Fifth IFSA World Congress (1993), 773-776

FUZZY ALGEBRAIC ADAPTIVE SYSTEMS BASED ON LINEAR COMBINERS

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Abstract

A design method for linear combiner type filters, based on a fuzzy variant of the usual design method, is introduced and analyzed. Design results are exemplified.

1. Introduction

The problem in hand is the design of a digital system that behaves as close as possible to a required manner.This means that the required system either similarly behaves to a "goal" system (the "model" problem), or that it processes in a desired way a given input signal (the "filter problem") These problems are classical ones in systems theory and electronic engineering. Under the frame of linear systems theory, a linear combiner configuration is used. The linear combiner equation is:

$$y_n = \sum_{i=0}^{L} w1_i * x_{n-i} + \sum_{i=1}^{L} w2_i * y_{n-i} \quad (1)$$

where: x_{n-i} - are samples of the input signal; $w1_i$, $w2_i$ - are (crisp) coefficients.

The linear combiner design method is as follows: the signal to be processed is input, and the output is compared with the desired signal. The error between the desired and the actual output signal is: $\varepsilon = d - y$

The weights Wk are then adjusted in accordance to:

$$\Delta W = \eta * \varepsilon * x$$

The above described adaptation method is in fact a gradient - type method, as the change of the parameters of the system (weights) is proportional to the gradient to the error surface, in the point where the system is at that moment of time [1]. The basic idea of this method is that the minimum/maximum of a functional is reached in a search procedure following the 'valley' direction. As the optimization criterion is that of the extremum of a given functional, the mentioned euristical search procedure founds many applications in design. The purpose of this paper is to prove by examples that using fuzzy quantities in finding the optimal solutions can help. Thus, we prove that the use of fuzzy quantities is benefic in optimization, design - and in general, in creative processes.

2. The design problem

The theoretical frame to define and analyze the filters is that of the algebraic fuzzy systems, shortly described in [2]. These filters are an extension of the digital linear filters (1).

Algebraic fuzzy systems have a transfer function defined by a recursive polynom with fuzzy coefficients and variables.

The extension of the crisp filters to the algebraic fuzzy systems is:

$$\tilde{y}_n = \sum_{i=0}^{L} \tilde{w}1_i * x_{n-i} + \sum_{i=1}^{L} \tilde{w}2_i * \tilde{y}_{n-i} \quad (2)$$

where: L is the number of delays (crisp delay values); x is the input vector (crisp or fuzzy input signal); $\tilde{w}1_i, \tilde{w}2_i$ are the fuzzy coefficients.

These systems require fuzzy numbers as coefficients as in (2) and are able to process fuzzy signals. In this paper we use triangular membership functions.

Both the non-recursive filter case and the recursive filter case were analyzed in this research.

In both cases, the system (2) is used in a feedback loop, to allow the adaptation. The error between the defuzzificed output signal of the system and the desired signal is used in the algebraic fuzzy systems adaptation.

The adaptation method used is a fuzzy extension of the LMS (least-mean-square) classic algorithm. Indeed, taking into account the fact that we deal with an adaptive fuzzy linear combiner and that each iteration value of input signal as well as the values of the desired response are available, the LMS-type algorithms were considered an appropriate tool.

The use of fuzzy quantities is equivalent to the existence of a 'fuzzy ravine' in the optimization process (see **Figure 1**).

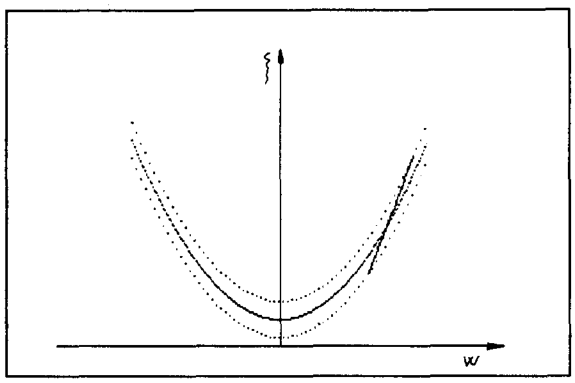

Figure 1

Figure 1: The 'fuzzy ravine': The gradient is steeper than in the crisp case

The following relations for determining the algebraic fuzzy systems are used:

