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FUZZY CONTROLS BY FUZZY SINGLTON-TYPE REASONING METHOD

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1. INTRODUCTION

This paper proposes new fuzzy control method called "fuzzy singleton-type reasoning method" in which the consequent part of a fuzzy control rule is a fuzzy singleton (that is, a real number with a positive weight) rather than a fuzzy set and a real number. This fuzzy reasoning method is a generalized version of a simplified fuzzy reasoning method [1,7] whose consequent part consists of a real number. By using a fuzzy singleton in the consequence part, emphatic and restrained effects can be realized on the fuzzy inference results.

It is shown that good control results are obtained by using the fuzzy singleton-type reasoning method as a fuzzy control method since this method can adjust subtly control results by changing the weights of fuzzy control rules.

2. SIMPLIFIED FUZZY REASONING METHOD AND FUZZY SINGLETON-TYPE REASONING METHOD

We shall first review a fuzzy reasoning method named *simplified fuzzy reasoning method* [1] which treats the following multiple fuzzy reasoning form whose consequent part consists of a real number z_i $(i=1,\cdots,n)$:

$$\begin{array}{ll} \text{Rule 1:} & A_1 \text{ and } B_1 \Rightarrow z_1 \\ \text{Rule 2:} & A_2 \text{ and } B_2 \Rightarrow z_2 \\ & \cdots\cdots\cdots \\ \text{Rule n:} & A_n \text{ and } B_n \Rightarrow z_n \\ \text{Fact:} & x_\circ \text{ and } y_\circ \\ \hline \text{Cons:} & \qquad z_\circ \end{array} \qquad (1)$$

where A_i is a fuzzy set in X; B_i in Y; and z_i, $z_\circ$ are real numbers and $x_\circ \in X$, $y_\circ \in Y$.

The consequence $z_\circ$ by the simplified fuzzy reasoning method is obtained as follows. The degree of fitness of the fact 「$x_\circ$ and $y_\circ$」 to the antecedent part 「A_i and B_i」 is given as

$$h_i = \mu_{A_i}(x_\circ) \cdot \mu_{B_i}(y_\circ) \qquad (2)$$

The degree of fitness, h_i, is regarded as the degree to which z_i is obtained. Thus, the final consequence $z_\circ$ of (1) is obtained as the weighted average of z_i by the degree h_i. Namely,

$$z_\circ = \frac{h_1 \cdot z_1 + h_2 \cdot z_2 + \cdots + h_n \cdot z_n}{h_1 + h_2 + \cdots + h_n} \qquad (3)$$

Note that the simplified fuzzy reasoning method is regarded as a special case of product-sum-gravity method [2-5], but not a special case of *min-max-gravity method* known as Mamdani's fuzzy reasoning method [6].

Usually, an identical fuzzy control rule is not used two or many times at a time in the execution of fuzzy controls. However, an identical fuzzy control rule can be used twice or more in the simplified fuzzy reasoning method and thus emphatic effects are obtained on fuzzy inference results.

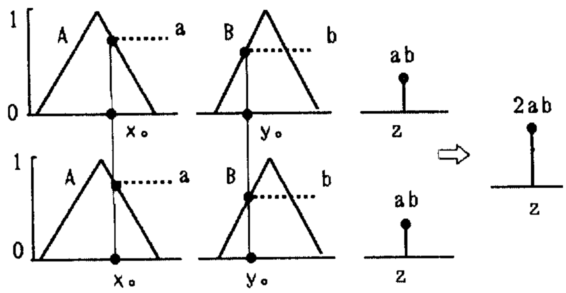


Fig.1 Emphatic effect by using two same rules

For example, suppose that a fuzzy control rule A and B => z is used twice, that is,

$$\begin{array}{ll} \text{Rule 1:} & A \text{ and } B \Rightarrow z \\ \text{Rule 2:} & A \text{ and } B \Rightarrow z \end{array} \qquad (4)$$

are used simultaneously, then the inference result obtained is double, i.e., 2ab in height as shown in Fig.1. Hence, by using the identical fuzzy rule twice, we can have an inference result at double height and thus double emphatic effect is given on the inference result. Therefore, the role of a fuzzy control rule is enhanced every time it is used twice or more at the same time.

