

The Optimal Tuning Algorithm for Fuzzy Controller

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Abstract

In this paper, an optimal tuning algorithm is presented to automatically improve the performance of fuzzy controller, using the simplified reasoning method and the proposed complex method. The method estimates automatically the optimal values of the parameters of fuzzy controller, according to the change rate and limitation condition of output. The controller is applied to plants with delay time and dead time. Then, computer simulations are conducted at step input and the performances are evaluated in the ITAE.

1. Introduction

The design object of the controller is to compensate the dynamic characteristics of the plant under control. Because of simplicity of the parameter tuning and the controller design, the PID type is well established and widely employed. However, the conventional PID controller with linear relations to plants becomes so sensitive to the control environment and the parameter changes, and the efficiency of its utility for the complex and nonlinear plants has been questioned. The fuzzy logic controller may be able to utilize a large number of the linguistic control rules based on the human experiences and knowledge, it has been proved that the fuzzy logic has been suitable for controlling the linear plants as well as the nonlinear plants. The linguistic control rules are difficult to express perfectly the human knowledges. One of the difficulties is to choose the linguistic control rules, scaling factors and membership functions of the object plants, which are the important elements of fuzzy logic controller, in order to improve control performances. The new algorithm for auto-tuning of the parameters, such as the linguistic control rules, the scaling factors and the membership functions by means of the analysis of the plant response should be developed.

In this study, an auto-tuning algorithm is presented to improve automatically the performance of the fuzzy logic controller utilizing the simplified reasoning method and the proposed complex method. The algorithm estimates and generates the optimal values of the linguistic control rules, the scaling factors and the membership functions, according to the rate of change and the limitation

condition of output. The algorithms are developed for the fuzzy logic controller including of fuzzy PI as well as fuzzy PID, the hybrid (fuzzy PID + PID, etc.), and hybrid with Smith-predictor, and are applied to the plants with delay time and dead time. Computer simulations are carried out for the step input and the system performances are evaluated in the ITAE (Integral of the Time multiplied by the Absolute value of the Error).

2. Hybrid fuzzy controller

The hybrid fuzzy controller consists of a fuzzy PID controller and a PID controller. The principal elements are scaling factors, membership functions, weighting coefficient and PID coefficients. The block diagram of hybrid controller is shown in figure 1.

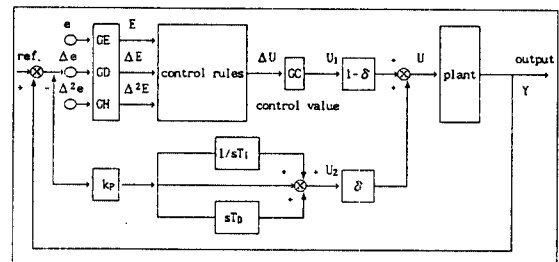


Fig.1 The scheme of hybrid fuzzy controller

The fuzzy controller with linguistic control variables consists of the N control rules which are implemented by the fuzzy logic implications as eqn. (1).

$$R_k : \text{IF } E \text{ is } A_k, \Delta E \text{ is } B_k \text{ and } \Delta^2 E \text{ is } C_k, \\ \text{Then } \Delta U \text{ is } D_k \quad (1)$$

where R_k : k -th control rule, ($k=1,2,\dots,N$)

N : the number of control rules

E : error

ΔE : change of error

$\Delta^2 E$: change of variation error

ΔU : change of plant control input

A_k, B_k, C_k and D_k : linguistic variables

If $E^0, \Delta E^0$ and $\Delta^2 E^0$ are substituted for the fuzzy variables $E, \Delta E$ and $\Delta^2 E$ of antecedent, truth value of antecedent in each rule is like eqn. (2).

$$W_i = \min\{\mu_{A1}(E^0), \mu_{B1}(\Delta E^0), \mu_{C1}(\Delta^2 E^0)\} \quad (2)$$

If the membership function D_i of consequent is not fuzzy set but singleton, the inferred value of eqn. (1) is simplified like eqn.(3), using the simplified reasoning method.

$$\Delta U = \frac{\sum_{i=0}^n W_i * D_i}{\sum_{i=0}^n W_i} \quad (3)$$

The PID controller consists of a conventional one with k_p , k_i and k_d . PID coefficients is tuned with parameters of fuzzy controller. The output of hybrid fuzzy controller is presented as eqn.(4). The weighting coefficient δ is assumed as a fuzzy variable. δ is tuned with parameters of fuzzy controller and decides weight. In the paper, all the parameters is automatically estimated and optimized by improved complex method. Optimal parameters are applied to fuzzy controller.