$$\tilde{y}_k = \sum_{l=0}^{L} \tilde{w}1_{lk} * x_{k-l} + \sum_{l=1}^{L} \tilde{w}2_{lk} * y_{k-l}$$

$$\tilde{\varepsilon}_k = d_k - \tilde{y}_k$$

$$\tilde{w}1_{k+1} = \tilde{w}1_k + 2*\mu*\varepsilon_k*x_{k-l} \quad (3)$$

$$\tilde{w}2_{k+1} = \tilde{w}2_k + 2*\mu*\varepsilon_k*y_{k-l}$$

where: d_k is the current sample of the desired output signal; ε_k is the error between the actual output value and the desired response; μ is the coefficient of the adaptation speed. To determine the weights, a gradient technique is applied. As a measure of the adaptation performance, one determines the MSE/mean-square-error both the full adaptation time and on the last analyzed period of the signal.

The use of the adaptive fuzzy filters is important in the crisp filters design problems, where the filter tolerance coefficients are known and are considered to give rise to a fuzzy-type uncertainty.

3. Simulation results

A computer aided design of the filters (1) and (2) was performed and the results were contrasted.

According to (3), as a result of the design, one gets the fuzzy values of the weights which, by defuzzification, are determining the crisp system weights, having the given tolerances.

The subsequent figures show examples of results of the design.

Figure 2 represents a schema of a non-recursive adaptive fuzzy filter with variable number of delays.

Figure 3 represents the input and the desired signals.

Figure 4 presents a comparison between the results for the fuzzy optimal filter and the crisp filter: the MSE for the fuzzy and crisp systems (MSE versus time).

Figure 5 is a comparison between the output signals from the fuzzy filter and the corresponding crisp filter.

4. Conclusions

The analysis performed has evidenced that the use of 'fuzzy' design can be superior to the deterministic one, even in a typical 'deterministic' problem, such as the design of a digital filter. This result, that seems hard to believe, has a number of explanations. Remember that the design is based on optimization. Using fuzzy quantities instead of crisp ones gives rise to a 'fuzzy ravine' that allows a better directioning of the search gradient,as sketched in Figure 2.

One can argue that the 'fuzzification method' should be considered in any design or research problem, to yield faster, or better results.

This design procedure of crisp filters with tolerances, based on the fuzzy filter generalization, in many cases allows better results, i.e. better optimized filters with zero tolerances.