In general, to obtain emphatic effect of w times, it is suggested to use a fuzzy rule whose consequent part is not a real number but a fuzzy singleton, that is, a real number z with grade w. Namely, we can define the following fuzzy rule of fuzzy singleton type:

$$A \text{ and } B \Rightarrow w/z \qquad (5)$$

where the notation w/z stands for a fuzzy singleton, that is, a real number z with weight w. It is assumed that the weight w is a real number such as $w \geq 0$. Clearly, when $w = 1$, a fuzzy rule of (5) reduces to an ordinary fuzzy rule.

Hence we can define a new fuzzy reasoning method called "*fuzzy singleton-type reasoning method*" or "*simplified fuzzy reasoning method with weight*" which treats the following fuzzy reasoning form:

$$\begin{array}{ll} \text{Rule 1:} & A_1 \text{ and } B_1 \Rightarrow w_1/z_1 \\ \text{Rule 2:} & A_2 \text{ and } B_2 \Rightarrow w_2/z_2 \\ & \cdots\cdots\cdots \\ \text{Rule n:} & A_n \text{ and } B_n \Rightarrow w_n/z_n \\ \text{Fact:} & x_\circ \text{ and } y_\circ \\ \hline \text{Cons:} & \qquad z_\circ \end{array} \qquad (6)$$

where the weight w_i is a real number such as $w_i \geq 0$. $w_i \geq 1$ means that the corresponding rule A_i and $B_i \Rightarrow w_i/z_i$ is "emphasized," and $0 \leq w_i \leq 1$ means that the rule is "restrained."

The consequence $z_\circ$ by the fuzzy singleton-type reasoning method is obtained as follows (see Fig.2). The degree of fitness h_i of the fact 「$x_\circ$ and $y_\circ$」 to the antecedent part 「A_i and B_i」 is given as (2), and the product $h_i w_i$ of the fitness

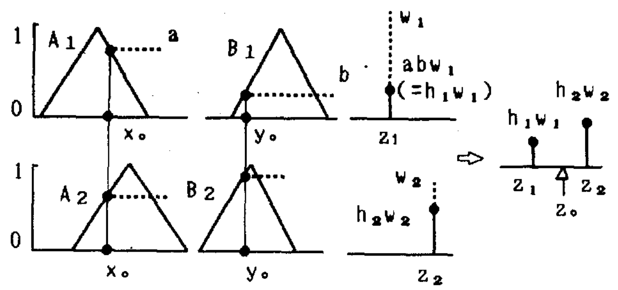


Fig.2 Fuzzy singleton-type reasoning method

h_i and the weight w_i of z_i is regarded as the degree to which z_i is obtained. Therefore, the final consequence z. of (6) is infered as the weighted average of z_i by the degree $h_i w_i$:

$$z. = \frac{h_1w_1z_1 + h_2w_2z_2 + \cdots + h_nw_nz_n}{h_1w_1 + h_2w_2 + \cdots + h_nw_n} \quad (7)$$

As a simple example, we shall consider the following fuzzy rules:

$$\begin{array}{ll} \text{Rule 1: } \underset{\sim}{x1} \Rightarrow w1/y1 & \\ \text{Rule 2: } \underset{\sim}{x2} \Rightarrow w2/y2 & (8) \\ \text{Fact: } \quad x. & \\ \hline \text{Cons: } \qquad\qquad y. & \end{array}$$

where $\underset{\sim}{xi}$ stands for a triangular fuzzy set representing "about xi" (see Fig.3). Let a be the grade at x. of fuzzy set $\underset{\sim}{x1}$, then the consequence y. is given from (7) as

$$y. = \frac{aw1y1 + (1-a)w2y2}{aw1 + (1-a)w2} \quad (9)$$

which leads to the following expression by dividing the denominator and numerator by w1 and by letting w = w2/w1 :

$$y. = \frac{ay1 + (1-a)wy2}{a + (1-a)w}, \quad a = \mu_{\underset{\sim}{x1}}(x.) \quad (10)$$

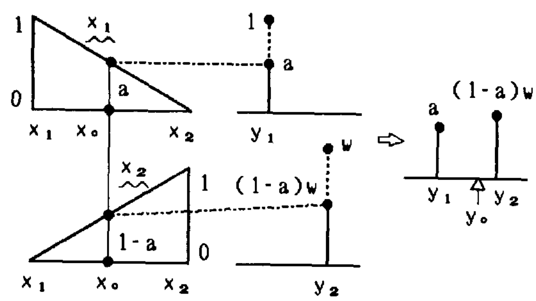


Fig.3 Fuzzy singleton-type reasoning for (10)

