Fifth IFSA World Congress (1993), 941 - 944

# PERFORMANCE EVALUATION OF FUZZY CONTROL THROUGH AN INTERNATIONAL BENCHMARK

Serge BOVERIE[(1)(2)], Bernard DEMAYA[(2)], Jean–Michel LEQUELLEC[(1)(2)], André TITLI[(2)(3)(4)]

(1) SIEMENS AUTOMOTIVE SA
Avenue du Mirail
BP 1149
31036 Toulouse Cedex
France

(2) Laboratoire MIRGAS
SIEMENS AUTOMOTIVE SA
Avenue du Mirail - BP 1149
31036 Toulouse Cedex
France

(3) INSA Toulouse
Avenue de Rangueil
31077 Toulouse Cedex
France

(4) LAAS–CNRS
7,Avenue du Colonel Roche
31077 Toulouse Cedex
France

## Abstract

In this paper, we present an international benchmark used in the adaptive control specialist community in order to evaluate the fuzzy control performances.
Before solving the corresponding problems, we introduce some improvements on a classic fuzzy controller in order to consider high order systems and time delays.
At the end of this paper, the simulation results obtained with the extended Fuzzy Controller will be compared with those obtained with a Supervised Adaptive Controller.

## 1.Introduction

Fuzzy control is a topic highly controversial in the field of automatic control. Therefore an objective evaluation of the performances of such a control is necessary. To do this, we have chosen to deal with an international benchmark used in the community of specialists in adaptive control and worldwide recognized as a difficult problem (even for adaptive control !). Of course, simple and conventional fuzzy controllers cannot be used directly. Some improvements are necessary to cope with high order processes and time delays for example. Aim of this paper is to present these improvements and the results obtained with them on the benchmark.

## 2. PRESENTATION OF THE BENCHMARK [6] & [7]

The overall diagram of the system is represented in Fig. 1 :

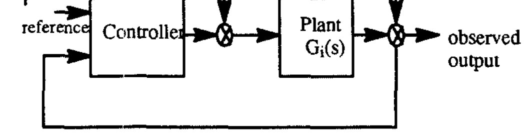


Fig.1 – *Description of the plant*

Three different linear processes are considered.They are described by the following models in table1.

| | |
|---|---|
| First order process : | $G_1(s) = \frac{K}{s+a}$ |
| Second order process : | $G_2(s) = \frac{K}{s^2 + a_1 s + a_2}$ |
| Third order process : | $G_3(s) = \frac{K}{s^3 + a_1 s^2 + a_2 s + a_3}$ |

***Table 1** : The different models*

For each process, the nominal values of the $a_i$ terms and the gain K are given below :

- first order : $a = 1 ; K = 1$
- second order : $a_1 = 1.4$ , $a_2 = 1 ; K = 1$
- third order : $a_1 = 1.75, a_2 = 2.15, a_3 = 1 ; K = 1$

### 2.1. *Description of the disturbances applied to the process:*

Following MASTEN and COHEN ([6] & [7]) , three kind of disturbances are applied to the process (see table 1):

- External disturbances : Bias on the control variable and white noise on the measurement.
- Change on the model parameters : poles, static gain.
- Neglected dynamics : Time delay, Unmodelled pole and zero.

| COMPLEXITY LEVEL | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Disturbance level in % of the nominal value | 100 % | 10 % | 60 % | 100 % |
| Sensor noise(R M S) | 0 | 0.02 | 0.16 | 0.2 |
| Unmodelled dynamic. (sec) | 0 | 0.10 | 0.25 | 0.33 |
| Time delay (sec) | 0 | 0.05 | 0.1 | 0.3 |
| Unmodelled plant zero (sec) (only for second and third order plants) | 0 | 0.10 | 0.2 | 0.3 |
| Pole variation max value | $\Delta a = \pm 2$ for first order<br>$\Delta a = \pm 3$ for 2nd, 3rd order | | | |
| Gain variation % of the nominal gain | -50 to 200 for first order<br>-50 to 300 for 2nd to 3rd order | | | |

