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Phase Compensation of Fuzzy Control Systems and Realization of Neuro-fuzzy Compensators

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Abstract

This paper proposes a design method of fuzzy phase-lead compensator and its self-learning by neural network. The main feature of the fuzzy phase-lead compensator is to have parameters for effectively compensating phase characteristics of control systems. An important theorem which is related to phase-lead compensation is derived by introducing concept of frequency characteristics. We propose a design procedure of fuzzy phase-lead compensators for linear controlled objects. Furthermore, we realize a neuro-fuzzy compensator for unknown or nonlinear controlled objects by using Widrow-Hoff learning rule.

1. Introduction

Fuzzy control was first introduced in the early 1970's by Mamdani [1]. However, we lack at present theoretical controller design methods although fuzzy control has been applied to many real industrial processes. This paper presents a theoretical compensation method of fuzzy control systems based on frequency characteristics.

The main purpose of controller design is to realize control system such that transient characteristics such as speed of response and damping characteristics are satisfied. The best way of realizing such a design is to introduce concepts of frequency characteristics such as gain crossover frequency and phase margin. Because transient characteristics are strongly related to frequency characteristics. For example, gain crossover frequency is related to speed of response, and phase margin is related to damping characteristics. Generally speaking, it is necessary to compensate phase characteristics of control systems in order to improve damping characteristics. It is, however, known that compensation of phase characteristics is not generally easy. In the field of fuzzy control, it has been said that a phase-lead compensation can be achieved if we use a coordinate transformation of e-ė phase plane, exactly speaking, rotation of e-ė phase plane [2,3,4,6]. However, we reported in a previous paper [5] that the coordinate transformation does not always realize an effective phase-lead compensation. We propose a new coordinate transformation of effectively realizing it.

An important theorem which is related to phase-lead compensation is derived by introducing concept of frequency characteristics. We propose a design procedure of fuzzy phase-lead compensators for linear controlled objects. Furthermore, we realize a neuro-fuzzy compensator for unknown or nonlinear controlled objects by using Widrow-Hoff learning rule which is a basic learning method in neural networks. The following symbols will be used in this paper.

G(s): A transfer function of a controlled object.
Gc(S): A transfer function of a linear PI controller $(a+b\cdot s)/s$.
Gc*(S): A transfer function of a linear PI controller $(a^*+b^*\cdot s)/s$.
wCG: A gain crossover frequency of open loop transfer function Gc(s)G(s).
θm: A phase margin of open loop transfer function Gc(s)G(s).
w0CG: A desired gain crossover frequency of open loop transfer function.
θ0m: A desired phase margin of open loop transfer function.
εp: A overshoot of transient response of control system.
Tp: A time to peak of transient response of control system.
ε0p: A overshoot of desired transient response of control system.
T0p: A time to peak of desired transient response of control system.
$g(w)$: A gain in the frequency w of Gc(s)G(s).
$\Psi(w)$: A phase in the frequency w of Gc(s)G(s).
$g^*(w)$: A gain in the frequency w of Gc*(s)G(s).
$\Psi^*(w)$: A phase in the frequency w of Gc*(s)G(s).

2. Frequency characteristics

Fig.1 shows an example of Bode diagram. Generally speaking, wCG and θm are related to speed of response and damping characteristics, respectively. Let us consider the following second order lag system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where ζ is damping ratio and wn is undamping natural frequency. It is known that wCG and θm can be represented using ζ and wn.

$$\theta_m = 90 - \tan^{-1}\sqrt{0.25\sqrt{4 + \frac{1}{\zeta^4}} - 0.5} \quad (1)$$

$$\omega_{CG} = \sqrt{\sqrt{4\zeta^4 + 1} - 2\zeta^2}\,\omega_n \quad (2)$$

Fig.2 shows a transient response of second order lag system. ζ and w_n can be represented by using ε_p and Tp as follows.

$$\zeta = \frac{(-1/\pi)\ln(\varepsilon_p/100)}{\sqrt{1 + ((-1/\pi)\ln(\varepsilon_p/100))^2}} \quad (3)$$

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} \quad (4)$$

The transient characteristics of a high order lag system with a overshoot εp and a time to peak Tp can be approximated by a second order lag system with ζ and w_n calculated by substituting εp and Tp into Eqs.(3) and (4).

