# A Fuzzy Logic Controller Using Error, Change of Error, and Control Input

Jihong Lee\* and Seog Chae\*

\* Dept of Control Engineering

Kum-oh National Institute of Technology

188 Shinpyung-dong, Kumi, Kyungbuk 730-701, Korea

#### ABSTRACT

We propose a new PI FLC which utilizes the error(e), change of error( $\Delta e$ ), and previous control(u(k-1)). It is shown by simulation that the proposed scheme gives better performance in steady state than the conventional PI FLC.

#### 1. Introduction

Recently, fuzzy control has emerged as one of the most fruitful research areas in fuzzy set theory[1]. Many practical applications to industrial process, as well as studies on the theory itself, have been reported in many works. The application area may be classified into two groups: one is the case where controlled process has so many variables that conventional control methods are difficult to be applied, and the other one is the case where, even if the controlled system is simple, conventional linear control algorithms show limits on performances.

Even though the details may differ with each other, two types of the structure of the fuzzy logic controller have been studied so far: the one is position-type fuzzy controller which generates control input(u) from error(e) and change of error( $\Delta e$ )[2-3], and the other is velocity-type fuzzy logic controller which generates incremental control input( $\Delta u$ ) from error and change of error  $\Delta e$  (or change of error rate  $\Delta \Delta e$  may be included)[4]. The former is called PD FLC and the latter is called PI FLC according to the characteristics of information that they process. In the view point that the FLC

is based on knowledges of human experts and that FLC is generally applied to unknown or partially known system, PI FLC is known to be more feasible than PD FLC. The PI FLC gives good performance in steady state, but gives poor performance in transient state. To improve the transient response of PI FLC is not easy especially for a system of order more than one. This may be one of main reason why such many works handling PI FLC have adopted first order system for simulations. Even in the work in which second order system was considered, the maximum variation of the incremental control input( $\Delta u$ ) is limited to somewhat small one to reduce overshoot of the transient response(but this approach cause a larger rise time).

One natural approach to overcome such difficult situation is to adopt change of error rate( $\Delta e$ ). If this quantity is adopted, the fuzzy controller may be called as PID FLC. It is not easy, however, to measure the instantaneous value of the quantity, and also it is hardly believed that an expert senses acceleration terms of the error at every instance in his control action. Even in the method of approximating  $\ddot{e}$  in terms of  $\Delta e(k)$  and  $\Delta e(k-1)$ , we can point out a problem that some information of previous sampling time should be memorized continuously.

In this work, we use the previous control input u(k-1) as an alternative quantity to  $\ddot{e}$ . Our motivation of introducing control input u instead of  $\ddot{e}$  stems from the following observations: i) an expert may fuzzily know the control input

exerted by himself at every sampling time, and ii) acceleration of a system is related to force exerted on the system.

A new FLC structure utilizing the error(e), change of error( $\Delta e$ ), and previous control(u(k-1)), is proposed and its usefullness is shown by computer simulation.

# 2. A New Fuzzy Logic Controller

The structure of the proposed FLC and the internal information processing are shown in Fig.1 and Fig.2, respectively. We know from Fig.2 that the proposed FLC consists of two parts. One is the conventional fuzzy PI part which calculates the normal incremental control input  $\Delta u(k)$ , and the other is the weighting mechanism which calculates the weight  $w_{ij}$  exerted on the normal incremental control  $\Delta u(k)$ .

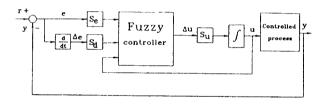


Fig. 1. Structure of proposed fuzzy logic controller

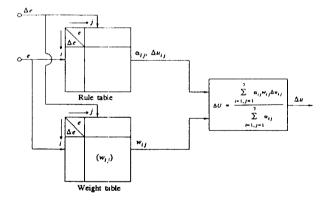


Fig. 2. Internal information processing of the proposed fuzzy logic controller.

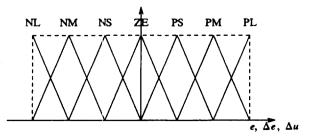


Fig.3: Fuzzy values of e,  $\Delta e$ , and  $\Delta u$ 

# 2-1. Conventional Fuzzy PI part[4]

The conventional fuzzy PI part plays a role of calculation of  $\Delta u_{ij}$  from e and  $\Delta e$ . The ij-th linguistic control rule is represented by Eq.(1) and the overall control rules are given in Table1.

 $R_{ij}$ : if e is  $A_{ij}$  and  $\Delta e$  is  $B_{ij}$  then  $\Delta u$  is  $C_{ij}$ , (1) where  $A_{ij}$ ,  $B_{ij}$ , and  $C_{ij}$  are shown in Fig.3.

Then  $\Delta u_{ij}$  is calculated by using Mamdani's method as follows[4].