Plant perturbations

***Table 2** : Disturbances applied on the models*

### 2.2. *Requirements:*

MASTEN and COHEN ( [6], [7]), have defined three ideal process behaviors in response to a reference step input, with respect to the order of the plants . They are introduced in the simulation as a reference model :

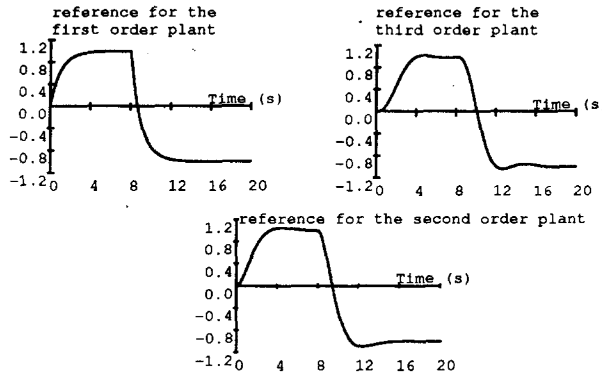


Fig.2 – *Reference model for the different processes*

## 3. SYNTHESIS OF THE FUZZY CONTROLLER

### 3.1. *Structure of a fuzzy controller for a first order process without pure time delay:*

The operation of a Fuzzy controller can be described by means of the following sequence [10] :

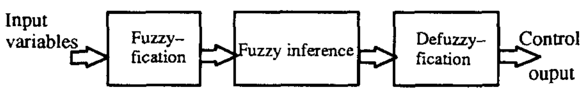


Fig.3 – *principle of the basic fuzzy controller*

Most fuzzy controllers use as inputs the deviation $\varepsilon$ between the set point and the process output, and of the error variation $\Delta\varepsilon$ ([1] & [2]).

- The fuzzyfication action enables to define a grade of membership to fuzzy sets characterizing input variables.
- The defuzzyfication action enables to calculate the effective control $\Delta u$ from all rules applicable to a given input set.
- Human expertise is translated in the form of rules of the following type:

  **If** < Fuzzy conditions on input variables >
  **Then** < Fuzzy control actions >

These rules are considered by the inference engine. The more standard set of rules is the diagonal type (Mac Vicar–Whelan rules) and is given in the following table (see [3]).

| x\Δx | NB | NM | NS | ZE | PS | PM | PB |
|---|---|---|---|---|---|---|---|
| PB | ZE | PS | PM | PB | PB | PB | PB |
| PM | NS | ZE | PS | PM | PB | PB | PB |
| PS | NM | NS | ZE | PS | PM | PB | PB |
| ZE | NB | NM | NS | ZE | PS | PM | PB |
| NS | NB | NB | NM | NS | ZE | PS | PM |
| NM | NB | NB | NB | NM | NS | ZE | PS |
| NB | NB | NB | NB | NB | NM | NS | ZE |

***Table 3*** : *rules base*

With the following fuzzy sets:
NB, NM, NS: Negative Big, Negative Medium, Negative Small ;
ZE : ZEro ;
PB, PM, PS: Positive Big, Positive Medium, Positive Small.

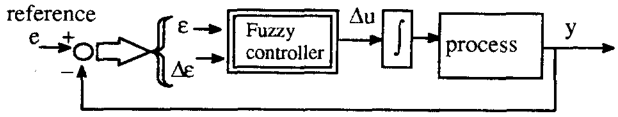


Fig.4 – *Structure of the basic fuzzy controller*

### 3.2. *Structure of a fuzzy controller for any order process without pure time delay:*

The conventional controllers based on the use of error and error variation as inputs (see 3–1) are unfortunately not able to control processes where order is higher than two.
To solve this problem, various solutions can be used.
In this paper we propose three different methods:

- The first one consists in a direct approach , designing a multi–input fuzzy controller,
- The second and the third ones present a compromising solution between performances and complexity of the structure with the use of a Fuzzy Incremental Controller (F.I.C.) and a Fuzzy Controller with Parallel Structure (F.C.P.S).