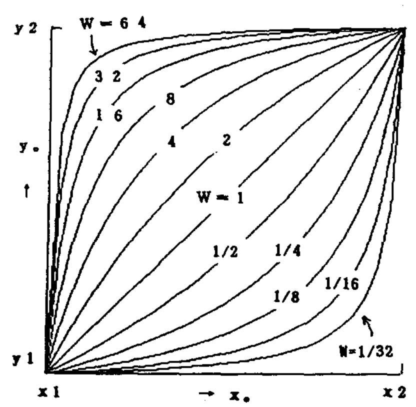


Fig. 4 Inference result y. of (10) when changing w (=w2/w1)

Clearly, this inference result y. comes from the following fuzzy reasoning form of fuzzy singleton type:

$$\begin{array}{ll} \text{Rule 1: } \underset{\sim}{x1} \Rightarrow 1/y1 & \\ \text{Rule 2: } \underset{\sim}{x2} \Rightarrow w/y2 & (11) \\ \text{Fact: } \quad x. & \\ \hline \text{Cons: } \qquad\qquad y. & \end{array}$$

Fig.4 indicates the diagrams of y. when changing w (= w2/w1). At w = 1 (i.e., w1 = w2), the diagram becomes linear, which corresponds to the case of a simplified fuzzy reasoning method. On the contrary, we can have nonlinear results at $w \neq 1$.

As the continuation of this example, we shall consider the following reasoning form of fuzzy singleton type (see Fig.5 and 8(a)):

$$\begin{array}{ll} \text{Rule 1: } \underset{\sim}{-x1} \Rightarrow 1/-y1 & \\ \text{Rule 2: } \underset{\sim}{0} \Rightarrow w/\ 0 & \\ \text{Rule 3: } \underset{\sim}{x1} \Rightarrow 1/\ y1 & (12) \\ \text{Fact: } \quad x. & \\ \hline \text{Cons: } \qquad\qquad y. & \end{array}$$

Fig.6 shows the inference results y. at w = 1/16, 1/4, 1, 4, and 16. It is found from the figure that the slope at x. = 0 can be adjusted subtly by changing the weight w of Rule 2. As will be shown in the next section, better control results can be obtained since it is possible to adjust finely the control in the vicinity of a setting point by changing the weights of fuzzy control rules.

When we want to obtain the inference results of Fig.6 by using a simplified fuzzy reasoning method of (1), more fuzzy rules will be needed in general. For example, seven fuzzy rules of (13) are needed to approximate piece-wise lineally the inference result at w = 1/16 (see Fig.7) which can be realized by just three fuzzy rules of (12) in the fuzzy singleton-type reasoning method.

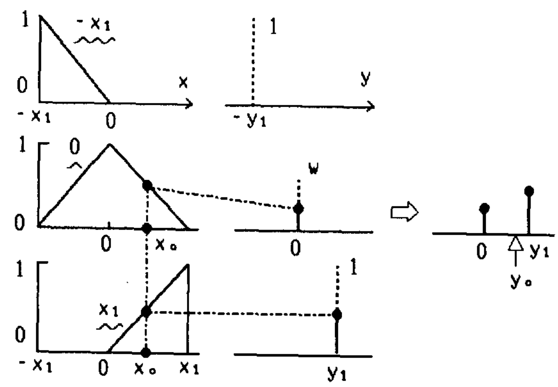


Fig.5 Fuzzy reasoning of (12)

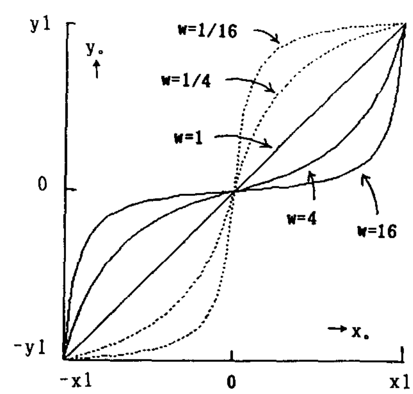


Fig. 6 Inference results of (12)

