Fifth IFSA World Congress (1993), 895-898

A Fuzzy Controller using Fuzzy Relations on Input Variables

Jihong Lee

Dept. of Control Engineering
Kum-oh National Institute of Technology
188 Shinpyoung-dong, Kumi, Kyoungbuk 730-701, Korea

ABSTRACT

Instead of Cartesian product for in combining multiple inputs for fuzzy logic controllers, a method using fuzzy relation in inference is proposed. Moreover, fuzzy control rule described by fuzzy relations is derived from given conventional fuzzy control rule by fitting concept. It will be shown through several examples that the proposed technique gives smoother interpolation than conventional ones.

1 Introduction

During the past several decades, fuzzy control has emerged as one of the most fruitful research areas in the application of fuzzy set theory. Many practical applications to industrial process, as well as studies on the theory itself, have been reported in many works. Even though the details may be different, the basic structure of announced fuzzy logic controllers is composed of fuzzification part, rule base, inference engine, and defuzzification parts. Among the parts, we will focus discussions on the inference method, and propose some modified method to the conventional ones.

As summarized in [1], the four most widely used inference methods use *min* operation in calculating firing strength of each rule for multiple input cases. Since fuzzy control rules, in general, are described by finite number of rules and finite number of fuzzy sets, some informations may be lost in *min* operation[2, 3]. Therefore, we propose a inference method for FLC adopting fuzzy relation in describing conditional part of fuzzy control rules.

Even though the fuzzy relation has some advantage over the *min* operator, it is not easy for experts of a system to summarize control rules in the form of fuzzy relation because the description should be arranged into some forms in which the input variables are coupled. Therefore, we proposed some fuzzy relation-fitting technique with fuzzy sets and fuzzy rules given by experts. For example, the following type of control rules

$$R_i : \text{if } (x, y) \text{ is } D_i \text{ then } z \text{ is } C_i$$

are fitted from the following type of control rules.

$$R_i : \text{if } x \text{ is } A_i \text{ and } y \text{ is } B_i \text{ then } z \text{ is } C_i.$$

Simulation results using our approach to a class of system will be given in comparison with the ones using *min* operators.

2 Fuzzy Inference Methods for FLC

For completeness let's review the four most widely used fuzzy inference methods assuming two fuzzy control rules with two-inputs and one-output[1]. The first three types process the following fuzzy control rules:

$$R_1 : \text{if } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } z \text{ is } C_1.$$
$$R_2 : \text{if } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } z \text{ is } C_2.$$

With the two rules, the firing strengths α_1 and α_2 of the first and second rules may be expressed as

$$\alpha_1 = \mu_{A_1}(x_0) \wedge \mu_{B_1}(y_0) \tag{1}$$

$$\alpha_2 = \mu_{A_2}(x_0) \wedge \mu_{B_2}(y_0) \tag{2}$$

where $\mu_{A_i}(x_0)$ and $\mu_{B_i}(y_0)$ play the role of the degree of partial match between the measured data and the data in the rule base.

1) *Fuzzy Reasoning of the First Type – Mamdani's Minimum Operation Rules as a fuzzy Implication Function:*

In this method, the ith rule leads to the control decision

$$\mu_{C_i'}(w) = \alpha_i \wedge \mu_{C_i}(w) \tag{3}$$

By the above calculation, the membership function μ_C of the inferred consequence C is pointwise given by

$$\mu_C(w) = \mu_{C_1'} \vee \mu_{C_2'} \tag{4}$$
$$= [\alpha_1 \wedge \mu_{C_1}(w)] \vee [\alpha_2 \wedge \mu_{C_1}(w)] \tag{5}$$

2)*Fuzzy Reasoning of the Second Type – Lasen's Product Operation Rule as a Fuzzy Implication Function:* This method is based on the use of Lasen's product operation rule as a fuzzy implication function, in which the ith rule leads to the control decision

$$\mu_{C_i'}(w) = \alpha_i \cdot \mu_{C_i}(w) \tag{6}$$

For the case of two rules

$$\mu_C(w) = \mu_{C_1'} \vee \mu_{C_2'} \ = [\alpha_1 \cdot \mu_{C_1}(w)] \vee [\alpha_2 \cdot \mu_{C_2}(w)] \tag{7}$$