#### 3.2.1. *Multi–input fuzzy controller:*

Qualitatively, the structure of the fuzzy controller is equivalent to that of a $PID^{(n-1)}$ controller. If u is the value of the control action provided by the fuzzy controller, we can show that this value is a non–linear function of the error $\varepsilon$, of the integral of error, and its (n–1) successive derivatives.

$$u = F\left(\int \epsilon, \epsilon, \dot{\epsilon}, \ldots, \epsilon^{(n-1)}\right) \qquad (1)$$

We find here an expression which is equivalent to those of a conventional controller. The control rules can be generated in a qualitative way, following the same approach as for the 1st order process. However, this method is complex to implement, because the number of rules increases in a near–exponential way with the order of the system. For example, for a first order system, the number of rules is equal to $m^2$ if m is the number of membership functions of the input variables. and for a $n^{th}$ order system, the number of rules is equal to $m^{n+1}$. We can easily understand how difficult it is to implement such controllers.

#### 3.2.2. *Synthesis of the fuzzy incremental controller (F.I.C):*

This structure is based on the association between a conventional fuzzy controller and an incremental block, and allows the adaption of the overall controller to the order of the process to be controlled.

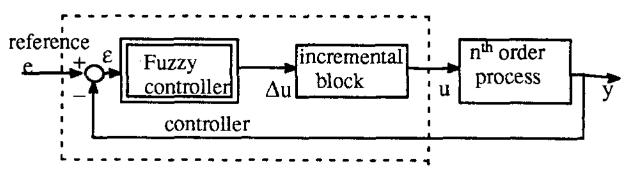


Fig.5 – Fuzzy Controller associated with an incremental block

For a $n^{th}$ order process, it is well known that the order of the control system must be (n–1), and, in this way, the structure of the controller can be described by the following symbolic form : $PID^{(n-1)}$. In our case, the structure of the fuzzy controller has been defined in 3–1, and $\Delta u$ is the output of the fuzzy controller. This fuzzy controller is a PD type; so, in order to respect the preceding condition, the incremental bloc must have a $PID^{(n-2)}$ form. From this, we can deduce the main shape of this control block .

$$U = k_0 \int_0^t \Delta u(\tau)d\tau + k_1\Delta u + k_2\Delta\dot{u} + \ldots + k_{n-1}\Delta u^{(n-2)} \qquad (2)$$

To ease the use of this formula and also its implementation in a Digital Unit, it is necessary to formulate a simplifying hypothesis and then, to get the discrete expression of the control :

$$U_j = k_0 \sum_{i=0}^{i=j} \Delta u_i + k_1\Delta u_j + k_2\Delta\dot{u}_j + \ldots + k_{n-1}\Delta u_j^{(n-2)} \qquad (3)$$

with : $\Delta \dot{u}_j = \frac{\Delta u_j - \Delta u_{j-1}}{\Delta t}$, $\Delta u_j^{(2)} = \frac{\Delta u_j - 2.\Delta u_{j-1} + \Delta u_{j-2}}{\Delta t^2}$

and : $\Delta u_j^{(n-2)} = \frac{f(\Delta u_j, \ldots, \Delta u_{j-n+2})}{\Delta t^{n-2}}$ (4)

Finally, we obtain the control structure:

$$U_j = k_0 \sum_{i=0}^{i=j} \Delta u_i + k'_1 \Delta u_j - k'_2 \Delta u_{j-1} + \ldots + (-1)^n k'_{n-1} \Delta u_{j-n+2} \quad (5)$$

$k^*_i > 0$

We notice that the incremental block has n degrees of freedom for a $n^{th}$ order process and therefore, we can suppose that this structure makes the tuning of the fuzzy controller more complex. But we have shown experimentally that the sensitivity of tuning parameters is very small thanks to the great robustness of the fuzzy controller and justify the association between a fuzzy controller and an incremental block.

