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FUZZY DECISION MAKING SYSTEM

Ephim Ja. KARPOVSKY, Prof., D.Sc.

Head of Computer Science Department,

Odessa Institute of National Economy, Ukraine

Abstract

This paper focuses on the usage of the fuzzy set theory in decision making systems. The approach to calculation of generalized membership function, based on application of method of principal components is proposed. For solving of the problem of fuzzy forecasting the development of Bayes procedure is used. The structure of decision making system, in which following procedures are fulfilled, is discussed.

Introduction

Problems of multicriteria choice & decision making are often characterized by non-fullness of volume and/or statistical heterogenity of initial data, using of fuzzy criteria and linguistic variables, In these cases it is prospective to use fuzzy sets theory for modelling of initial situation, evaluating of alternative decisions and forecasting of results of realization of decision taken. That's why nowadays fuzzy decision making systems (FDMS) are spread, experience of using them has revealed hardships, connected with problem of generalization (composition) of membership functions (MF) and fuzzy forecasting of the results of realization of decision taken.

In this work the approach to calculation of generalized MF, based on application of method of principal components is proposed. For solving of the problem of fuzzy forecasting the development of Bayes procedure is used. Then the structure of FDMS, in which following procedures are fulfilled, is discussed:

measurement of initial situation with the help of numeric and linguistic variables;

transferring of numeric meanings into fuzzy (L-R) numerals and setting of MF for linguistic variables (the set of L-R numerals and MFs characterized vector fuzzy model of initial situation is created);

generation of the list of alternatives for calculated meanings of generalized MF;

counting of values of gain (loss) for every alternative and choice of preferences;

if fuzzy forecasting of results of realization of chosen alternative doesn't improve the initial situation, then the work of FDMS continued; in the opposite case, the information about the initial situation, methods recommended and devices of it's improvement with commentaries on every step of process of support of decision taken and fuzzy forecast of results of it's realization is put to the screen of display.

1. Calculation of generalized membership function.

Let the linguistical variables (LV) vector, which characterizes different qualities of the one and the same situation (object, process), is measured. Such a task appears, for example, during the estimation initial situation. As the LV vector belongs to the one situation (object), in this case, as a rule, some interlinks (linguistical, correlational, functional) between linquistical meanings of separate LV exist. For example, the more expensive automobile usually is more comfortable; short human's weight is, as a rule, small, etc. The counting of the additional information about interlinks between different LV, characterizing the one and the same object, allows us to suggest formalized procedures of the generalized MF drawing up, which delete subjectivism, peculiar for the most of such procedures.

The generalized MF drawing up for getting of the characteristic of the situation (object, process), is described in this work in cases when:

a) Situation (object, process) is simulated with the help of LV vector $L=\{l_i\}$, $i=\overline{1,N}$; N>2, where every one from l_i LV characterizes one of the object fuzzy characteristics;

b) The meaning l-s LV set up with the help of the membership function $\mu(u) \in [0;1]$, $u \in [0;1]$, and the existence of the correlational links between the meaning $\mu_i(U)$ and $\mu_j(U)$ $i \neq j$, determined for example, with the help of the correlational coefficient $r_{ij}(\ (U))$,

So the initial model of the object (process) has the form of vector of LV. It is necessary to determine the object's integral fuzzy characteristic by counting generalized MF.

Let the fuzzy variables to be setted with the help of the corresponding MF like μ_i and μ_j, where N is the general number of the fuzzy variables. Matrix $M=||\mu_i(u)||$ consists of the set of MF and characterizes the fuzzy model of the researched situation (object) in general. Then integral generalize membership function are suggested to be counted as the linear functions in the form of ,

$$F=A^{-1}M,$$

where

$A=U*Lambda^{1/2}$ - matrix of the scale coefficients of the principal components;

Lambda - eigenvalue vector of the matrix R;

U - matrix of eigenvalue vectors of matrix R;

$R=\frac{1}{N}MM^{T}$ - correlation matrix of the MF for an object.

To calculate the elements of the vector Lambda and matrix U following matrix equations are solved:

$$|R-Lambda*E|=0$$

and

$$(R-Lambda*E)U=0$$

where $E=||e_{ij}||$ is the matrix, in which $e_{ii}=1$; $e_{ij}=e_{ji}=0$, $i \neq j$.