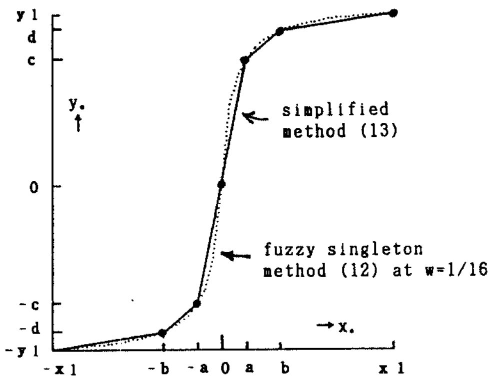
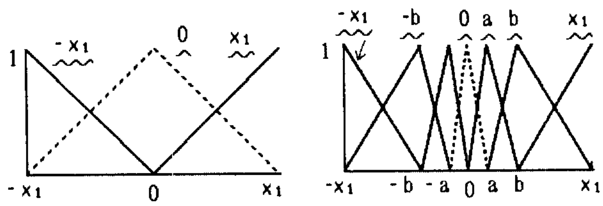


Fig. 7 Inference results of (12) at w=1/16 by fuzzy singleton-type reasoning method and (13) by simplified reasoning method



(a) 3 fuzzy sets (b) 7 fuzzy sets

Fig.8 Fuzzy sets partitioning $[-x_1, x_1]$

$$
\begin{array}{llll}
\text{Rule 1:} & \underset{\sim}{-x1} & \Rightarrow & -y1 \\
\text{Rule 2:} & \underset{\sim}{-b} & \Rightarrow & -d \\
\text{Rule 3:} & \underset{\sim}{-a} & \Rightarrow & -c \\
\text{Rule 4:} & \underset{\sim}{0} & \Rightarrow & 0 \\
\text{Rule 5:} & \underset{\sim}{a} & \Rightarrow & c \\
\text{Rule 6:} & \underset{\sim}{b} & \Rightarrow & d \\
\text{Rule 7:} & \underset{\sim}{x1} & \Rightarrow & y1
\end{array}
\tag{13}
$$

where seven fuzzy sets $\underset{\sim}{-x1}$, $\underset{\sim}{-b}$, ⋯, $\underset{\sim}{x1}$ in the antecedent parts which partition the interval [-x1, x1] are shown in Fig.8(b), with a an b being appropriate points in [0,x1], and c and d being the values (that is, inference results) at a and b, respectively, as shown in Fig.7.

We shall next indicate control results by the fuzzy singleton-type reasoning method.

3. FUZZY CONTROL RESULTS

In this section we shall show that control results by the fuzzy singleton-type reasoning method are better than those by a simplified fuzzy reasoning method.

We shall now consider a plant model $G(s) = e^{-2s}/(1+20s)$ with first order delay. Fuzzy control rules for the plant model are shown in the table in Fig.10 and interpreted as

R1: e is NB and Δe is NB => Δq is PB
R2: e is NB and Δe is NS => Δq is PB (14)
………………
R25: e is PB and Δe is PB => Δq is NB

where e is error, Δe is change in error and Δq is change in action. NB, NS, ⋯, PB are fuzzy sets in Fig.9 which divide the interval [-6, 6] into five fuzzy partitions. The conditions are as follows:

Time constant = 20; Dead time = 2;
Scale factor of Δe = 1.2; Scale factor of Δq = 2.5

Now we shall begin with the case of using fuzzy control rules of emphatic type.

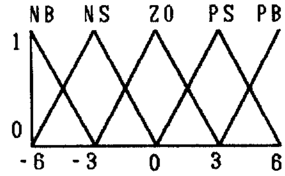


Fig.9 Fuzzy sets for 5 divisions

In order to aim emphatic effect around the setting point 40, the following fuzzy control rule of fuzzy singleton type is used:

$$\text{e is ZO and } \Delta e \text{ is ZO} \Rightarrow \Delta q \text{ is } w/0 \tag{15}$$

The notation 0^w found at the center of fuzzy control table of Fig.10 indicates this fuzzy control rule of (15). Clearly, when w = 1, (15) reduces to the ordinary fuzzy control rule of

