have common points (the intersection of their supports is not empty set).

The defuzzified output $y_{CG'}$, shown in Figure 3 is also determined by on equation similar to (5), but with different coefficients. Figure 4 presents the transfer curves of the FLC for a complete description -curve (1) - $y_{CG}(x)$, and for incomplete description - curve (2) - $y_{CG'}(x)$ according to the sets of rules (3) and (6) respectively.

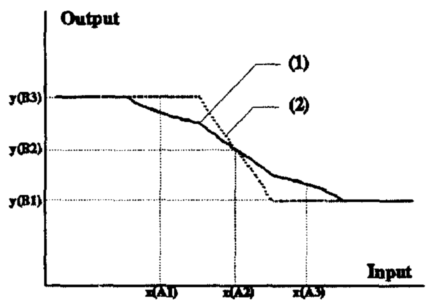

Fig.4 Transfer Characteristics

The slope of the transfer characteristic for FLC with a complete description (curve (1)) is smoother than the slope of the transfer characteristic for FLC with an incomplete description (curve (2)), because the maximum distance $d(x(A_i)/y(B_k), x(A_{i+1})/y(B_{k+1}))$ of FLC with incomplete description is larger than the maximum distance $d(x(A_i)/y(B_k), x(A_{i+1})/y(B_{k+1}))$ of FLC with complete description. Here, $x(A_i)$, (i=1..3) stand for the values of the peak/middle of the triangle membership functions corresponding to the input linguistic degrees and $y(B_k)$, (k=1..3) stand for the peak/middle values of the triangle membership functions of the output linguistic degrees.

Discussion and conclusions

A complete linguistic description needs a significantly higher number of fuzzy rules versus an incomplete linguistic description to cover the same Euclidian input domain, i.e to allow a complete description in D. The number of rules can be reduced by about 50%, with acceptable degradation of the transfer characteristic. This reduction is ***not got by interpolation***, although the procedures have some similarity. For the fuzzy controller for a PLL circuit, described in [2], it was possible a reduction of the number of the fuzzy rules from 25 to 13, without significant degradation of the control. This quality of fuzzy controllers with incomplete description presents a major interest for hardware implementation.

In the case of fuzzy controller of the PLL circuit, [2], the phase error for complete description of FLC is $7*10^{-5}$ radian and the phase error for incomplete description of FLC is $5*10^{-4}$.

The dynamic response of the incompletely described FLC in [2] was faster than of the completely described FLC. The incompletely described FLC for PLL circuit is locked after 6 iterations, while the completely described FLC is locked after 8 iterations, [2].

References

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