Let us discuss an example. Let a model of situation (object, process) has three linguistics variables:

{cost},{reliability},{efficiency}.

Suppose that the situation has following lunguistic estimations:

{efficiency}<---->{nearly GOOD};
{cost}<---->{AVERAGE};
{reliability}<---->{GOOD}.

The corresponding membership functions are:

$\mu_1=\mu$ {efficiency}=0|0+0|.2+.6|.4+ +.8|.6+.9|.8+1|1;

$\mu_2=\mu$ {cost}=0|0+0|0+.6|.4+1|.6+ +.6|.8+0|1;

$\mu_3=\mu$ {reliability}=0|0+0|0+.4|.6+ +.8|.8+1|1.

Calculation by the principal component method gives following results:

Compo-nent	Eigen-value	Percentage of variability
1	2.262	75.4
2	.735	24.5
3	.003	.1

The values of the first principal component weighing coefficients are $a_1=.384$; $a_2=.253$; $a_3=.363$. Then generalized membership functions by the first principal component (75.4%) is

$\bar{\mu}_1$ =0|0+0|.2+.53|.4+.78|.6+ +.79|.8+.75|1.

The second principal component (24.5%) is

$\bar{\mu}_2$ =.56|0+.56|.2+.79|.4+1|.6+ +.58|.8+0|1.

These two components are quite fully determined the composed membership function in such case.

Moreover, we may show that in the most cases not more than three components are needed to determine the generalized membership function. This suggestion is true if the membership functions are determined by the basis of three linguistics values ({BAD},{AVERAGE},{GOOD}) and its modifications with the L.Zade terms ({VERY},{NEARLY},etc.) help.

So, if the initial characteristics of situation are measured by a redundant set of linguistics values and these characteristics are highly correlative,then this object may be determined with the help of not more than three linear combinations of membership functions, i.e. principal components.

The set of principal components is a fuzzy composed model of situation. This model is used not only for estimation fuzzy situation, but for the Bayesian prediction of the results of realization of chosen alternative

2. The Bayesian procedure.

The presented Bayesian procedure is the same as in work [1], but it is assumed that a priori and a posteriori estimates of probabilities and an elements of matrix C of incomes are (L-R) fuzzy numbers.

So,let fuzzy numbers $P(b_j)$, which correspond to estimates of probabilities of fuzzy situations $b_j \in B$ to be given. $P(b_j)$ is calculated with the help of fuzzy composed model of situation as a set of principal components of membership functions. Also information is known about fuzzy numbers $x_j \in X$, which characterize possible values of real estimation of situations. Let us assume also that we have a set $A=\{a_i\}$ of possible alternatives $(i=1,\dots,n)$ and a mat- rix $C=||c_{ij}||$ of incomes. What we need is to determine proper alternative.

To describe the method of solving we will employ:

- fuzzy numbers adding $\tilde{\oplus}$;
- fuzzy numbers multiplication $\tilde{\circledast}$;
- fuzzy probabilities estimates multiplication $\tilde{\ast}$;
- fuzzy probabilities estimates adding $\tilde{+}$;
- fuzzy probabilities estimates division $\tilde{/}$.

The introduced operations allow to calculate a fuzzy number of a priori estimate of an average possible income for the i-th alternative:

$$z_i(B)=\tilde{\sum_j} c_{ij} \tilde{\ast} P(b_j). \qquad /1/$$

Similarly a posteriori estimate can be determined as follows:

$$z_i(B/X)=\tilde{\sum_j} c_{ij} \tilde{\ast} P(b_j|X), \qquad /2/$$

where

$P(b_j|X)$-fuzzy number, which characterizes a posteriori probability of the j-th situation.

Using eguations /1/ and /2/ we can generalize the Bayesian formula for fuzzy numbers for the case when more than two mutually exclusive hypotheses forming the complete set of events are considered. For this let us pick out some single hypothesys as $(b_j)$and group all the rest into $(\overline{b_j})$.

