Fifth IFSA World Congress (1993), 1044-1046

On the behavioural dependence of fuzzy concepts

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Abstract

The notion of behavioural dependence of fuzzy concepts is introduced. Examples are given along with first results concerning classical aggregation operators.

Key words: Fuzzy sets, behavioural dependence.

1 Introduction

Not every concept, that is a judgement criterion, has its own ontological status. There exist concepts that take their meaning from others by means of simple alterations. The goal of this paper is to introduce a mathematical formalization of these alterations in the framework of Fuzzy Logic.

A fuzzy concept **C** (e.g **good, tall, young** etc.) can be associated to a fuzzy set C defined over a universe of discourse U in such a way that for all $u \in U$, $\mu_C(u)$ represents the estimated degree to which u can be classified as **C**. Such an estimated degree does not give any information about the structure of u but it provides a characterization of **C** (and in turn of C) in the universe U.

A general limitation of Fuzzy Set Theory and any other mathematical theory is that they can not fully formalize the semantic of a concept but only characterize by means of the membership values the relations with other concepts defined on the same universe.

Therefore all we can say about **C** (and in turn of C) is limited to μ_C, i.e. the behaviour of C in the universe U.

It may then be the case that two concepts whose meanings are completely different can have on some universes (or possibly on all universes) the same behaviour. For instance, on the universe of *human beings good dancers* and *bad drivers* might have the same behaviour though semantically very different.

With such a limitation in mind, two fuzzy sets will be considered equal when they have the same behaviour and not when they represent the same concept.

Therefore, studies of conceptual alterations in mathematical formalizations are reduced to studies of the most adequate formalizations of the alterations. We will provide here one possible formalization.

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2 Problem Setting

Our starting points will be classical alterations such as *negation* or modifiers such as *very, quite, etc.*. Our goal will be to point out common characteristics of alterations and formalize behavioural dependence according to a *possible* criterion.

Our formalization will take into account that

- since (fuzzy) concepts are judgement criteria, their alterations cannot depend upon the universe of discourse;
- alterations cannot depend upon the concept to which their applied;
- judgement criteria are defined on a continuous universe and so will their alterations;
- for every concept **C'** which is semantically dependent on the concept **C** the behaviour of C' must be strongly connected to the behaviour of C on any universe U. Such a connection will be expressed by imposing coherent variations of the values of the membership functions.

Our underlying belief is that it is possible to describe a universe by considering only its independent concepts. Dependence is defined as follows.

DEFINITION 2.1 *Let $f : [0,1] \to [0,1]$ be a continuous, monotonic function.*

Given two fuzzy concepts **A** *and* **B** *with associated fuzzy sets A and B defined over U by the membership functions μ_A and μ_B, we say that* **B** *is* behaviourally dependent *(b-d for short) on* **A** *with alteration f if for any universe U and for all $x \in U$ we have*

$$\mu_B(x) = f(\mu_A(x)).$$

□

Immediate examples of behavioural dependence are given by negation operators also called fuzzy complements (see [3] and [2] pg. 38). Indeed, a negation operator $N : [0,1] \to [0,1]$ satisfies the following conditions

(n1) $N(0) = 1$ and $N(1) = 0$;

(n2) N is monotonic nonincreasing, i.e. if $a < b$ then $N(a) \geq N(b)$;

(n3) N is a continuous function;

(n4) N is involutive, i.e. $N(N(a)) = a$.

In order to test the usefulness of our criterion, let us consider T-norms and T-conorms. They represent general aggregation operators that can be used to combine fuzzy sets (see [1] for a comprehensive study of aggregation operators and their properties).

In particular, a T-norm is a map $T : [0,1] \times [0,1] \to [0,1]$ such that

(T1) $T(a,b) = T(b,a)$

(T2) $T(a, T(b,c)) = T(T(a,b),c)$

(T3) $T(a,b) \geq T(c,d)$ if $a \geq c$ and $b \geq d$

(T4) $T(a,1) = a$

T-norms generalize the concept of logical *and* or equivalently of set intersection.

A T-conorm is a map $S : [0,1] \times [0,1] \to [0,1]$ such that

(S1) $S(a,b) = S(b,a)$

(S2) $S(a, S(b,c)) = S(S(a,b),c)$

(S3) $S(a,b) \geq S(c,d)$ if $a \geq c$ and $b \geq d$

(S4) $S(a,0) = a$

T-conorms generalize the concept of logical *or* or equivalently of set union.

Our first result is the following.

THEOREM 2.1 *If B is b-d on A with alteration f and C is b-d on A with alteration g then $T(B,C)$ is b-d on A with alteration $T(f,g)$ and $S(B,C)$ is b-d on A with alteration $S(f,g)$ for any T t-norm and S T-conorm, where*

$$\begin{aligned} T(B,C)(u) &= T(\mu_B(u), \mu_C(u)) \\ T(f,g)(x) &= T(f(x), g(x)) \end{aligned}$$

■

3 Final Remarks

At this stage, our research is oriented towards the formalization of invariants of particular behavioural dependences, their characterization and their consistency with the already existing fuzzy set operations. We are also trying to obtain significative results on the global theory by using behavioural dependence as an interpretational instrument.

Future perspectives appear to be quite good if we will be able to define classes of concepts generated only by a finite collection of concepts. In this case, all information contained in the class will be captured by the relation among the generators and their intrinsic properties.

References

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