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Uncertainty Fusion of Sensory Information Using Fuzzy Numbers

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Abstract

The Multisensor Fusion Problem (MFP) deals with the methodologies involved in effectively combining together homogeneous or non-homogeneous information obtained from multiple redundant or disparate sensors in order to perform a task more accurately, efficiently, and reliably. The inherent uncertainties in the sensory information are represented using Fuzzy Numbers, $\mathcal{F}$-numbers, and the Uncertainty-Reductive Fusion Technique (URFT) is introduced to combine the multiple sensory information into one consensus $\mathcal{F}$-number. The MFP is formulated from the Information Theory perspective where sensors are viewed as information sources with a fixed output alphabet and systems are modeled as a network of information processing and propagating channels. The performance of the URFT is compared with other fusion techniques in solving the 3-Sensor Problem.

1 The Multisensor Fusion Problem

The Multisensor Fusion Problem (MFP) deals with the methodologies involved in effectively combining together homogeneous or non-homogeneous information obtained from multiple redundant or disparate sensors in order to perform a task more accurately, efficiently, and reliably. A single sensor is never sufficient enough to provide the complete reliable information needed by an intelligent system. Multiple sensors are needed in order to confirm each other's measurements, thus increasing the reliability of the sensory information, and to provide a more complete information about the world by combining partial information from each of the sensors. MFP involves choosing an uncertainty representation for the sensory information, deciding on the level of the information processing hierarchy at which the fusion process is to take place, maintaining the kinematic and feature correspondence among the sensory information, and finally fusing the multiple sensory information into one representative consensus information.

The inherent uncertainties in the sensory information, which can arise due to noise, limitations of the sensor, changes in the environment, or changes in the system itself, can generally be represented in 4 different ways [7] [8]: no uncertainty representation, probabilistic representation, representation using belief measures, and possibilistic representation. We use Fuzzy numbers and possibility distributions [3] [17] to represent the sensory information. When a sensor provides a measurement m about a feature f of the world, the sensor is actually providing a linguistic proposition of the form "the feature f is *about* m". A possibility distribution $\pi(x)$ can be determined to represent the degree of possibility that any x in the referential set X might actually be the true value f. Using Dubois and Prade's definition of the LR-type Fuzzy number [4] [18], the mean-value m_i of the $\mathcal{F}$-number $\tilde{F}_i$ represents the actual sensor measurement and the spread α_i of the $\mathcal{F}$-number represents the uncertainty level of the $\mathcal{F}$-number (see the next section for the explanation).

The fusion process can be implemented at the data level, the feature level, the knowledge level or the decision level in the information processing hierarchy. The performance of the fusion operation is dependent on the fusion level chosen. We perform the fusion at the feature level where the fusion process is divided into two fusion subprocesses which can run in parallel: the mean-value fusion process $g_m(\cdot)$ and the uncertainty-fusion process $g_u(\cdot)$. Together they form the overall fusion process $G(\cdot)$ which is called the Uncertainty-Reductive Fusion Technique (URFT). There is a need to fuse the uncertainties inherent in the sensory information as well as the measurement values in order to provide some sort of confidence or reliability measure about the information the system is processing. Any decision made or action taken by the system can be based on these confidence measures which would increase the dependability and the correctness of the decisions or actions.

The Multisensor Fusion Problem can be formally stated as:

> **Given a *sheaf* of Fuzzy-numbers, $\tilde{F}_0, \tilde{F}_1, \ldots, \tilde{F}_N$, which constitutes a sensory information vector for one feature f, determine the consensus Fuzzy-number $\tilde{F}_C$ which "best" represents the information conveyed by all the N Fuzzy-numbers and accurately estimates the true feature value.**

We approach the MFP from the Information Theory perspective [1] [6] where the sensors are modeled as information sources with a fixed output alphabet Γ and systems are modeled as networks of information processing and propagating channels. Before the fusion process, the sheaf of N $\mathcal{F}$-numbers are at the maximum uncertainty level. However, as the overlapping or confirmative information between the $\mathcal{F}$-numbers is identified and subtracted from the initial uncertainty level, the uncertainty of the overall system decreases until all the provided information are used and the minimum attainable uncertainty level is reached. The uncertainty of the system after the fusion process yields the uncertainty level of the consensus $\mathcal{F}$-number.