Knowing that the sum of all a posterioiri probabilities equals 1 we will get

$$P(b_j|X)=(P(X|b_j) \tilde{\ast} P(b\ )) \tilde{/}$$

$$\tilde{/}(\tilde{\sum_j}(P(X|b_j) \tilde{\ast} P(b_j)). \qquad /3/$$

3. Decision making algorithm

The above presented approach is implemented in the following procedure:

Step 1. Firstly we should calculate values of a priori functions of belonging $\mu(i)^+$ and of not belonging $\mu(i)^-$ of the i-th value a_i to the $A_p$set of recommended alternatives for this situation:

$$\mu(i)^+=\mu(a_i \in A_p\ |z_i(B))=$$

$$=1 \tilde{\ominus} d_i(B) \tilde{/} \tilde{\sum} d_i(B), \qquad /4/$$

$$\mu(i)^-=1 \tilde{\ominus} g_i(B) \tilde{/} \tilde{\sum_j} g_i(B), \qquad /5/$$

where
$d_i(B)=(z_i(B) \tilde{\ominus} z_{min}(B)) \tilde{/} z_{min}(B)$,
$g_i(B)=(z_{max}(B) \tilde{\ominus} z_i(B)) \tilde{/} z_{max}(B)$.

Having the results of step 1 we can draw n plots of fuzzy numbers (corresponding to the number of possible alternatives) that characterize belonging and not belonging of the i-th value to the A_ρ set of recommended alternatives.

Step 2. Values a_i are put in descending (by $z_i(B)$ value) order to find such a number i=k for which following equations are valid:

$\mu(k-1)^- >= \mu(k-1)^+$,

$\mu(k+1)^- <= \mu(k+1)^+$, k=2,...,n-1

Then $A_\rho(B)$ set will consist of those alternatives (put in the descending order by $z_i(B)$) whose numbers are greater or equal than k, i.e.

$A_\rho(B)=\{a_i(B)\}$, i=k,...,n.

Step 3. By means of formulae similar to /4/ and /5/ the set of $A_\rho(B|X)=\{a_i(B|X)\}$ is determined.

Note. When comparing $\mu(i)^+$ with $\mu(i)^-$ on the steps 2 and 3 we assume that fuzzy numbers differ only when division rate /3/ exceeds some critical level.
If division rate is smaller then it may be assumed that $\mu(i)^+$ and $\mu(i)^-$ are equal.

Step 4. Sets $A_\rho(B)$ and $A_\rho(B/X)$ are analyzed. If

$A_\mu = A_\rho(B|X) \setminus A_\rho(B) = 0$

then decision is made. If so, income of chosen alternatives determined by values $a_i \in A_\rho(B|X)$.

If not, next iteration is fulfilled with the step 1-4. If the r-th iteration gives $|A_\mu(r)|$ greater than the (r-1)-th one, i.e.

$|A_\mu(r)| / |A_\mu(r-1)| > 1$,

then the set of alternatives is unsufficient.

4. Fuzzy decision making system

The FDMS is performing following procedures:

- descriptions of microsituations with the help of numeric and linguistic indicies,
- transformation of numbers into fuzzy (L-R) numbers, creation of membership functions for linguistic indicies (the set of (L-R) numbers and membership functions characterizes reasons of microsituations appearance),
- situation (a certain set of microsituations) is modelling with the help of special procedure of construction of fuzzy composed models,
- generation of a list of alternatives,
- calculation of generalized membership function for every alternative, choose the preferable alternative and it's prediction,
- if prediction of the results of realization of chosen alternative doesn't improve situations, which is fuzzy diagnosted, then FDMS work continues, otherway information about diagnosis of situation, recommended methods and means of its changes with the explanations of every stage of decision making and prediction of it's results is displayed to the user.

FDMS consists of three subsystems:

- fuzzy modelling subsystem (FMSS);
- computing subsystem (CSS);
- fuzzy decision making subsystem (FDMSS).

Unity of these subsystems and their interaction are provided by means of their common parts:

- data bases;
- knowledge bases;
- end user and expert interfaces;
- data processing programs.

References.

1. Karpovsky E.,Nsowah-Nuamah N. Computer and Information Sciences No. 3, vol. 35, 1988.