The compatibility  $\alpha_{ij}$  of the ij-th control rule for given e and  $\Delta e$  is calculated by Eq.(2).

$$\alpha_{ij} = \mu_{A_{ij}}(e) \wedge \mu_{B_{ij}}(\Delta e) \qquad (2)$$

The consequence  $C'_{ij}$  is calculated by Eq.(3).

$$\mu_{C'_{ij}} = \alpha_{ij} \wedge \mu_{C_{ij}} \tag{3}$$

 $\Delta u_{ij}$  is calculated by using the center of gravity method.

$$\Delta u_{ij} = cog(C'_{ij}) \tag{4}$$

# 2-2. Weighting Mechanism

This part calculates the weight  $w_{ij}$  for  $\Delta u_{ij}$ . We know from Table1 that the control rule  $R_{ij}$ ,  $j \ge -i + 8$  plays a role of acceleration. Since the fuzzy control rule in Table1 does not consider the previous value of control u(k-1), u(k) may be increased even though u(k-1) is sufficiently large. It may

	j							
*· *	Δε	NB	NM	NS	ZE	PS	PM	PB
	NB	NB	NB	NB	NB	NM	NS	ZE
	NM	NB	NB	NB	NM	NS	ZE	PS
	NS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	РВ
	PS	NM	NS	ZE	PS	РМ	РВ	PB
	PM	NS	ZE	PS	PM	РВ	РВ	РВ
	PB	ZE	NS	PM	РВ	PB	PB	РВ

Table 1: Linguistic control rules

arise a large overshoot. To obtain a good performance, we can intuitively say that the incremental control input  $\Delta u(k)$  should be dependent on the previous control value u(k-1). Specifically, this relationship can be described as follows.

If e(k) is PB and u(k-1) is larger than or equal to PB, then reduce  $\Delta u(k)$ ,

If e(k) is PM and u(k-1) is larger than or equal to PM, then reduce  $\Delta u(k)$ ,

If e(k) is PS and u(k-1) is larger than or equal to PS, then reduce  $\Delta u(k)$ .

To achieve the above idea, we use the ij-th weighting function  $w_{ij}$  for  $\Delta u_{ij}$  described by Eq.(5) and shown in Fig.4.

$$w_{ij} = \begin{cases} 0, & \text{if } u(k-1) > r_j \\ 1, & \text{if } u(k) < l_j \\ \frac{r_j - u(k-1)}{r_j - l_j}, & \text{otherwise} \end{cases}$$
 (5)

where u(k-1) is the previous control,  $r_j$  and  $l_j$  are given as follows:

$$r_j = (2j-6) \times \frac{U_{\text{max}}}{6},\tag{6}$$

$$l_j = (2j-9) \times \frac{U_{\text{max}}}{6}$$
 (7)

where  $U_{\text{max}}$  is the maximum value of the universe of discourse for control u(k).

On the other hand, the control rule  $R_{ij}$ ,  $j \le -i + 8$  plays a role of decelleration and its corresponding weighting function  $w_{ij}$  for  $\Delta u_{ij}$  is described by Eg.(8) and is shown in Fig.4.

$$w_{ij} = \begin{cases} 0, & \text{if } u(k) < I_j \\ 1, & \text{if } u(k-1) > r_j, \quad j \le -i+8, \quad i, j=1, \cdots, 7. \\ \frac{I_i - u(k-1)}{I_j - r_j}, & \text{otherwise} \end{cases}$$
 (8)

where  $r_j$  and  $l_j$  are given by the following equations:

$$r_j = (2j-8) \times \frac{U_{\text{max}}}{6},\tag{9}$$

$$l_j = (2j-11) \times \frac{U_{\text{max}}}{6}$$
 (10)

#### 2-3. Calculation of control input u(k)

The incremental control input  $\Delta u$  and control input u(k) are finally calculated as follows:

$$\Delta u = \frac{\sum_{i=1, j=1}^{7} \alpha_{ij} w_{ij} \Delta u_{ij}}{\sum_{i=1, j=1}^{7} \alpha_{ij}}, \qquad (11)$$

$$u(k) = u(k-1) + \Delta u(k).$$
 (12)

### 3. Computer Simulation

Consider a plant described by the following equation.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s(s+1)}$$
 (13)

We know from many simulations that it is difficult to find out the control parameters of PI FLC for the above plant such that give the desirable responses.

The unit step responses of the conventional PI FLC system and the proposed PI FLC system are shown in Fig.5 with  $(S_e, S_d, S_u) = (1, 0.2, 0.5)$ . We observe from Fig.5 that the

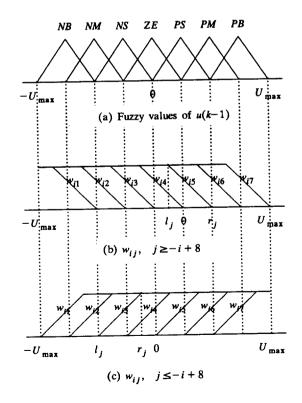


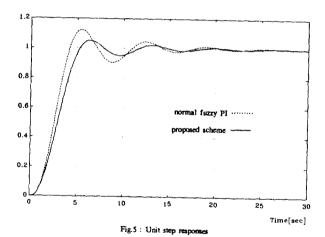
Fig. 4: Weighting functions  $w_{ij}$ 

proposed scheme gives better performance in steady state that the conventional PI FLC scheme.

# 4. Concluding Remarks

In this work, we have proposed a new PI FLC, in which utilizes the error(e), change of error( $\Delta e$ ), and previous control(u(k-1)). It is shown by simulation that this scheme gives better performance in steady state than the conventional PI FLC.

But this scheme has a disadvantage: Since the steady state value of control u is dependent on the reference and the load disturbance, it is difficult to determine the weighting functions given by Eq.(3) -Eq.(5) without using the information about plant to be controlled.



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