#### 3.2.3. *Fuzzy controller with parallel structure of second order blocks (F.C.P.S.):*

We can easily show that a $PD^{n-1}$ controller can be decomposed in a parallel association of (n–1) PD structures. Each elementary block can be described by the following equation:

$$\Delta u_i = \left(k^*_{2i} + k^*_{2i+1} \frac{d}{dt}\right) \epsilon^{\,i} \quad (6)$$

The global control action can be given by :

$$\Delta u = \Sigma_i \Delta u_i = k_0 \epsilon + k_1 \dot{\epsilon} + \ldots + k_{n-1} \epsilon^{(n-1)} \quad (7)$$

The global structure of such a controller is given on figure 6.

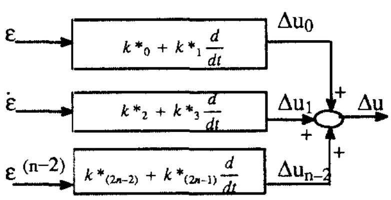


Fig.6 – *parallel block structure– linear controller*

As for a $PD^n$ linear controller the use of elementary PD blocks can be extended to the fuzzy control of high order systems. This structure is described by the figure 7. The equation of an elementary block will be:

$$\Delta u_i = F_{uzz_i}(\epsilon_i, d\frac{\epsilon_i}{dt}) \quad (8)$$

The general expression of the fuzzy control action is given by:

$$\Delta u = \sum_{i=0}^{i=(n-2)} \Delta u_i \quad (9)$$

The control rules can be expressed in a qualitative way in ( n–1 ) two input tables. These tables are similar and directly related to those described in III–1.

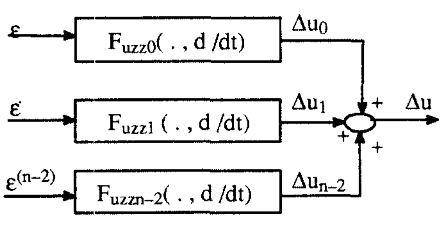


Fig.7 – *F.C.P.S. description*

***Note:*** This parallel structure of (n–1) elementary blocks has no integral term. This control is not able to supress the static error. In order to take this into account we can use the following modified structure:

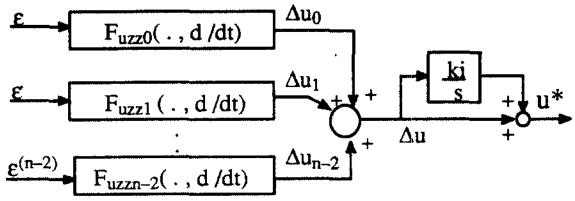


Fig.8 – *F.C.P.S. with integrator*

### 3.3. *Structure of a fuzzy controller for any order process with pure time delay:*

The different structures defined in the former paragraphs give generally good results as long as the process to be controlled presents small time delay. When the time delay increases, it is necessary to take it into account. In order to tackle with this problem, we have introduced a Fuzzy Predictive Controller (FPC), based on the use of an internal model (see Fig. 9). This structure is composed of three parts :

- The first one is a conventional fuzzy controller as defined in paragraphs 3–1 and 3–2,
- The second one consists in a simple fuzzy model representing an approximation of the process behavior to be controlled, without time delay,
- The last one consists in an approximation of the time delay.

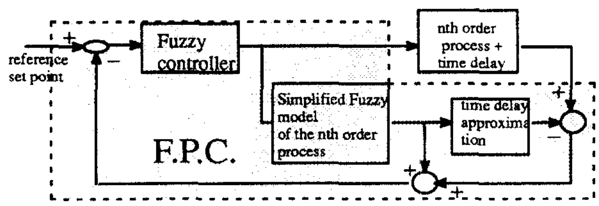


Fig.9 – *Fuzzy Predictive Controller Description*

Developement of the fuzzy model:

We suppose that we know the order of the system to be controlled. The development of the fuzzy model is equivalent to those of a state space representation. We consider the state space representation of a $n^{th}$ order process (10).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = F(x_1, x_2, \ldots x_n, u) \end{cases} \quad (10) \Rightarrow \quad \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = F_{uzz}(x_1, x_2, \ldots x_n, u) \end{cases} \quad (11)$$

Where $x = x_1$ is the state variable and $x_2, \ldots, x_n$ are the $(n-1)^{th}$ successive derivatives of the state. F can be a non–linear function that we can approximate by a fuzzy function, and the fuzzy state space representation becomes (11).