3) *Fuzzy Reasoning of the Third Type - Tsukamoto's Method with Linguistic Terms as monotonic Membership Functions:* Under the assumption that the membership functions of fuzzy set A_i, B_i, and C_i are monotonic, the result inferred from the first rule is α_1 such that $\alpha_1 = C_1(y_1)$ and α_2 from second rule such that $\alpha_2 = C_2(y_2)$. Also, a crisp control action may be expressed as the weighted combination

$$z_0 = \frac{\alpha_1 y_1 + \alpha_2 y_2}{\alpha_1 + \alpha_2}. \tag{8}$$

4) *Fuzzy Reasoning of the Fourth Type - The Consequence of a Rule is a Function of Input Linguistic Variables:* For the simple case of two-input, one-output, and two control rule system, the fuzzy control rules are described as follows

R_1: if x is A_1 and y is B_1 then $z = f_1(x, y)$
R_2: if x is A_2 and y is B_2 then $z = f_2(x, y)$.

The inferred value of the control action from the first rule is $\alpha_1 f_1(x_0, y_0)$. The inferred value of the control action from the second rule is $\alpha_2 f_2(x_0, y_0)$. Finally the crisp control action is given by

$$z_0 = \frac{\alpha_1 f_1(x_0, y_0) + \alpha_2 f_2(x_0, y_0)}{\alpha_1 + \alpha_2}. \tag{9}$$

In the above four methods, *min* operator is used to calculate the firing strength α_i. Graphical interpretation of the *min* operator is shown in Fig. 1.

As shown in the figures the *min* operator does not distinguish points in some region(see the line A in the Fig. 1). This means that a controller using *min* operator in combining multiple input variables inherently has some limits on control performances especially in the case where control laws should couple the input variables, which was pointed out in [3] on the view point of function generating capabilities of fuzzy logic controllers.

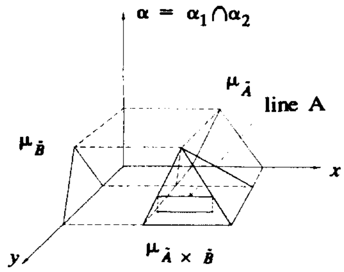


Figure 1: Result of *min* operation for two convex fuzzy sets.

Moreover, suppose the rule of Fig. 2 be given by an expert to control a system. If we focus on the parts where the conclusion parts are ZE's, we can know that the intention of the expert is to give control action ZE around the line l_{ZE} in the Fig. 3. But it necessary to analyze what happens if the measured state is mapped to the point P in Fig. 3.

ė \ e	NB	NM	NS	ZE	PS	PM	PB
PB	ZE	PS	PM	PB	PB	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PS	NM	NS	ZE	PS	PM	PB	PB
ZE	NB	NM	NS	ZE	PS	PM	PB
NS	NB	NB	NM	NS	ZE	PS	PM
NM	NB	NB	NB	NM	NS	ZE	PS
NB	NB	NB	NB	NB	NM	NS	ZE

Figure 2: A fuzzy control rule

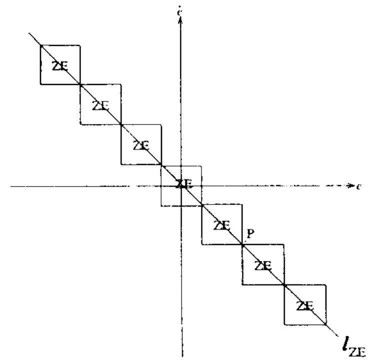


Figure 3: Part of control rule corresponding to ZE control action

Since the the point P is very near to the line l_{ZE}, the true intention of the expert is to mainly exert control ZE to such situation. However if we use the following type of rules and *min* operation to combine two input variables, the main control is a fuzzy variable corresponding to box containing the point P which may be possibly different from ZE.

R_k: if x is A_k and y is B_k then z is C_k.

These observations give us a motivation to device a fuzzy logic controller using fuzzy relation(i.e., fuzzy sets defined in the whole space of input variables) to overcome these difficulties. At this point we summarize the definition of the fuzzy relations[3] for the simple binary case:

Definition : Let $X, Y \subseteq R$ be universal sets then

$$\tilde{R} = \{((x, y), \mu_{\tilde{R}}(x, y)) | (x, y) \subseteq X \times Y\} \tag{10}$$

is called a fuzzy relation on $X \times Y$.