Of course, this algorithm also allows the

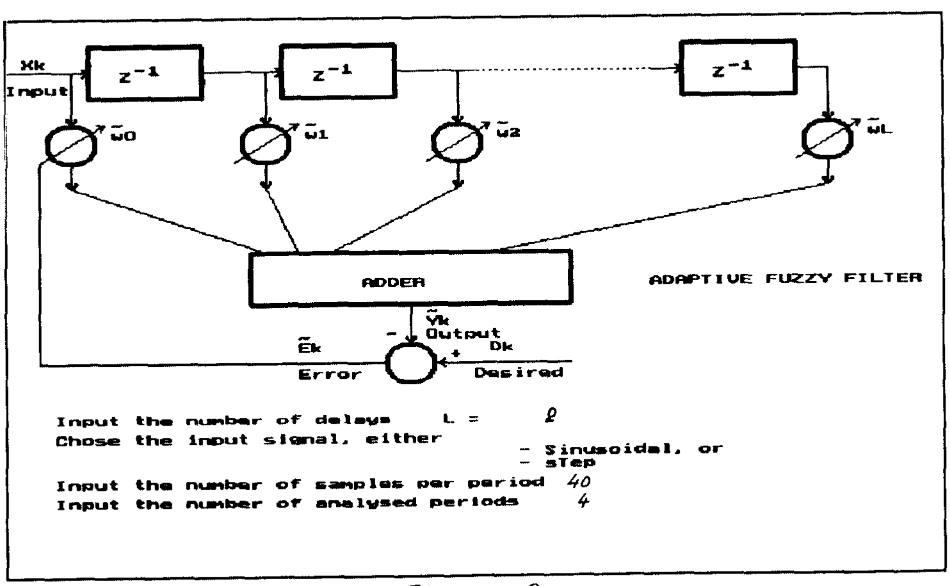

Figure 2

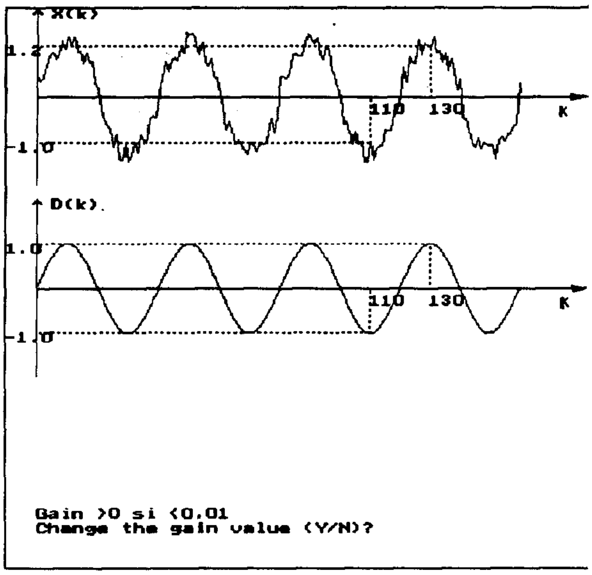

Figure 3

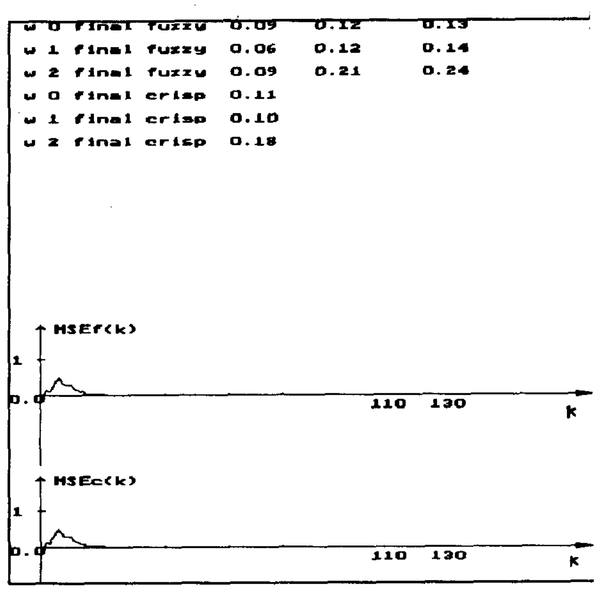

Figure 4

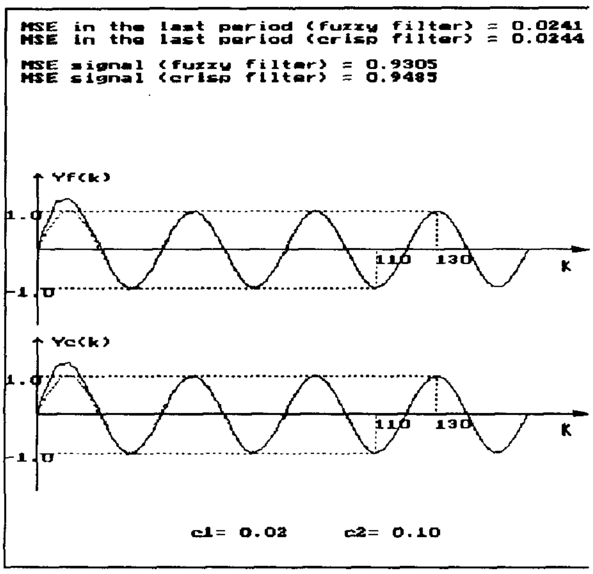

Figure 5

computation of the algebraic fuzzy system with given response to a given input signal (the modelled system order being given).

The same algorithm may be used to design non-linear systems (next case study).

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