$$U = (1-\delta) \cdot U_r + \delta \cdot U_{pid} \quad (4)$$

3. Autotuning by improved complex method

Consider the optimal control to make the error minimum, using ISE or ITAE which shows an error characteristic of the control response to the step input and be a cost function to evaluate the optimal tuning state. Fuzzy controller also has an object to minimize ISE or ITAE as cost function. But, as we regulate the scaling factors, weighting coefficient, membership functions and PID coefficients in order to minimize ISE or ITAE, the cost function of fuzzy controller has the nonlinear dynamic characteristics that can not be formulated. Also fuzzy controller has a difficult problem to apply general optimal techniques, because it is difficult to obtain the cost function and differential value of ISE or ITAE. In order to solve the problems, the autotuning algorithm using improved complex method, a kind of nonlinear program that abstract scaling factors, weighting coefficient, membership functions and PID coefficients for the minimum error, is suggested. The variables of cost function are given by scaling factors, membership functions, weighting coefficient and PID coefficients. After we select ITAE as cost function, we try to minimize the cost function at the step input. Since ISE is also a single optimal value, it can be chosen as the value of the cost function. But, even if ISE satisfies the minimum values, the optimal parameters of ISE are somewhat different from those of ITAE. Hence overshoot and reaching time etc. are a little different. When the difference is small, low-order plant can use ISE or ITAE as the cost function. However, as the difference is relatively big in the high-order plant, ISE or ITAE is chosen according to the object of control. The scheme of system to autotune all the parameters is like figure 2. After the control output is calculated in off-line, ITAE is obtained. Such a series of values are repeatedly calculated by improved complex method until the standard deviation of ITAE is smaller than some prescribed small quantity. The parameters of optimal ITAE is stored as the new parameters of scaling factors, weighting

coefficient, membership functions and PID coefficients.

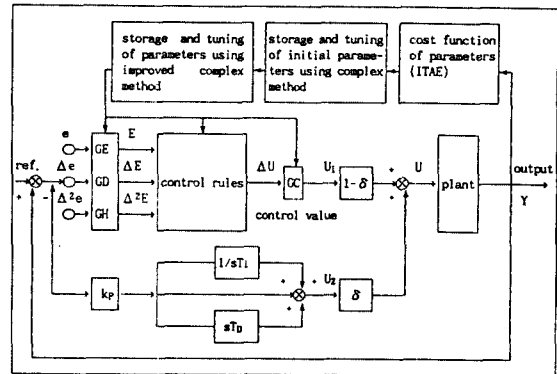


Fig.2 The scheme of autotuning hybrid fuzzy controller

We realize the algorithm to expand the simplex concept to the complex method - constrained optimization technique - as follows, and the flowchart of the proposed optimal tuning algorithm is described in Fig.3.

<step 1>

The set of the initial values for the parameters is prepared more than the number of variables, arbitrarily. The parameters mean scaling factors, weighting coefficient and membership functions. They are defined as $X_k = (x_1^k, x_2^k, \dots, x_n^k; k = 1, 2, \dots, n, n+1, \dots, m)$ in n dimension space.

<step 2>

The initial values of α , β and γ is specified using the Reflection, Expansion and Contraction of simplex concept as follows:

$$i) \text{ Reflection: } X_r = X_0 + \alpha(X_0 - X_h) \quad (5)$$

$$ii) \text{ Expansion: } X_e = X_0 + \gamma(X_r - X_0) \quad (6)$$

$$iii) \text{ Contraction: } X_c = X_0 + \beta(X_h - X_0) \quad (7)$$

<step 3>

X_h and X_l are the vertices corresponding to the maximum function value $f(X_h)$ and the minimum function value $f(X_l)$. X_0 is the centroid of all the points X_i except $i=h$. Reflected point X_r is given by $X_r = X_0 + \alpha(X_0 - X_h)$. If X_r may not satisfy all the constraints, a new point X_r is generated by $X_r = (X_0 + X_r)/2$. This process is conducted repeatedly until X_r satisfies all the constraints. A new simplex is started.

<step 4>

If a reflection process gives a point X_r for which $f(X_r) < f(X_l)$, i.e. if the reflection produces a new minimum, we expand X_r to X_e by $X_e = \gamma X_r + (1-\gamma)X_0$. If X_e may not satisfy all the constraints, a new point X_e is generated by $X_e = (X_0 + X_e)/2$. This process is conducted repeatedly until X_e satisfies all the constraints. If $f(X_e) < f(X_l)$, we replace the point X_h by X_e and restart the process of reflection. On the other hand, if $f(X_e) > f(X_l)$, we replace the point X_h by X_r , and start the reflection process again.