Also we show an example of the fuzzy relation as follows: Let $X = Y = R$ and $\tilde{R}$:= "considerably larger than." The membership function of the fuzzy relation, which is, of course, a fuzzy set on $X \times Y$, can then be :

$$\mu_{\tilde{R}}(X, Y) = \begin{cases} 0 & for\ x \leq y \\ (1 + (y - x)^{-2})^{-1} & for\ x > y \end{cases} \tag{11}$$

By the fuzzy relation we propose fuzzy control rules described in the following forms:

$$R_i : \text{if } (x, y) \text{ is } A_i \text{ then } z \text{ is } C_i.$$

Note that A_i is a fuzzy relation defined on $X \times Y$, and "twice bigger than" may be an example. With the fuzzy relation we can protect the loss of information contained in the measured data pair (x_0, y_0) caused by the *min* operation. Extension to n input variable case is straightforward.

3 Fuzzy Inference based on Fuzzy Relation

Even though the fuzzy relation has some advantage over the Cartesian product method using *min* operator in combining multiple variables, it is not easy for experts of a system to summarize the control rules in the form of fuzzy relation because the description should be arranged into some forms in which the input variables are coupled. Therefore, we have found some fuzzy relation-fitting technique with the decomposed fuzzy sets and fuzzy rules given by experts, i.e., a technique generating the following type of control rules and fuzzy relations

$$R_i : \text{if } (x, y) \text{ is } D_i \text{ then } z \text{ is } C_i$$

from the following type of control rules and fuzzy sets.

$$R_i : \text{if } x \text{ is } A_i \text{ and } y \text{ is } B_i \text{ then } z \text{ is } C_i.$$

To do so, we first assume that all the fuzzy sets in the controller are fuzzy numbers which satisfy convexity and have only one maximum point. Also we denote $\hat{A}$ as a real number that gives maximum membership grade of fuzzy set A, i.e.,

$$\hat{A} = \mu_A^{-1}(1) \tag{12}$$

and call such point as *maximum point* of fuzzy number A. Then we summarize the whole procedures;

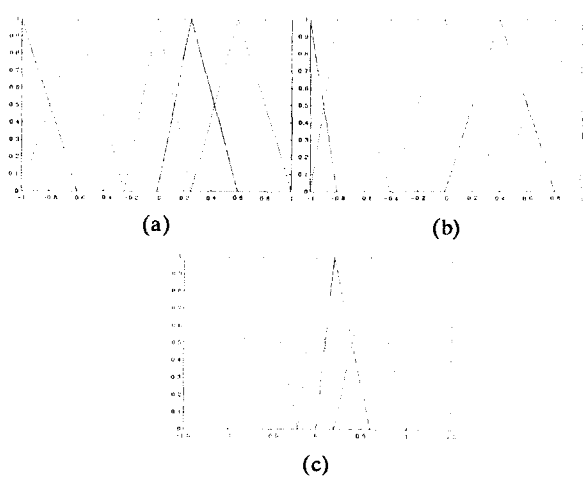


Figure 4: Fuzzy sets used in simulation for (a) error, (b) change of error, and (c) control input.

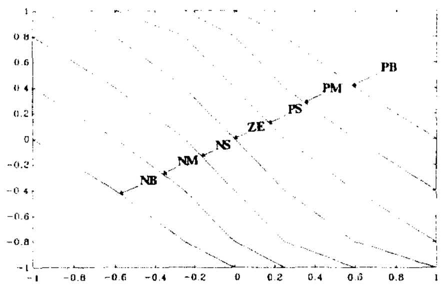


Figure 5: Lines formed by connecting maximum points

(a1) Gather all rules that have same conclusion part C_k, $k = 1, \ldots, K$, where K is the number of fuzzy sets defined on the space of control variable.

(a2) Gather the points $(\hat{A}_{j_k}, \hat{B}_{j_k})$ corresponding to the conditional part of j_kth rule having C_k in conclusion part.

(a3) Construct piecewise linear line l_{C_k} by connecting adjacent two points of (a2) by straight lines.

With the fuzzy sets of Fig. 4 and fuzzy rules of Fig. 2, the resultant lines are shown in the Fig. 5.

From now on, a method of inference using fuzzy relations is described. Even though there may be many ways

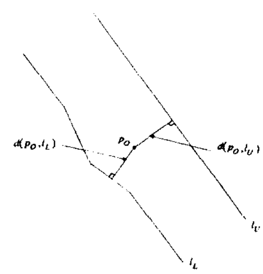


Figure 6: Minimum distances between lines and measured state

to implement such inferences, we adopt a simple method under the assumption that every region is covered by two fuzzy relations, i.e., only two fuzzy relations give nonzero membership grade for every measured state.

