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On the Interpretation of Fuzzy Controllers

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Abstract

In the last years fuzzy control has grown up to an important methodology of control engineering. In spite of the successful realizations of the underlying concepts in industrial products there has only been little effort regarding a semantical foundation of the prevailing heuristics that are used in fuzzy control.

For this reason we want to outline promising approaches to an interpretation and better mathematical justification of fuzzy control, where the fundamental ideas of using equality relations to specify fuzzy environments for crisp data are presented. It turns out that Mamdani's classical max-min-inference is a consequence of our model.

1 Introduction

In most cases classical methods of control engineering are related to *physical modelling*, where the considered control task is characterized by a mathematical model which often consists of a system of differential equations to be (numerically) solved. If a solution has been found, then well-known techniques of approximation, linearization, validation, and stability analysis are applied to obtain an appropriate control function which quantifies the relationships between the input values and the corresponding output values of the given control system.

Whenever physical modelling is difficult to realize because of complexity problems, or the available information turns out to be rather vague and uncertain than crisp, so that there only exists a partial integration of knowledge into the mathematical models used in classical control engineering, then the alternative consideration of *cognitive modelling* seems to be reasonable, if experience has shown that the underlying control task is feasible by human experts.

Cognitive modelling develops a semiformal description of the control strategy and formalizes it by a cognitive model instead of the unknown physical model. Similar to the physical model the iterative procedure of tuning, validation, and stability analysis has to be executed to get an appropriate control function. Using a cognitive approach it is therefore not intended to get a model of the process, but rather a model of the expert who is in the position to specify the most important properties of the process.

Current cognitive models are, for example, *neural networks* and *truth–maintenance systems*. The idea of *fuzzy control* [14, 16] is to simulate the behaviour of a human expert (who is able to solve the given control problem) by translation of his (linguistic) inference rules into a control function. Benchmark tests in the field of adaptive control showed that cognitive methods have surprising positive robustness properties and are therefore acceptable as reasonable alternatives to modern methods of control engineering (e.g. supervised adaptive control) [1].

2 Approaches to Cognitive Modelling

In order to understand how and why fuzzy control is an appropriate control technique, it is necessary to provide a well-founded semantic background for the applied concepts, enabling us to explain what specific fuzzy sets mean, where they come from and how to operate with these fuzzy sets. Some of the concepts applied in fuzzy control are based on a rather intuitive understanding of fuzzy set theory without having a clear model which motivates and justifies these concepts. As an example consider the classical version of the max–min controller [14, 16] which tries to handle the four steps of fuzzification, inference, combination, and defuzzification in different mathematical structures. While the inference mechanism may be clarified by logics, the center of area defuzzification method (which gives good results due to its good interpolation properties) cannot be justified in a logical calculus. Hence there is a need to provide underlying semantics and a reasonable integrating formal environment.

One promising way to free ourselves of the heuristic semblance of fuzzy control refers to the concept of equality relations [9, 11] which reflect a mathematical characterization of the indistinguishability of objects. A second way to encounter the mentioned semantical problems is to use methods of approximate reasoning. Typical approaches to the foundation of fuzzy control interpret the control rules as inference rules leading to a conjunctive combination of rules and, in case of a possibilistic interpretation of the involved fuzzy sets, resulting in the application of the Gödel relation for the inference procedure [2, 13]. The interpretation of the rules and the fuzzy sets is beyond question a crucial point for providing a semantical background for fuzzy control. Since there are various interpretations for the rules as inference schemes [3] and for the origin of fuzzy sets [5, 6], the corresponding model has to be chosen carefully. In this connection one starting point for interpretation aspects is the context model [6, 7], a generalized random set approach to develop an integrating framework for the representation, interpretation, and operational composition of data that are expected to contain three special kinds of imperfect knowledge, which are imprecision, competition, and uncertainty. Since the clarification of the context model and the existing relationships between the mentioned approaches is beyond the scope of this paper, as one interesting result it should be emphasized that the application of the concepts provided by the context model (vague characteristics, their information compressing transformation to possibility functions, correctness– and sufficiency–preservation [8]) induce the same final results as they are also obtained by the use of equality relations.

An alternative methodology which is not based on inference methods applied in approximate reasoning is to embed fuzzy control in the classical interpolation and approximation theoretic approaches [4]. From this point of view fuzzy control helps to define the input–output function by using additional expert information such as linguistic rules, approximate input–output tuples etc, but this area of "knowledge based interpolation" is yet not fully developed.