<step 5>

If the reflection process gives a point X_r for which $f(X_r) > f(X_l)$, for all i except $i=h$, and $f(X_r) < f(X_h)$, then we replace the point X_h by X_r . In this case, we contract the simplex as follows: $X_c = \beta X_h + (1-\beta)X_0$. If $f(X_r) > f(X_h)$, we will use $X_c = \beta X_h + (1-\beta)X_0$ without changing the previous point X_h . If X_c may not satisfy all the constraints, a new point X_c is generated by $X_c =$

$(X_0+X_1)/2$. This process is conducted repeatedly until X_0 satisfies all the constraints. If the contraction process produces a point X_c for which $f(X_c) < \min[f(X_h), f(X_r)]$, we replace the point X_h by X_c and proceed with the reflection again. On the other hand, if $f(X_c) > \min[f(X_h), f(X_r)]$, we replace all X_i by $(X_i+X_1)/2$, and start the reflection process again.

<step 6>

The method is assumed to have converged whenever the standard deviation of the function at the vertices of the current simplex is smaller than some prescribed small quantity ϵ like eqn.(8).

$$Q = \left\{ \frac{\sum_{i=1}^{n+1} [f(X_i) - f(X_0)]^2}{n+1} \right\}^{1/2} \leq \epsilon \quad (8)$$

If Q may not satisfy eqn.(8), we go to step 3.

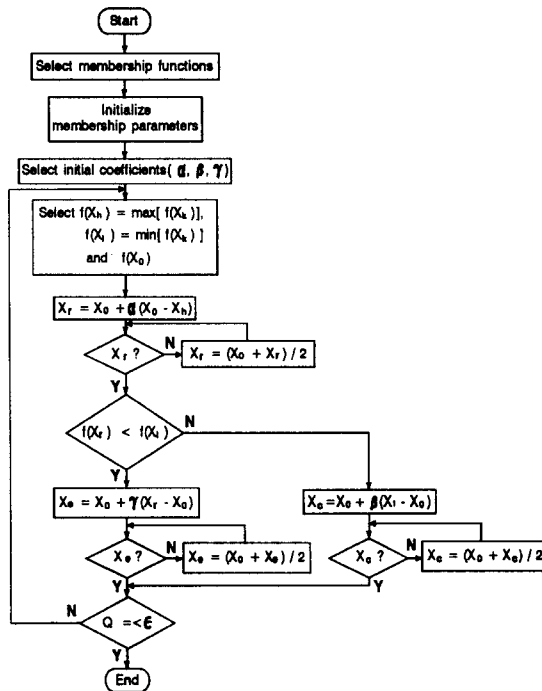


Fig. 3 Flowchart of optimal tuning algorithm

4. Computer simulation and results

To evaluate the performances and characteristics of hybrid controller with optimal autotuning algorithm, the plants with time-delay is given in eqn. (9)-(10). Computer simulation is conducted at the step input of sampling time 0.5[s]. We analyze the various cases of fuzzy PID and hybrid controllers in examples. Table 1 and figure 4 are initial linguistic control rule and membership functions using in examples.

$$\text{PLANT 1 : } \frac{Y(s)}{U(s)} = \frac{e^{-2s}}{s+1} \quad (9)$$

$$\text{PLANT 2 : } \frac{Y(s)}{U(s)} = \frac{e^{-0.8s}}{(s+1)(s+2)} \quad (10)$$

Table 1. linguistic control rules for 3-fuzzy variables

(a) $\Delta^2 E = N$				(b) $\Delta^2 E = Z$				(c) $\Delta^2 E = P$			
	N	ΔE Z	P		N	ΔE Z	P		N	ΔE Z	P
N	NB	NB	NM	N	NB	NM	NS	N	NM	NS	ZE
E Z	NM	NS	ZE	E Z	NS	ZZ	PS	E Z	ZE	PS	PM
P	ZE	PS	PM	P	PS	PM	PB	P	PM	PB	PM