(b1) For measured value $p_0 = (x_0, y_0)$, determine two enclosing lines, upper-line l_U and lower-line l_L.

(b2) Calculate minimum distances between the measured point and the lines, $d(p_0, l_U)$ and $d(p_0, l_L)$.

(b3) Calculate the firing strengths α_U and α_L as

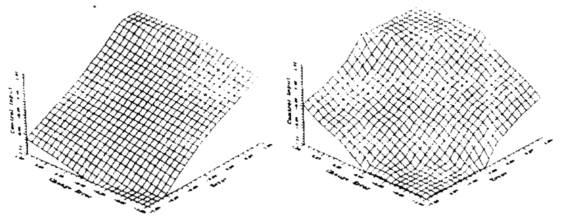
$$\alpha_U = \frac{d(p_0, l_L)}{d(p_0, l_U) + d(p_0, l_L)} \tag{13}$$

$$\alpha_L = \frac{d(p_0, l_U)}{d(p_0, l_U) + d(p_0, l_L)} \tag{14}$$

(b4) With these firing strengths, apply the simplified inference method[3] to calculate control input.

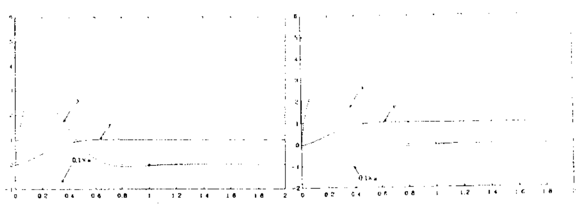
Part of the procedure is shown in Fig. 6.

With the fuzzy sets of Fig. 4 and the control rules of Fig. 2, the proposed methods gave the result of inference as Fig. 7-(a) while conventional method gave the result as Fig. 7-(b). Note that simplified defuzification method[2] was adopted in the comparison simulation. As shown in the figure, the new method gives smoother variation of calculated control input than conventional methods.



(a) (b)

Figure 7: results of defuzzification of (a) proposed method and (b) conventional method.



(a) (b)

Figure 8: Responses of $G_1(s)$ by (a) new method and (b) conventional method.

4 Illustrative Examples

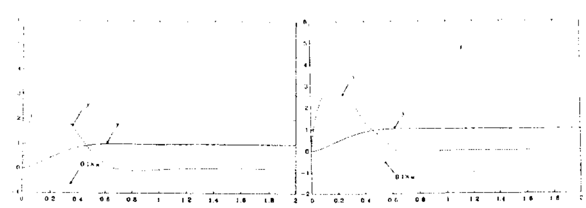
We have applied the proposed method for plant described as

$$G_1(s) = \frac{1}{s(s+1)} \tag{15}$$

and got the result of Fig. 8. Also we have applied the proposed method for the following plant and got the result of Fig. 9.

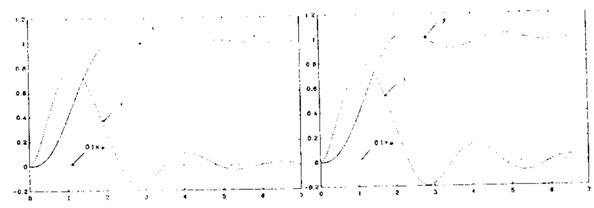
$$G_2(s) = \frac{1}{s^2 + 0.2s + 1} \tag{16}$$

As shown in the two figures, the new inference method gives smoothly varying control inputs. In the example of the following plant, slightly more stable response was obtained by the new method than by conventional methods.



(a) (b)

Figure 9: Responses of $G_2(s)$ by (a) new method and (b) conventional method.



(a) (b)

Figure 10: Responses of $G_3(s)$ by (a) new method and (b) conventional method.

$$G_3(s) = \frac{1}{s(s+0.1)(s+2)} \tag{17}$$

5 Concluding Remarks

An inference method for FLC adopting fuzzy relation in describing conditional part of fuzzy control rules was proposed instead of conventional ones using *min* operator in combining multiple inputs. Moreover, a fuzzy relation-fitting technique with fuzzy sets and fuzzy rules described in conventional forms which can be easily given by experts, is proposed.

Each control rule of proposed method is summarized as the following form

$$R_i : \text{if } (x, y) \text{ is } D_i \text{ then } z \text{ is } C_i$$

and is proven to give smoother interpolation result than conventional methods using Cartesian product of fuzzy sets for multiple input case.

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