Finally we obtain a function $F_{uzz}$ with (n+1) inputs and one output. As long as the process is equivalent to a first order system, this function is easy to synthesize, but when the order of the process grows, the size of this function becomes discriminatory. With some restrictive hypotheses (which are valid in most cases), it is possible to reduce the complexity of this function.

In all cases, the function F can be written as a Taylor's serie expansion:

$$F(x_1, x_2, \ldots x_n, u) = \frac{\partial F}{\partial u}.u + \frac{\partial F}{\partial x_1}.x_1 + \ldots + \frac{\partial F}{\partial x_n}.x_n \quad (12)$$

With $F(0, \ldots, 0) = 0$

If they are not coupling terms in F , we can write :

$$F(x_1, x_2, \ldots x_n, u) = F_0.u + F_1.x_1 + \ldots + F_n.x_n \quad (13)$$

and the fuzzy equivalent function $F_{uzz}$ can be written as follows:

$$F_{uzz}(x_1, x_2, \ldots x_n, u) = F_{uzz_0}.u + F_{uzz_1}.x_1 + \ldots + F_{uzz_n}.x_n \quad (14)$$

We obtain in this case a parallel structure of n single–input / single–output fuzzy functions.

For a second order process the fuzzy model is described in Fig. 10:

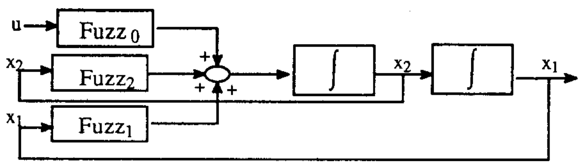


Fig. 10 – *Structure of the fuzzy model for a second order process*

If the terms are combined two by two, it is also possible to define a simple structure. In this case, we have a parallel structure of (n–1) two inputs / one output fuzzy functions.

## 4. APPLICATION TO THE BENCHMARK AND CONCLUSION

All these techniques have been used to solve the benchmark problem. Significant and selected results are given below (see Fig. 11 to 15). The fuzzy control performances are compared with those obtained by M'Saad et al.v in [8] & [9] using supervised adaptive control (adaptive control results are not available for the third order system). All the results show that even if it is not so competitive as a sophisticated method such as a supervised adaptive control, fuzzy control gives some satisfactory performances in a more simple manner.

Obviously, fuzzy control with some improvements is then able to deal with high–order systems really disturbed and it appears to be a robust technique. This situaion encourages us to pursue our efforts in the development and use of fuzzy control.

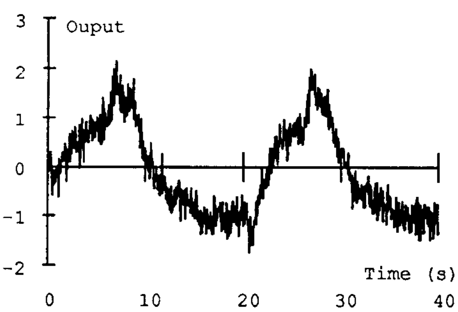


*Fig.11 – First Order Process / K=1, a=3*
*Fuzzy Control (level 3)*

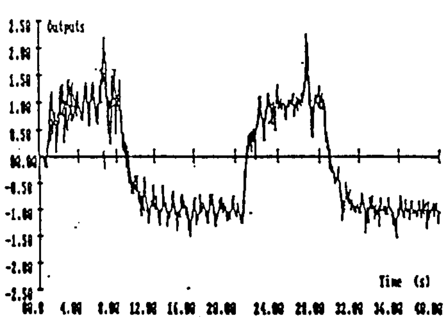

*Fig.12 – First Order Process / K=1, a=3*
*Supervised Adaptive Control (level 3)*

## REFERENCES

[1] – S.BOVERIE, B.DEMAYA, A.TITLI, Fuzzy logic control compared with other automatic control approaches, Proceedings of the 30th IEEE conference on decision control, december 1991, pp. 1212–1216.