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Preliminary Concepts and Definitions

The measure of uncertainty associated with a $\mathcal{F}$-number is determined using Higashi and Klir's U-uncertainty measure [9] [11]. U-uncertainty measure is a measure of possibilistic uncertainty which is derived from Harteley's Measure of Information and is based more on the concepts of the number of choices rather than probabilities. The inherent uncertainty associated with sensory measurements are based on imprecision rather than randomness or entropy; therefore, the U-uncertainty measure is ideal in determining the uncertainty level of a $\mathcal{F}$-number representing sensory information.

$$u_i = U(\tilde{F}_i) = \sum_{j=1}^{l}(\rho_j - \rho_{j+1})log|A_{\rho_j}| \quad (1)$$

where ρ_j $(j = 1, 2, \ldots, l)$ is the ordered possibility distribution with $\rho_{l+1} = 0$, $\rho_1 = 1$ and $\rho_j \geq \rho_{j+1}$. Note also that ρ_j is equivalent to the α-levels for the α-level sets A_α.

Some properties of the U-uncertainty measure which are relevant to our formulation are: 1) shift invariance, that is, $U(\pi(x)) = U(\pi(x+k))$ for some $k \varepsilon \Re$ (this means that the uncertainty measure is independent of the mean-value m of the $\mathcal{F}$-number and only dependent on the spread α of the $\mathcal{F}$-number) and 2) monotonicity, that is, $U(\pi_1(x)) \leq U(\pi_2(x))$ if and only if $\pi_1(x) \leq \pi_2(x)$ for all $x \varepsilon X$ (this means that $U(\tilde{F}_1) \leq U(\tilde{F}_2)$ if and only if $\alpha_1 \leq \alpha_2$).

Definition 1.1 *The confidence level c_i of a $\mathcal{F}$-number $\tilde{F}_i$ is defined as the reciprocal of the uncertainty level,*

$$c_i = 1/u_i \quad (2)$$

A measure of confidence or reliability is more appropriate in basing decisions upon than the measure of uncertainty since the term uncertainty is too vague and too broad.

2 Uncertainty Fusion

The fusion of the N $\mathcal{F}$-numbers into a consensus $\mathcal{F}$-number involves two fusion subprocesses $g_m(\cdot)$ and $g_u(\cdot)$ where the former fuses the mean-values of the $\mathcal{F}$-numbers, which is equivalent to the fusion of the measurement values of the N sensors, and the latter fuses the uncertainty levels or the confidence levels of the N sensors. The overall fusion process $G(\cdot)$ is called the Uncertainty-Reductive Fusion Technique (URFT) and it is designed to meet the following general Fusion Criteria:

1. The fusion subprocesses $g_m(\cdot)$ and $g_u(\cdot)$ must be convex.
2. If $U(\tilde{F}_1) < U(\tilde{F}_2)$, then $d(m_C, m_1) < d(m_C, m_2)$, where $d(\cdot)$ = distance measure between two real numbers.
3. $\lim_{n\to\infty} G(\tilde{F}_1, \tilde{F}_2, \ldots, \tilde{F}_n) = f$, where f is the true value of the feature being measured.

The interpretation of the Fusion Criteria is as follows : 1) the aggregation functions must be convex in order to ensure the convergence of the algorithm, 2) the mean-value of the consensus $\mathcal{F}$-number should be closest to the mean-value of the most confident $\mathcal{F}$-number, and 3) the consensus should be an unbiased and consistent estimate of the measured feature f. The meaning of criterion 3 can be better understood by dividing it into two separate limits: $\lim_{n\to\infty} g_m(m_1, m_2, \ldots, m_n) = f$ and $\lim_{n\to\infty} g_u(\alpha_1, \alpha_2, \ldots, \alpha_n) = 0$.

The mean-value fusion is performed using the Linear Opinion Pool with confidence weighting,

$$g_m(m_1, m_2, \ldots, m_N) = m_C = \sum_{i=1}^{N} w_i m_i \quad (3)$$

where the weights $w_i = c_i$ and $\sum_{i=1}^{N} w_i = 1$.