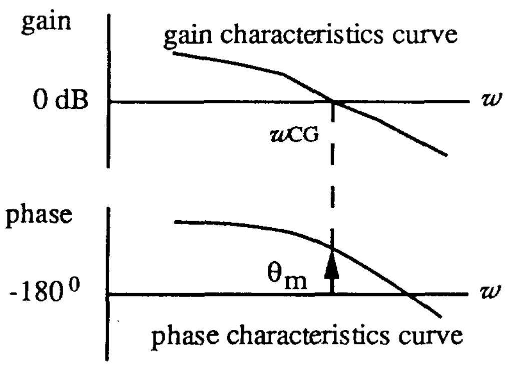

Fig.1 Bode diagram

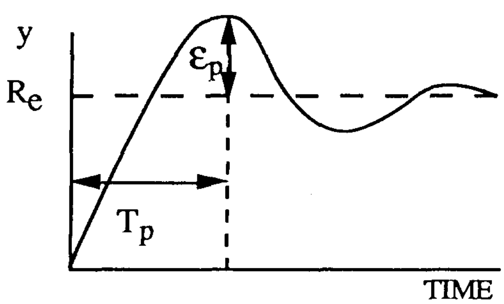

Fig.2 Transient response

3. Fuzzy phase-lead compensation

We derive an important theorem which is related to phase-lead compensation by introducing concept of frequency characteristics.

[Theorem 3.1]

If we use

$$G^*c(s)=(a^*+b^*\cdot s)/s$$

instead of

$$Gc(s)=(a+b\cdot s)/s,$$

where

$$[a^*\ b^*] = [a\ b]\ \mathbf{T}(\theta_c, w_{CG}), \quad (5)$$

$$\mathbf{T}(\theta_c, w_{CG}) = \begin{bmatrix} \cos(-\theta_c) & -(1/w_{CG})\sin(-\theta_c) \\ w_{CG}\sin(-\theta_c) & \cos(-\theta_c) \end{bmatrix}, \quad (6)$$

then the gain crossover frequency and the phase margin of open loop transfer function of G*c(s)G(s) *become* w_{CG} *and* $\theta m+\theta c$, *respectively.*

(proof)The proof is omitted due to lack of space.

We construct the following fuzzy phase-lead compensator by using the transformation matrix of Eq.(6).

Rule 1:IF Φ is about "-π or 0 or π"
THEN

$$\dot{u}_1 = [a\ b]\begin{bmatrix} e \\ \dot{e} \end{bmatrix},$$

(8)

Rule 2: IF Φ is about "-π/2 or π/2"
THEN

$$\dot{u}_2 = [a\ b]\,\mathbf{T}(\theta_c, w_{0CG})\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = [a^*\ b^*]\begin{bmatrix} e \\ \dot{e} \end{bmatrix},$$

where $\Phi = \tan^{-1}(\dot{e}/e)$. The final output of this controller is calculated as

$$\dot{u} = \frac{w_1\dot{u}_1 + w_2\dot{u}_2}{w_1 + w_2}, \quad (9)$$

where w_1 is a membership value of the fuzzy set, about "-π or 0 or π", of Rule 1 and w_2 is a membership value of the fuzzy set, about "-π/2 or π/2", of Rule 2. Fig.3 shows the fuzzy sets, where p is a premise parameter of the fuzzy sets.

