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Fuzzy control for a flexible arm manipulator

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Abstract.

In this paper a fuzzy controller for a flexible arm with one degree of freedom is presented. Goal of the control is to drive the manipulator to the position θ_0 avoiding the oscillations due the elasticity of the arm.

The performances of the fuzzy controller are evaluated through a series of simulations that shows appreciable results both for the transient and the steady behaviour .

1. Introduction

Lightweight robots present evident advantages over the conventional rigid robots in terms of higher speed and low energy consumption, therefore in terms of higher productivity and efficiency. However the use of lighter materials in robot manufacturing introduces flexibility problems: such robot models typically show highly vibratory poles and low damping factors, that describe their attitude to deflection and vibration. The controller design for these systems is thus required to eliminate the elastic oscillations. The main problem in the controller design is in that an uncertainty $\Delta G(s)$ derives from the reduction of the arm model order. A flexible arms is in fact a distributed parameter system of infinite order whose identification leads however to an approximated finite order model. The strategy of control for this system must therefore take into account the real uncertainty that may raise instability problems.

The suitability of fuzzy control for flexible arms is supported by its intrinsic characteristic of robustness to the uncertainty $\Delta G(s)$ in the plant. Tracking performances are guaranteed even in the presence of parameters disturbs, provided their width be bounded, since the fuzzy controller acts with a sliding mode action [1]. The model of the flexible arm is introduced in Section 2 . The fuzzy controller and the methodology of design are described in Section 3 and 4. Simulation results are shown in Section 5 and compared with other techniques applied on to the same plant.

2. The robot system

The flexible arm is constituted by a single continuous beam driven by a dc motor ; its scheme is shown in Fig. 1.

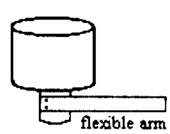

Fig. 1

The flexible arm is modelled as a distributed system whose transfer function from the input torque to the rotation angle is constituted by an infinite sum of second order terms:

$$G(s) = \frac{A_0}{s^2} + \sum_{i=1}^{\infty} \frac{A_i}{s^2 + 2\xi_i \omega_i + \omega_i^2} \qquad (1)$$

The identified model is constituted by an integration term representing the rigid body motion and few second order terms representing the main elastic modes; further non identified elastic modes are grouped into an additive uncertainty term. The transfer function of the manipulator may assume then the following expression:

$$G(s) = G_0(s) + \Delta G(s) \qquad (2)$$

with

$$G_0(s) = \frac{A_0}{s^2} + \sum_{i=1}^{2} \frac{A_i}{s^2 + 2\xi i \omega i + \omega i^2} \qquad (3)$$

and

$$\Delta G(s) = \sum_{i=3}^{\infty} \frac{A_i}{s^2 + 2\xi i \omega i + \omega i^2} \qquad (4)$$

$G_0(s)$ being the nominal model and $\Delta G(s)$ being the perturbation. The model of the flexible arm has been taken from literature [2]. The numerical values of the identified parameters are reported in Tab. I.

i	ω_i	$\xi_i\omega_i$	A_i
0			2.043
1	20.02	0.4774	6.446
2	61.55	1.468	12.96
3	118.3	2.823	-19.49
4	186.4	4.447	-19.44
5	279.2	6.659	-13.49

Tab. I

3. The fuzzy controller

A fuzzy controller can be described through a table of *fuzzy conditional rules.* Each rule is in the form:

IF x_1 IS A_1 ... AND x_n IS A_n THEN $y^1 = y_0^1$

x_j (j=1,..,n) being the input variables defined in their respective *Universe of Discourse* U_j, A_j the fuzzy set for the j-th variable and y^i the output variable computed by each rule R_i. Each fuzzy set A_j is characterized by a *membership function* μ_{Aj}: U→[0,1] which associates to x_j a number in [0,1], indicating its grade of membership to the set A_j (Fig. 2)

Generally the input variables to the fuzzy controller are the errors e_i between the system state variables and their respective setpoints. If the number of fuzzy sets E_i for each premise variable e_i is fixed to n_i, then the maximum number of fuzzy rules constituting the controller is given by all the possible combinations of the fuzzy sets:

$$m = n_1 * \ldots * n_n$$

The fuzzy controller looks therefore like the following m-rule based table.