The fusion of the uncertainty levels is not so straight forward since determining the amount of overlapping information among the N $\mathcal{F}$-numbers is very complicated and in some sense non-standard. The use of conventional techniques such as fuzzy integrals or the simple t-norm minimum operator are not satisfactory because the confidence of the resulting solution is not updated. Instead of identifying and determining all the overlapping information among the N $\mathcal{F}$-numbers a more global and macro approach is taken.

Definition 2.1 *Let $U_O(\tilde{F}_1, \tilde{F}_2)$ be the measure of overlapping information or redundant information provided by two $\mathcal{F}$-numbers $\tilde{F}_1$ and $\tilde{F}_2$. Then $\tilde{F}_1$ and $\tilde{F}_2$ are said to be "confirmative" or are said to provide "overlapping" information if $U_O(\tilde{F}_1, \tilde{F}_2) > 0$.*

The Information Transmission [11] is defined as,

$$T(\tilde{F}_1, \tilde{F}_2) = U(\tilde{F}_1) + U(\tilde{F}_2) - U_O(\tilde{F}_1, \tilde{F}_2). \quad (4)$$

which is equal to the uncertainty of the consensus $\mathcal{F}$-number when $\tilde{F}_1$ and $\tilde{F}_2$ are fused together.

$$U(\tilde{F}_C) = U(G(\tilde{F}_1, \tilde{F}_2)) = U(\tilde{F}_1) + U(\tilde{F}_2) - U_O(\tilde{F}_1, \tilde{F}_2). \quad (5)$$

Note that $U(\tilde{F}_1) + U(\tilde{F}_2)$ is the uncertainty of the system before the fusion process and so,

$$U_{init} = U(\tilde{F}_1) + U(\tilde{F}_2) \quad (6)$$

Therefore, if there is no overlapping information, that is, $U_O(\cdot)$ is equal to zero, then $U(\tilde{F}_C) = U_{init}$. If $U_O(\cdot) > 0$, then this means there is some overlapping information between the two $\mathcal{F}$-numbers so $U(\tilde{F}_C) < U_{init}$ and if $U_O(\cdot)$ provides maximum overlapping information, then $U(\tilde{F}_C) = U_{min}$ which is determined using the Principle of Maximum Confirmation.

Let λ be the degree of confirmation or overlap. For no confirmative information, $\lambda = 0$ and for maximum confirmative information, $\lambda = 1$. Therefore, as λ varies from 0 to 1, the measure of overlapping information among all the N $\mathcal{F}$-numbers varies from 0 to $[U_{max} - U_{min}]$. Therefore, the overlapping information measure is given by,

$$U_O(\tilde{F}_1, \tilde{F}_2, \ldots, \tilde{F}_N) = \lambda(U_{max} - U_{min}). \quad (7)$$

There are a lot of different ways to define the degree of confirmation, λ, such as using similarity measures or cardinality measures; however, in conforming with the Information Theory approach, we define it as,

$$\lambda = U(\pi_1 * \pi_2 * \ldots * \pi_N)/U_{ref} \quad (8)$$

where $*$ is the algebraic product operator and U_{ref} is the normalizing factor which is equal to the the uncertainty of the product $\pi_1 * \pi_2 * \ldots * \pi_N$ for the maximum confirmative state.

Proposition 2.1 *(The Principle of Maximum Confirmation) For a sheaf of N $\mathcal{F}$-numbers, the maximum confirmation occurs when $m_1 = m_2 = \ldots = m_N$.*

The uncertainty update or the uncertainty of the consensus $\mathcal{F}$-number is given by,

$$U(\tilde{F_C}) = U_{init} - \lambda[U_{max} - U_{min}] \quad (9)$$

or

$$U(\tilde{F_C}) = (1-\lambda)U_{init} + \lambda U_{min} \quad (10)$$

where $U_{init} = U_{max} = \sum_{i=1}^{N} U(\tilde{F_i})$ and $U_{min} = U_{ref}$. This is simply a linear weighting between U_{max} and U_{min}.