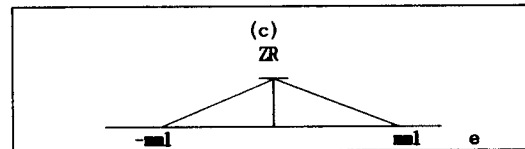
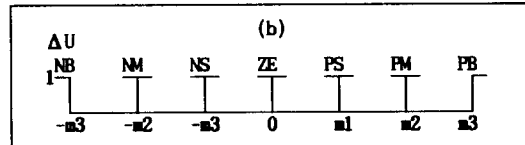
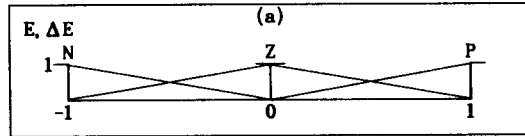
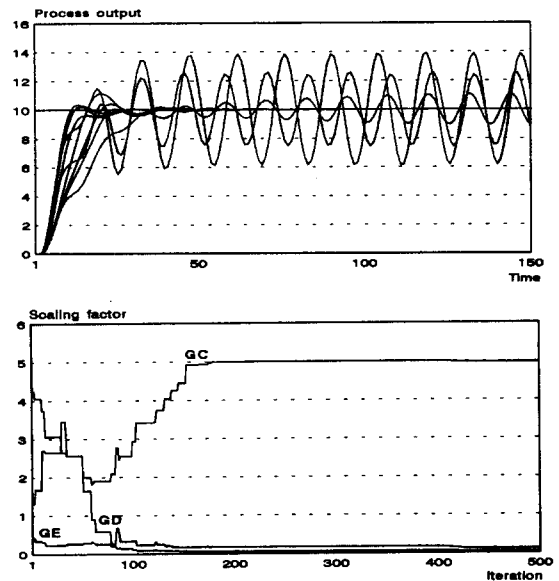


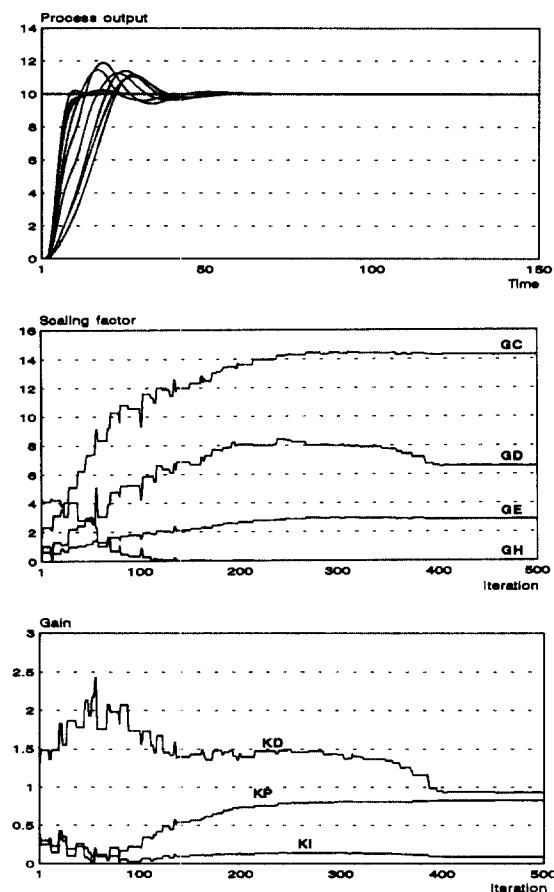
Fig. 4 (a), (b) : Membership functions of linguistic control rule
(c) : Membership function of weighting coefficient

The results are like figures 5-6.



Initial ITAE	203.05			Tuned ITAE	40.0			
Initial value	GE	0.5	GD	1.5	GH		GC	4.57
Tuned value	GE	0.045	GD	0.118	GH		GC	4.977
Initial rule	m1	0.33	m2	0.667	m3	1.0		
Tuned rule	m1	0.46	m2	0.947	m3	1.30		

Fig. 5 Output of plant 2 with fuzzy PID controller



Initial ITAE	84.595			Tuned ITAE			32.215		
Initial value	GE	0.41	GD	0.45	GH	4.15	GC	1.6	
Initial value	KP	0.1	KI	0.145	KD	1.150			
Tuned value	GE	2.870	GD	6.562	GH	0.002	GC	14.297	
Tuned value	KP	0.825	KI	0.09	KD	0.934			
Initial rule	m_1	0.333	m_2	0.667	m_3	1.0			
Tuned rule	m_1	0.086	m_2	0.607	m_3	1.064			
Weighting factor	10								

Fig. 6 Output of plant 2 with hybrid controller

5. Conclusions

This paper presents an optimal algorithm to autotune the scaling factors, membership functions, PID coefficients and weighting coefficient, using the proposed complex method. In order to improve control performance of fuzzy controller, the method is applied to the plants with delay time and dead time.

Some results are drawn from computer simulation as follows:

1. The scaling factors converge to the optimal values, according to adjusting the scaling factors through the proposed method, iteratively.
2. It is easy to autotune the linguistic control rules and the scaling factors, using the autotuned scaling factors as the initial values.
3. Because the optimal parameters are tuned automatically, under the change rate and limitation condition of

output, the proposed algorithm be applied to the real plant.

4. The optimal parameters are obtained by not only the determination of the initial parameters (as Chien Hrones Reswkr and Cohen Coon methods), but also the choice of the initial ill-condition, using the proposed algorithm.
5. In the step response of plants with delay time and dead time, the hybrid controller with smith predictor is excellent in comparison with the conventional fuzzy controller.

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