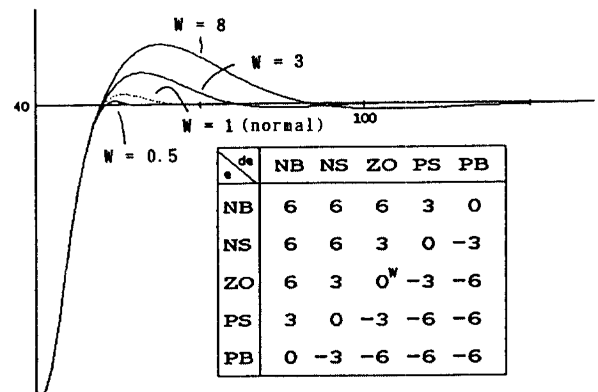
$$\text{e is ZO and } \Delta e \text{ is ZO} \Rightarrow \Delta q \text{ is } 0 \tag{16}$$

In order to observe the emphatic effect near the setting point 40, we shall show the control results in Fig.10 obtained by changing the weight w of (15).

At first, let w be large as is w = 3 and 8 to realize the emphatic effect at the setting point, which corresponds to make Δq = 0 forcedly (namely, to make the control action q unchangeable), then the control result becomes bad as shown in Fig. 10(a) since the change in error Δq is emphasized to be zero near the set point and thus the response turns to be dull.

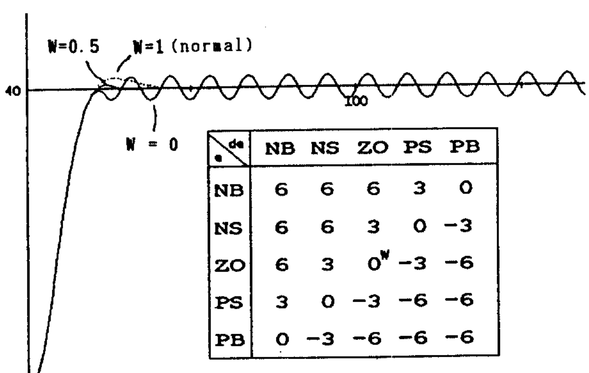
It is noted that dotted line in Fig.10 shows the control result at w = 1, which corresponds to the control result by a simplified fuzzy reasoning method.

On the contrary, when w is small, the restrained effect works and the best control result is found at w = 0.5 as in Fig.10(a) and (b). However, at w = 0, the control result



e \ Δe	NB	NS	ZO	PS	PB
NB	6	6	6	3	0
NS	6	6	3	0	-3
ZO	6	3	0^w	-3	-6
PS	3	0	-3	-6	-6
PB	0	-3	-6	-6	-6

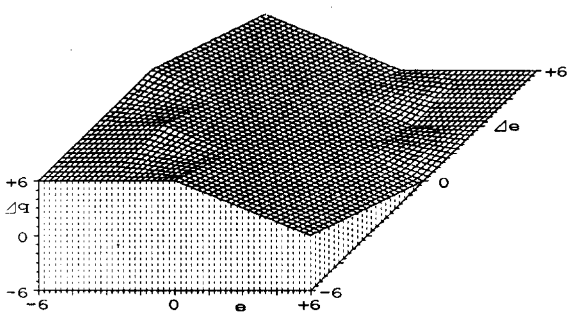
(a) Case of large w (emphatic case)



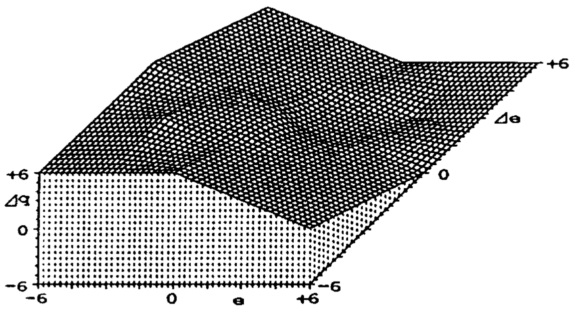
e \ Δe	NB	NS	ZO	PS	PB
NB	6	6	6	3	0
NS	6	6	3	0	-3
ZO	6	3	0^w	-3	-6
PS	3	0	-3	-6	-6
PB	0	-3	-6	-6	-6

(b) Case of small w (restrained case)

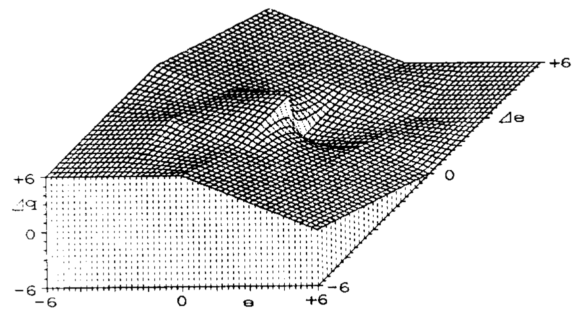
Fig. 10 Control results when w of (15) is changed