[2] – S.BOVERIE, B.DEMAYA, R. KETATA, A.TITLI, Performance evaluation of a fuzzy controller, Proceedings of the IFAC SICICA in Malaga, May 1992 .

[3]– L. FOULLOY, Projet commande symbolique et neuromimétique, Pôle Automatisation Intégrée, GR Automatique, Centre national de la Recherche Scientifique, Rapport 91–2, novembre 91.

[4] – HOLMBLAD, L. P., ∅STERGAARD, J.J., Control of a Cement Kiln by Fuzzy Logic, North Holland, 1982, Fuzzy Information and Decision Processes, pp. 389–399

[5] – KICKERT, W.J., VAN NAUTA LEMKE, R., Application of a Fuzzy Controller in a Warm Water Plant, Automatica, Vol. 12, pp. 301–308

[6] – MASTEN, M. K., COHEN, H. E., A Showcase of Adaptive Controller Designs, International Journal of Adaptive Control and Signal Processing, 1989, Vol. 3, pp. 95–101

[7] – MASTEN, M. K., COHEN, H. E., An advanced Showcase of Adaptive Controller Designs, International Journal of Adaptive Control and Signal Processing, 1990, Vol. 4, pp. 89–98

[8] – M'SAAD, I.D. LANDAU, M. SAMAAN, Further Evaluation of Partial State Model Reference Adaptive Design, International Journal of Adaptive Control and Signal Processing, 1990, Vol. 4. pp. 133–148

[9] – M'SAAD, I.D. LANDAU, M. DUQUE, M. SAMAAN, Example applications of the partial state model reference adaptive design technique, International Journal of Adaptive Control and Signal Processing, 1989, Vol. 3. pp. 155–165

[10] – L.A. ZADEH, Fuzzy logic, IEEE computer, 1988, pp. 83–93.

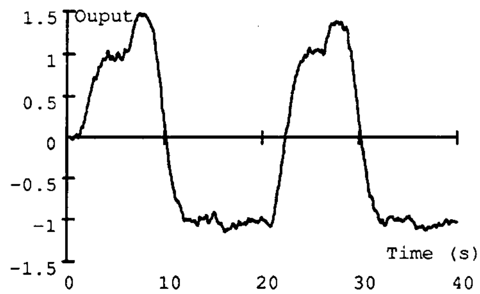


*Fig.13 – Second Order Process / K=0.5, $a_1$=1.4, $a_2$=1*
*Fuzzy Control (level 3)*

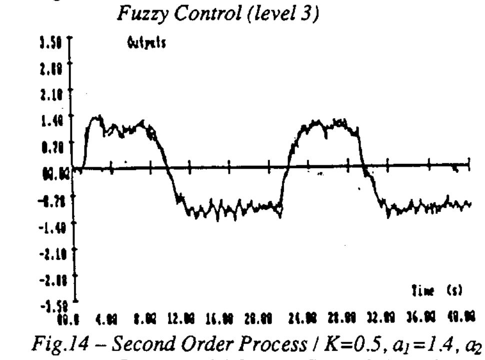

*Fig.14 – Second Order Process / K=0.5, $a_1$=1.4, $a_2$=1*
*Supervised Adaptive Control (level 3)*

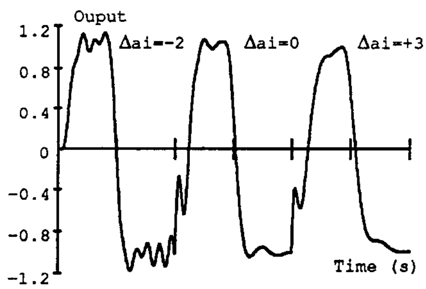


*Fig.15 – Third Order Process / K=0.5 / Fuzzy Control (level 2)*