The same concepts are extended to the fusion of more than 2 $\mathcal{F}$-numbers; however, unlike the fusion of 2 $\mathcal{F}$-numbers, as the number of $\mathcal{F}$-numbers increases the number of possible combinations of pairs or groups of $\mathcal{F}$-numbers providing overlapping information grows exponentially. So it is almost impossible and inefficient to try to identify and quantize all the overlapping information provided by the N $\mathcal{F}$-numbers.

2.1 Generation of the $\mathcal{F}$-numbers

As a precursor to the fusion process, there are 3 stages of pre-processing which need to be performed. First, the sensor measurements need to be fuzzified; in other words, a $\mathcal{F}$-number needs to be associated with each sensor's output. Second, the $\mathcal{F}$-numbers need to be transformedin order to maintain the kinematic correspondence between the different sensors which means that all the sensory information need to be referenced from a common reference frame. The feature correspondence also needs to be checked in order to make sure that only the information about the same feature is identified and fused together. Finally, the fusible information and the unfusible information need to be separated in order for the fusion process to be robust and accurate. This section illustrates the ideas behind the first pre-processing stage and the next section deals with the third stage. The second stage of information correspondence is a research area of its own right; refer to [5] [15] for the kinematic correspondence problem in sensor fusion and papers concerning feature matching in stereo imaging or image registration in computer vision for the feature correspondence problem.

There are basically 3 different ways of fuzzifying sensory information. The first is to use probabilistic data. The spread of the $\mathcal{F}$-number can be defined to be twice the standard deviation of the sensor [12] or some sample measurements can be taken by the sensor to transform the probabilistic data to membership functions or possibility distributions [2] [16]. The second method is more subjective in nature where a reliability index is associated with each of the sensors' outputs depending on the reliability or the accuracy of each physical sensor. This method is justifiable since the mean-value fusion is a Linear Opinion Pool with reliability weighting. The third method is a method developed and currently being researched by the authors. The method uses Maximum Uncertainty Possibility Distributions (MUPD) which are determined using Entropy or Uncertainty Optimization Principles [10] and uses fusion techniques to learn the membership functions,that is, to self-adapt to a representative membership function as sample measurements are taken. This is a concept of dynamic generation of $\mathcal{F}$-numbers.

2.2 Clustering and the Fusibility Condition

All N of the $\mathcal{F}$-numbers cannot be fused together at all times. The multisensor fusion system is designed to fuse all the N sensory information; however, due to the unpredictability of the world, only a subset M of the sheaf of $\mathcal{F}$-numbers can be fused at any given time. The Fusibility Condition is used to distinguish between fusible and unfusible sets of information.

Definition 2.2 *(Fusibility Condition) The sheaf of N $\mathcal{F}$-numbers are said to be fusible if and only if $U_C < min[U(\tilde{F_1}), U(\tilde{F_2}), \ldots, U(\tilde{F_N})]$.*

The Fusibility Condition is an extension of Swinburne's Theory of Confirmation [13] and it states that we should only fuse multiple information together if the resultant consensus $\mathcal{F}$-number is more reliable than any of the input $\mathcal{F}$-numbers. If the fusion process yields a consensus which is less reliable than one of the input $\mathcal{F}$-numbers, then there is no advantage in fusing the multiple information over simply picking the minimum uncertainty input $\mathcal{F}$-number as the consensus $\mathcal{F}$-number. URFT is designed to reduce the overall uncertainty of the system or at least not introduce more uncertainty into the system than before the fusion process, so the Fusibility Condition is an integral part of the URFT. Another reason for clustering the set of N $\mathcal{F}$-numbers into fusible and unfusible elements is to ensure that "faulty" information, that is, the information from faulty sensors, is not fused to form the consensus value.

Let $U(\tilde{F}_{min}) = min[U(\tilde{F_1}), U(\tilde{F_2}), \ldots, U(\tilde{F_N})]$. The decision to fuse or not to fuse is given by

Fuse if and only if $\lambda > [U_{max} - U(\tilde{F}_{min})]/[U_{max} - U_{min}]$.