R1: IF e1 IS $E1^1$ ... AND en IS En^1 THEN $u^1 = u0^1$

.... (5)

Rm: IF e1 IS $E1^m$ ... AND en IS En^m THEN $u^m = u0^m$

The input variables to the fuzzy controller for the proposed flexible arm are the error *e* between the desired position and the actual position of the tip and the rate of the error *de*, coinciding with the angular speed of the arm since the setpoint is constant. The output variable is constituted by the torque *u*.

For every input variable three fuzzy sets, *small*, *medium*, *large*, have been selected; the shape of their membership function is triangular or trapezoidal as in Fig. 2.

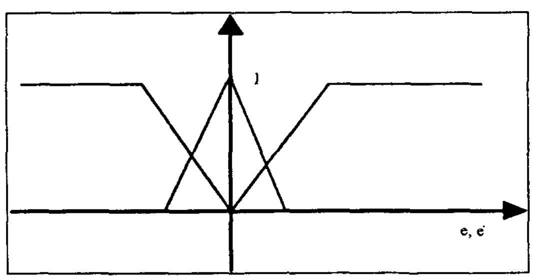

Fig. 2

The output variable of each rule is a crisp value instead than a fuzzy one. The i-th rule of the proposed fuzzy controller is therefore in the form:

If e *is* E_1^i *and* de *is* A_2^i *then* $u^i = u_{0i}$

E_k^i being one among the fuzzy sets *small, medium, large.* A total number of nine rules, deriving from the combination of all possible fuzzy values of e and de, constitutes the controller.

The control action u is evaluated from the outputs of the 9 rules by the following *inference* method:

$$u = \frac{\sum_{k=1}^{9} \mu(R_k)\, u^k}{\sum_{k=1}^{9} \mu(R_k)} \qquad (6)$$

$\mu(R_k)$ being the *activation value* of the k-th rule, computed as $\mu(R_k) = \mu_{E_1^k}(e) * \mu_{E_2^k}(de)$.

If the fuzzy sets of the input variables are *a priori* fixed in number, position in the universe of discourse and membership function shape, the only parameters that have to be designed in the fuzzy controller are the outputs of each rule u_0^k.

In the case of the proposed manipulator, since a symmetry is achieved with respect to the main diagonal in the phase space e, de, only four parameters have been identified.

4. The methodology for the design of the fuzzy controller

In order to identify these parameter vector a Cell-to-Cell approach has been used [3]. The main idea in this strategy is to perform a discretization of the premise variable space (in our case: e, de), limited by the ranges of interest of each variable, into a finite number of cells. An optimization procedure is then used to compute, for each cell, the control value u_{opt} that minimizes an adequate cost function representing the desired performances of the control action. The choice of the cost function is strictly problem-dependent and greatly influences the performance of the controller.

Once the optimal control policy is obtained for each cell, the fuzzy control rules can be identified from this discrete map through the following procedure of identification [4]. Let us indicate with β_k the following term:

$$\beta_k = \frac{\mu_{E_1^k}(e) \Lambda \mu_{E_2^k}(de)}{\sum_{k=1}^{9} \mu_{E_1^k}(e) \Lambda \mu_{E_2^k}(de)} \qquad (7)$$

Then (6) can be formulated as:

$$u = \sum_{k=1}^{9} \beta_k u_0^k \qquad (8)$$

If a suitable number t of cells is considered, a system with more equations than unknown quantities derives from equation (8): the solution $[u_0^1\ u_0^2 \ldots u_0^m]$ can be generally obtained using a least square algorithm, as explained below. Indicating by

$$\mathbf{U}_{opt} = [u_1^{opt}, u_2^{opt} \ldots\ldots u_t^{opt}]$$

the optimal control action vector relative to each cell, by B the following matrix:

$$\begin{matrix} \beta_{11} & \cdots & \beta_{19} \\ \beta_{t1} & \cdots & \beta_{t9} \end{matrix}$$

the matrix of the activation values, with β_i^j relative to the i-th cell and j-th rule, and by

$$\mathbf{u} = [u_0^1\ u_0^2 \ldots u_0^m]$$

the vector of the unknown parameters, the solution **u** can be computed as:

$$\mathbf{u} = (B^T B)^{-1} B^T \mathbf{U}_{opt} \qquad (9)$$

The fuzzy controller is therefore completely identified.