3 Fuzzy Control Based on Equality Relations

A widespread technique of fuzzy control is to use systems of linguistic control rules, which are interpreted by fuzzy relations. Each of the underlying control rules specifies a relationship between vague input values and vague output values. On the other hand we notice that the inference mechanism of max–min controllers is directed to the handling of fuzzified crisp input values and to the calculation of defuzzified crisp output values. Hence it seems to be reasonable to refer the specification of a fuzzy controller to crisp data in a fuzzy environment characterized by equality relations, but to avoid the consideration of fuzzy data. This idea has tradition in quantum physics, where also the problem arises that one cannot speak of two different points if the distance between these points is less than ε [15]. An in-depth presentation of the application of equality relations for the semantical foundation of common heuristic methods of fuzzy control is given in [12] and [10]. In this paper we only adress basic ideas.

Let $\xi_1, \ldots, \xi_n$ be n input variables with ranges $X_1, \ldots, X_n$, respectively, and for reasons of simplicity, η a single output variable with range Y. Furthermore we assume that an expert knows the correct output values $y^{(j)}$ that correspond to a number of special tuples $(x_1^{(j)}, \ldots, x_n^{(j)})$, $j = 1, \ldots, k$, of input values. The k given pairs of input– and output–values refer to the linguistic control rules

If ξ_1 *is* (approximately) $x_1^{(j)}$ *and* ... *and*
ξ_n *is* (approximately) $x_n^{(j)}$,
then η *is* (approximately) $y^{(j)}$, $j = 1, \ldots, k$.

The involved vagueness is characterized by the equality relations $E_1, \ldots, E_n$ and F on the sets $X_1, \ldots, X_n$ and Y, respectively. An *equality relation* E on a set X (with respect to a t–norm $\sqcap$) is a mapping $E : X \times X \to [0,1]$ that satisfies the axioms of total existence ($E(x,x) = 1$), symmetry ($E(x,y) = E(y,x)$), and transitivity ($E(x,y) \sqcap E(y,z) \le E(x,z)$). The unknown control function which is partly defined by the given pairs $((x_1^{(j)}, \ldots, x_n^{(j)}), y^{(j)})$, $j = 1, \ldots, k$, can be characterized by a *morphism* from $(X_1, E_1) \times \cdots \times (X_n, E_n)$ to (Y, F), i.e. a mapping $\phi : \bigtimes_{i=1}^{n} X_i \times Y \to [0,1]$ that satisfies the *extensionality axioms* $\phi(\mathrm{x}, y) \sqcap E(\mathrm{x}, \mathrm{x}') \le \phi(\mathrm{x}', y)$ and $\phi(\mathrm{x}, y) \sqcap F(y, y') \le \phi(\mathrm{x}, y')$, where E is assumed to denote the equality relation $(E(x_1, \ldots, x_n), (x_1', \ldots, x_n')) = E(x_1, x_1') \sqcap \cdots \sqcap E(x_n, x_n')$ induced by $E_1, \ldots, E_n$ on the cartesian product $X_1 \times \cdots \times X_n$. Furthermore ϕ shows the *singleton property* $\phi(\mathrm{x}, y) \sqcap \phi(\mathrm{x}, y') \le F(y, y')$ and the ***total definiteness*** $\sup\{\phi(\mathrm{x}, y) \mid y \in Y\} = 1$.

When ϕ is specified and the input values $\xi_i = x_i$, $i = 1, \ldots, n$, are available, then the fuzzy set $\mu[x_1, \ldots, x_n] : Y \to [0,1]$, defined by $\mu[x_1, \ldots, x_n](y) = \phi((x_1, \ldots, x_n), y)$ characterizes the vague output value. Additionally $x_1^{(j)}, \ldots, x_n^{(j)}, y^{(j)}$, $j = 1, \ldots, k$, induce the *singletons (extensional fuzzy sets)* $\mu_{x_i^{(j)}} : X_i \to [0,1]$, $\mu_{x_i^{(j)}}(x) = E_i(x, x_i^{(j)})$, $i = 1, \ldots, n$.

Given arbitrary input tuples $(x_1, \ldots, x_n)$ it is now easy to calculate a lower bound for $\mu[x_1, \ldots, x_n]$ according to the extensionality of ϕ. For all $y \in Y$, we obtain [12]:

$$\mu[x_1, \ldots, x_n](y) \ge \max_{j=1,\ldots,k} \{\mu_{x_1^{(j)}}(x_1) \sqcap \cdots \sqcap \mu_{x_n^{(j)}}(x_n) \sqcap \mu_{y^{(j)}}(y)\}.$$

In the special case $\sqcap = \min$ the most specific fuzzy set $\mu[x_1, \ldots, x_n]$ exactly coincides with the result that a max–min–controller applies for its defuzzification process. Hence we have found a justification of max–min–controllers by the mathematical concept of equality relations.

Regarding the semantical analysis of defuzzification strategies we refer to [12, 10].

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