The actual clustering algorithm is :

1. Let $M = N$ and place all the N $\mathcal{F}$-numbers in the *fuse-list*. Try fusing the N $\mathcal{F}$-numbers. If fusible, then done.
2. Let $M = M - 1$. If $M = 1$, then done.
   Or else, take out the first element of the *fuse-list* and and classify it as an unfused element. Try fusing the remaining M $\mathcal{F}$-numbers in the *fuse-list*. For each of the M elements in the *fuse-list*, replace it with the unfused element and perform the fusion for the new set of M $\mathcal{F}$-numbers.
3. If some set of M $\mathcal{F}$-numbers are fusible, then choose the set which produced the minimum $U(\tilde{F_C})$ value and place the elements of this set in the *fuse-list* and the remaining one unfused element in the *conflict-list*. Then done.
   Or else, choose the set of M $\mathcal{F}$-numbers which produced the largest measure of overlapping information and place the element of this set in the *fuse-list* and the remaining unfused element in the *conflict-list*. Go to step 2.

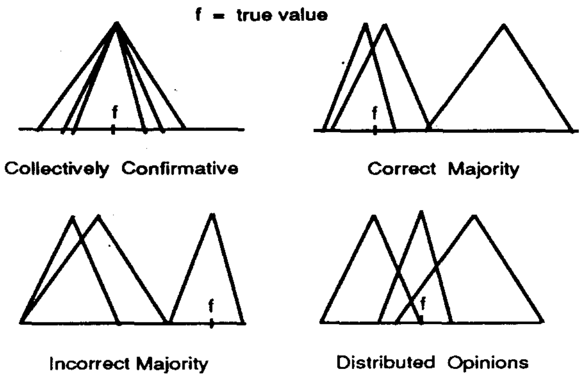

Figure 1: The 3-Sensor Problem

3 Simulation Results

The performance of the fusion technique can be evaluated by comparing the technique with other existing fusion methods. The problem we will consider is the 3-sensor Problem which is stated as follows : let s_1, s_2, and s_3 be the 3 sensors which provide outputs m_1, m_2, and m_3, respectively. Let the uncertainty of the sensors be represented using the variances $\sigma_1^2 = 0.1$, $\sigma_2^2 = 0.2$, and $\sigma_3^2 = 0.3$ or the uncertainty $u_1 = 1.977$ ($\alpha_1 = 0.5$), $u_2 = 2.929$ ($\alpha_2 = 1.0$), and $u_3 = 3.903$ ($\alpha_3 = 2.0$). Let $f = 5.0$ be the true value of the feature being measured. The 3-sensor problem has 4 cases to consider.

Case **I.** Collectively Confirmative
$m_1 = m_2 = m_3 = 5.0$
Case **II.** Correct Majority
$m_1 = 4.9$ $m_2 = 5.2$ $m_3 = 7.5$
Case **III.** Incorrect Majority
$m_1 = 4.9$ $m_2 = 3.4$ $m_3 = 3.7$
Case **IV.** Distributed Opinion
$m_1 = 5.0$ $m_2 = 4.0$ $m_3 = 6.0$

The fusion techniques considered are :

1. **Mean filter**
2. **Majority filter**
3. **Durrante-Whyte's Bayesian Updating [5]**
4. **URFT**

Table 1: Comparison of Fusion Techniques

case	method	m_c	confidence	method	m_c	confidence
I	1	5.0		3	5.0	$s_c^2 = 0.055$
I	2	5.0		4	5.0	$u_c = 1.679$
II	1	5.87		3	5.0	$s_c^2 = 0.067$
II	2	5.20		4	5.02	$u_c = 2.224$
III	1	4.0		3	3.52	$s_c^2 = 0.120$
III	2	3.7		4	3.53	$u_c = 2.782$
IV	1	5.0		3	4.91	$s_c^2 = 0.055$
IV	2	5.0		4	5.33	$u_c = 3.588$

4 Conclusion

The use of $\mathcal{F}$-numbers to represent sensory information proved to be very effective. The need for fusing uncertainties associated with sensor measurements was illustrated and the performance of the proposed Uncertainty-Reductive Fusion Technique was shown compared to other fusion techniques.

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