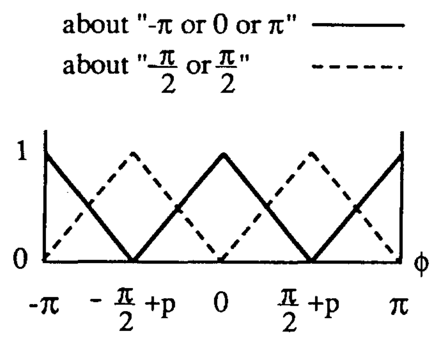

Fig.3 Fuzzy sets

The controller design is to determine a, b, a*, b* of Eq.(8) and p of Fig.3. The design procedure for linear controlled objects will be given below.

[Design Procedure]

Assume that a controlled object G(s) is linear. The procedure consists of five parts.

[Step 1]

Select a desired transient response of control system with a overshoot $\varepsilon 0p$ and a time to peak

TOp. From the values of $\varepsilon 0p$ and TOp, derive $\zeta 0$, $\omega 0n$, $\theta 0m$ and $\omega 0CG$ of the desired transient response by Eqs.(1) ~ (4).

[Step 2]
Determine a and b of Rule 1 such that speed of response is satisfied, that is, $Tp \le T0p$.

[Step 3]
Calculate the phase margin θm and the gain crossover frequency ω_{CG} of Gc(s)G(s). Next, derive a* and b* from Eq.(5), where $\theta c = \theta 0m - \theta m$.

[Step 4]
Assume that p=0. Next, investigate whether $Tp \le T0p$ and $\varepsilon p \le \varepsilon 0p$ or not. If both of them are satisfied , then end, else go to [Step 5].

[Step 5]
Adjust the premise parameter p. The value of p decreases if $Tp > T0p$ and $\varepsilon p \le \varepsilon 0p$. Conversely, it increases if $Tp \le T0p$ and $\varepsilon p > \varepsilon 0p$. If $Tp \le T0p$ and $\varepsilon p \le \varepsilon 0p$, then end, else go to [Step 2] and select other values of a and b.

We illustrate an example of the above design method.

[Example 3.1]
Let us consider the following controlled object.

$$\ddot{y} = -4\dot{y} - 4y + 3u$$

[Step 1]
We select a desired transient response such that $\varepsilon 0p$ = 5.0[%] and TOp =1.5[sec.]. From Eqs.(1) ~ (4), we obtain that $\zeta 0 = 0.69$, $\omega 0n = 2.89$, $\theta 0m$ = 64.63[deg] and $\omega 0CG$ = 1.89[rad/sec].

[Step 2]
a=6.70 and b=2.74.

[Step 3]
ω_{CG}=1.89.
θm = 39.94.
$\theta c = \theta 0m - \theta m$ = 64.63 - 39.94 = 24.69.
Therefore, a*=3.92 and b*=3.97.

[Step 4]~[Step 5]
TOp = 6.9[sec.] and $\varepsilon 0p$ =0.02[%] when p=0[rad]. So, we attempt to adjust the premise parameter p. TOp = 1.5[sec.] and $\varepsilon 0p$ =4[%] when p = $-60\pi/180$[rad]. Fig.4 shows control result.

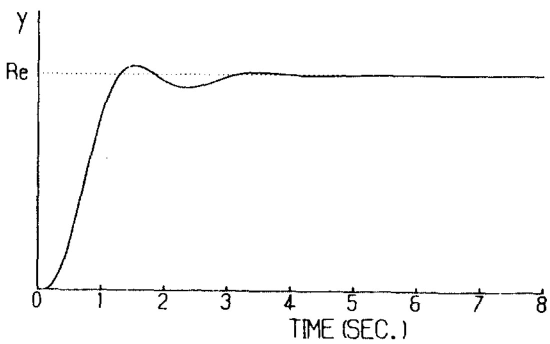

Fig.4 Control result.

4. Neuro-fuzzy compensators

The design procedure discussed in Section 3 cannot be applied to unknown or nonlinear systems since frequency characteristics cannot be used in this case. One of the possible approaches for such a case is to introduce concept of self-learning. In this section, we introduce a self-learning control by Widrow-Hoff learning rule which is a basic learning method in neural networks. The idea which optimizes parameters of fuzzy model by using Widrow-Hoff learning rule was first introduced by Ichihashi [7,8].