5. Results

The performances of the proposed fuzzy controller applied to the flexible arm have been evaluated through a series of simulations . In Fig. 3 the position behavior of the arm is shown in the cases where the nominal model $G_0(s)$ and a perturbed one $G_0(s)+\Delta'G(s)$ are adopted. The perturbation $\Delta'G(s)$ is constituted by three additive elastic modes:

$$\Delta' G(s) = \sum_{i=3}^{5} \frac{Ai}{s^2 + 2\xi i\omega i + \omega i^2}$$

It may be observed from Fig. 3 that in the case of perturbed model, the position behavior reveals only a slight overshoot with respect to the non perturbed one. A satisfactory fast response with a slight overshoot is observed in both cases. The adopted sample rate is 300 Hz. The fuzzy control action is provided in Fig.4.

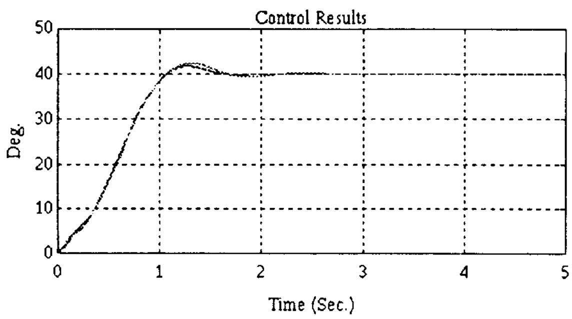

Fig. 3

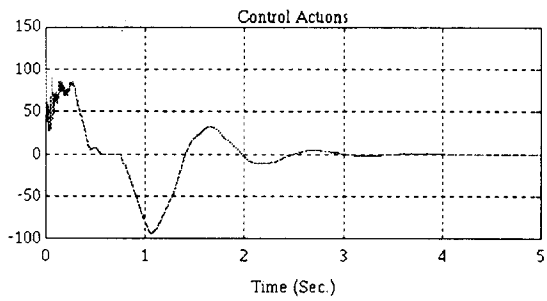

Fig. 4

The performances of the fuzzy controller have been compared to the ones of a Linear Quadratic Regulator designed for the same flexible arm. In Fig. 5 a simulation shows the position behaviour of the arm controlled by the LQR when the nominal model is used.

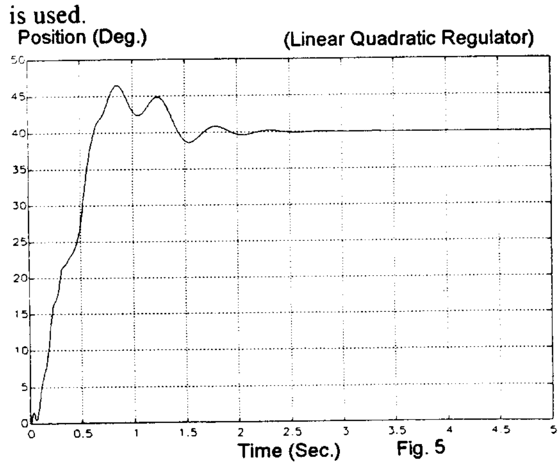

Fig. 5

If compared to the fuzzy controlled plant behavior it reveals a significantly greater overshoot.

However when the perturbation $\Delta' G(s)$ is added to G0(s), the plant with the LQR becomes unstable.

The performances of the fuzzy controller have been compared either to the ones of the robust controller designed for the same flexible arm in [2]. With respect to the fuzzy controller, for a setpoint of 40 degree, the robust controller notwithstanding a comparable rising time, cannot avoid a greater overshoot and a more oscillatory transient in the position curve .

6. Conclusion

In this paper a fuzzy controller for a flexible arm with one degree of freedom is presented along with a general methodology used in its design. The fuzzy controller shows appreciable results in terms of low rising time, smooth transient and slight overshoot in the arm position curve. An additive perturbatory term, constituted by higher oscillatory modes of the arm, in the model does not appreciably alter the performances of the fuzzy control policy.

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