(a) Case of w = 1



(b) Case of w = 0.5



(c) Case of w = 0

Fig.11 Illustrations of e, Δe → Δq at w = 1, 0.5, and 0 for the control table in Fig.10

becomes periodical. w = 0 means no existence of fuzzy control rule (16), so an actual Δq is decided by the surrounding control rules and thus the periodical control result occurs.

Fig.11 shows the inference results Δq at e and Δe in the cases of w = 1, 0.5 and 0 for the control table in Fig.10. Fig.11(a) shows the case of w = 1 which corresponds to the ordinary case by simplified fuzzy reasoning method. Fig.11(b) is the case of w = 0.5 which gives the best control result and the vicinity of center of the figure is not a plane but changes slightly, which causes good control result. Fig.11(c) is the case of w = 0 which indicates no existence of fuzzy rule (16).

Finally, Fig.12 indicates control results when a fuzzy control rule of fuzzy singleton type:

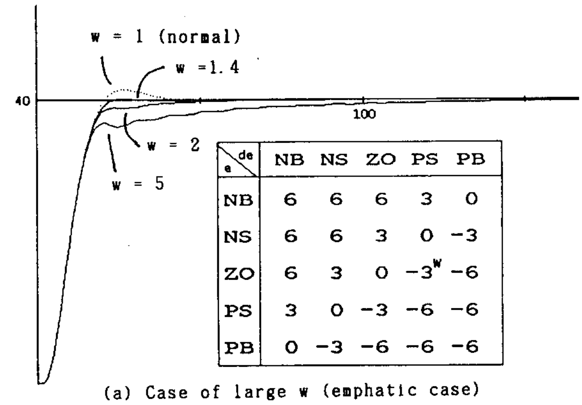
$$e \text{ is ZO and } \Delta e \text{ is PS} \Rightarrow \Delta q \text{ is } w/-3 \qquad (17)$$

is used. The best control result is obtained at w = 1.4.

As is found from these control results, we can get fine control results by changing the weight w of fuzzy control rules of fuzzy singleton type.

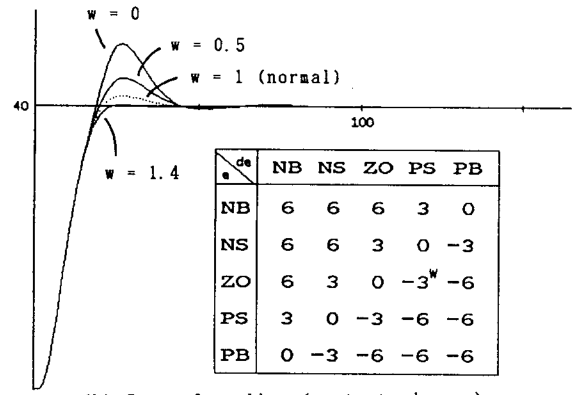
4. CONCLUSION

This paper defined new fuzzy control method called fuzzy singleton-type reasoning method and showed that better control results are obtained



e \ de	NB	NS	ZO	PS	PB
NB	6	6	6	3	0
NS	6	6	3	0	-3
ZO	6	3	0	-3^{w}	-6
PS	3	0	-3	-6	-6
PB	0	-3	-6	-6	-6

(a) Case of large w (emphatic case)



e \ de	NB	NS	ZO	PS	PB
NB	6	6	6	3	0
NS	6	6	3	0	-3
ZO	6	3	0	-3^{w}	-6
PS	3	0	-3	-6	-6
PB	0	-3	-6	-6	-6

(b) Case of small w (restrained case)

Fig. 12 Control results when w of (17) is changed

compared with those by simplified fuzzy reasoning method. Namely, this new fuzzy control method can adjust subtly control results by changing the weights wi of fuzzy control rules, and thus is more powerful than and as simple and fast in the execusion as the simplified fuzzy reasoning method which corresponds to the case of wi = 1 and is used widely as the tool of neuro-fuzzy techniques [7]. Therefore, the fuzzy singleton-type reasoning method may be considered as an ultimate fuzzy control method.

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