The performance function is defined as

$$J = \frac{1}{2}(Re - y)^2,$$

where Re is setpoint and y is the output of controlled object. From Eqs.(6), (8) and (9), we can obtain

$$\dot{u} = K\cdot\left[\left(w_1\cdot T + w_2\cdot\{T\cdot\cos(-\theta_c) + \sin(-\theta_c)\}\right)\cdot e + \left(w_1 + w_2\cdot\{\cos(-\theta_c) - T\cdot\sin(-\theta_c)\}\right)\cdot\dot{e}\right],$$

where K=b, T=a/b and ωCG=1. We should notice that the value of ωCG can not be calculated since a controlled object is unknown or nonlinear. So, we assume that ωCG=1. Even if it is assumed that ωCG=1, we can improve phase characteristics in the all frequency range by changing the value of θc. By partially differentiating J with respect to each controller parameter, we obtain

$$\frac{\partial J}{\partial K} = \frac{\partial J}{\partial y}\frac{\partial y}{\partial \dot{u}}\frac{\partial \dot{u}}{\partial K} = -(Re - y)\cdot\frac{\partial y}{\partial \dot{u}}\cdot p_K,$$

$$\frac{\partial J}{\partial T} = \frac{\partial J}{\partial y}\frac{\partial y}{\partial \dot{u}}\frac{\partial \dot{u}}{\partial T} = -(Re - y)\cdot\frac{\partial y}{\partial \dot{u}}\cdot p_T,$$

$$\frac{\partial J}{\partial \theta_c} = \frac{\partial J}{\partial y}\frac{\partial y}{\partial \dot{u}}\frac{\partial \dot{u}}{\partial \theta_c} = -(Re - y)\cdot\frac{\partial y}{\partial \dot{u}}\cdot p_{\theta_c},$$

where

$$p_K = \left(w_1\cdot T + w_2\cdot\{T\cdot\cos(-\theta_c) + \sin(-\theta_c)\}\right)\cdot e + \left(w_1 + w_2\cdot\{\cos(-\theta_c) - T\cdot\sin(-\theta_c)\}\right)\cdot\dot{e},$$

$$p_T = K\cdot\left[\left(w_1 + w_2\cdot\cos(-\theta_c)\right)\cdot e - \left(w_2\cdot\sin(-\theta_c)\}\right)\cdot\dot{e}\right],$$

$$p_{\theta_c} = K\cdot\left[\left(w_2\cdot\{-\cos(-\theta_c) - T\cdot\sin(-\theta_c)\}\right)\cdot e + \left(w_2\cdot\{T\cdot\cos(-\theta_c) - \sin(-\theta_c)\}\right)\cdot\dot{e}\right].$$

Therefore, the learning law is defined as follows.

$$K^{NEW} = K^{OLD} + \varepsilon_1\cdot\Delta K,$$

$$T^{NEW} = T^{OLD} + \varepsilon_2\cdot\Delta T,$$

$$\theta_c^{NEW} = \theta_c^{OLD} + \varepsilon_3\cdot\Delta\theta_c,$$

where

$$\Delta K = e\cdot\left(w_1\cdot T^{OLD} + w_2\cdot\{T^{OLD}\cdot\cos(-\theta_c^{OLD}) + \sin(-\theta_c^{OLD})\}\right)\cdot e + \left(w_1 + w_2\cdot\{\cos(-\theta_c^{OLD}) - T^{OLD}\cdot\sin(-\theta_c^{OLD})\}\right)\cdot\dot{e},$$

$$\Delta T = K^{OLD}\cdot\left[\left(w_1 + w_2\cdot\cos(-\theta_c^{OLD})\right)\cdot e - \left(w_2\cdot\sin(-\theta_c^{OLD})\}\right)\cdot\dot{e}\right],$$

$$\Delta\theta_c = K^{OLD}\cdot\left[\left(w_2\cdot\{-\cos(-\theta_c^{OLD}) - T^{OLD}\cdot\sin(-\theta_c^{OLD})\}\right)\cdot e + \left(w_2\cdot\{T^{OLD}\cdot\cos(-\theta_c^{OLD}) - \sin(-\theta_c^{OLD})\}\right)\cdot\dot{e}\right],$$

and e=Re-y and $\varepsilon 1$, $\varepsilon 2$ and $\varepsilon 3$ are learning factors.

We show some examples of this self-learning control. In these examples, $\varepsilon 1=0.01$, $\varepsilon 2=0.01$, $\varepsilon 3=0.000001$, Top=1[sec.] and $\varepsilon 0p=5[\%]$.

[Example 4.1]

Let us consider the following controlled object.

$$\ddot{y} = -10\dot{y} - 16y + 16u$$

Fig.5 shows a result of self-learning control, where the numbers denote iteration numbers.

[Example 4.2]

Let us consider the following nonlinear controlled object.

$$\ddot{y} = -10\dot{y} - 16y + \{-5\cdot\sin(0.5\pi\cdot u)+16\}\cdot u$$

Fig.6 shows a result of self-learning control, where the numbers denote iteration numbers.

[Example 4.3]

Let us consider the following nonlinear controlled object.

$$\ddot{y} = -10\dot{y} - \{-5\cdot\cos(4\pi\cdot y)+16\}\cdot y + 16u$$

Fig.7 shows a result of self-learning control, where the numbers denote iteration numbers.

5. Conclusion

The fuzzy phase-lead compensation based on frequency characteristics have been discussed. An important theorem which is related to phase-lead compensation has been derived by introducing concept of frequency characteristics. We have proposed a design method of fuzzy phase-lead compensators for linear controlled objects. Furthermore, we have attempted to design a neuro-fuzzy compensator for unknown or nonlinear controlled objects by using Widrow-Hoff learning rule.

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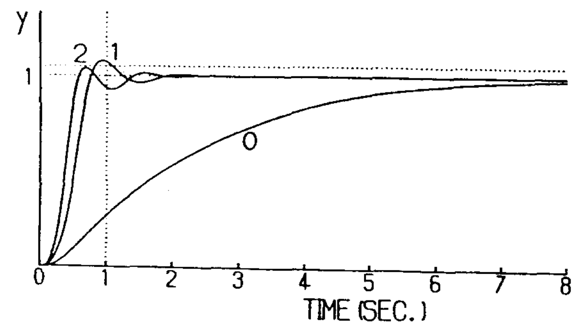

Fig.5 Control result (Example 4.1)

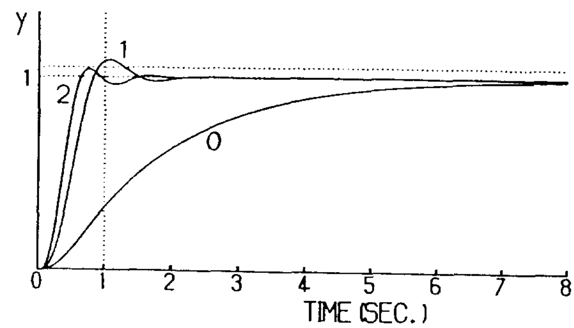

Fig.6 Control result (Example 4.2)

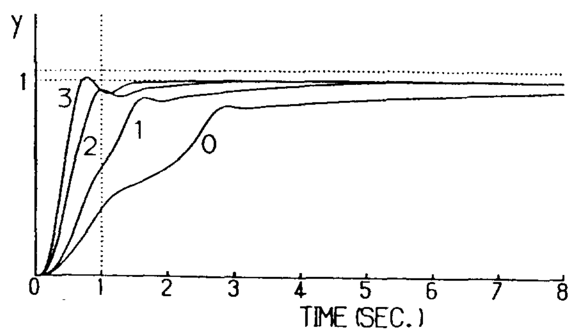

Fig.7 Control result (Example 4.3)