1993년도 한국음향학회 학술논문발표회 논문집(제 12권 1(s)호)

Decision-Theoretic Approach to Source Direction Finding in Array Sensor Systems

Wan-Sup Cheung(O), Moon-Jae Jho, and Hee-Joon Eun

Acoustics and Vibration Lab.
Korea Research Institute of Standards and Science (KRISS)
P.O. BOX 3, TAEDOK SCIENCE TOWN, TAEJON 305-606

Abstract

A decision-theoretic concept is introduced to investigate whether targets of interest in array sensor systems are present at some steering direction or not. The solutions to this problem are described as a set of simple numbers 0 or 1 corresponding to the direction under consideration. This coded number representation is transplanted in the optimisation technique based on the Hopfield neural network, which may provide a new aspect of determining the direction of arrival (DOA) of sources. To cast the perspectives of the proposed approach and illustrate its effectiveness in source direction finding in array sensor systems, simulation results and related discussions are presented in this paper.

I. Introduction

In the last decade, high resolution array signal processing methods have appeared that are commonly based on the eigenstructure of the correlation matrix. These methods [1-6] consist of estimating the correlation matrix from the measurements of equi-spaced array sensors and decomposing the matrix into the signal subspace and the noise subspace. The orthogonality between the subspaces is exploited to achieve the high resolution spectral distribution over the steering angle. Those techniques have provided basics and fundamentals for approaching acoustic radar or sonar systems problems - the early monitoring of stationary or moving sources (aircraft or sea vessels). Generally, the concepts and fundamentals are based on linear algebra, i.e. singular value decomposition [7], which enables us to estimate the 'best-fitted' direction-of-arrival of radiating or reflecting sources in the least squares sense.

However, we have observed another aspect arising from the array sensor systems, that is the early warning system. It involves a classical problem of deciding whether sources are present at the steering direction or not (in details discussed in Chapter 2 of reference [8]). When a source at the steering angle is present, the solution corresponding to the position is one, and otherwise it becomes zero. Obviously, the solutions to the problem are the set of simple numbers, 0's or 1's. It is indeed a decision-theoretic problem that accompanies a 'nonlinear' mapping of processed information about the DOA onto the decision space whose state is generally described as a set of binary numbers. The above eigenstructure methods may provide the basis for obtaining related information, but do not lead to any logical approach to the nonlinear decision mapping problem. This fact implies the possibility of approaching the direction finding problem from different viewpoints.

This paper exploits the optimisation technique using the Hopfield neural network models [9,10], which have proven to be very successful in combinatorial (NP-completeness) optimisation problems: the traveling salesman problem of finding the shortest route connecting multiple cities [11] and the Hitchcock problem of distributing a product from several sources to numerous locations in such a way to miminising the transportation cost [12]. One of the important properties of the Hopfield model-based optimisation method is the ability to simultaneously consider a large number of alternative hypotheses and at remarkable speed make adequate decision on them for given data. This feature has provided the major motivation of investigating the effectiveness of the Hopfield model-based optimisation in source direction finding. In Section II, basic ideas behind the Hopfield models are described and linked to the above decision problem in the array sensor systems. In Section III, we map this decision problem onto the Lyapunov candidate function of the Hopfield model and make modifications for improving the possibility of convergence to the better solutions. Simulation results and discussions are presented in Section IV. Finally, concluding remarks are summarised in Section V.

II. Fundamentals in Hopfield Neural Network Models

1. Hopfield Models

The Hopfield models [9,10] consist of a number of mutually interconnected computation units, called the neurons, whose states are defined by their outputs $\{v_i\}$. Each neuron state can be described as a discrete value, i.e. 0 or 1. Fig. 1 shows the schematic setup of Hopfield neural network models.

Each neuron i receives multiple inputs, denoted by the vector $v = [v_1, v_2, \dots, v_N]^T$, projects them onto its interconnection weight vector $\mathbf{w}_i = [w_{i1}, w_{i2}, \dots, w_{iN}]^T$, and then adds an externally supplied bias input b_i to the weighted value. This result represents the internal potential u_i of neuron i

$$u_i = \sum_{j=1}^{N} w_{ij} \cdot v_j + b_i \qquad (1)$$

where N is the number of neurons.

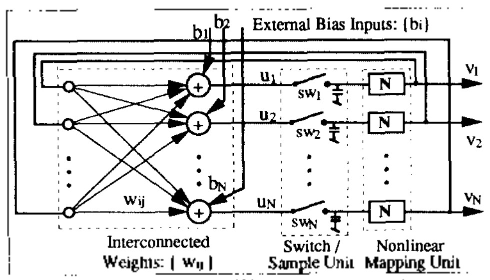

Fig. 1 Schematic setup of Hopfield neural networks

When switches sw_i in Fig. 1 turn on at some discrete time, the sampled internal potentials u_i are sent to the nonlinear activation units to change or leave neuron outputs according to a threshold rule performed by the nonlinear units N_i, that is

$$v_i(n) = N(\, u_i(n)\,) = \mathrm{stp}(\, u_i(n)\,) \qquad (2)$$

where n is discrete time index and stp(u) denotes a unit step function which is 1 for $u \geq 0$ and 0 for $u < 0$. Thus neurons take binary values 0 or 1. These binary outputs are feed back to the input junction of interconnected weights so that neurons gradually evolve to one of stable states in N-dimensional discrete space.

Hopfield [9,10] showed that if neuron weights w_{ij} are symmetric ($w_{ij} = w_{ji}$) then neurons in the network model evolve to one of stable states in such a way of minimising a Lyapunov candidate function, called the energy function,

$$\mathcal{E} = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\cdot v_i \cdot v_j \; - \; \sum_{j=1}^{N} b_i \cdot v_i . \qquad (3)$$

In fact, the evolution of neurons given in (1) is seen to be identical to the negative gradient of the energy function (3) with respect to the neuron states $\{v_i\}$. Here, one point is clear that only when we define a cost function of interest in our problems that is equivalent to the energy function (3) we can find a solution to the problems by updating the neuron states according to (1) and (2). This aspect is well illustrated in previous work such as the traveling salesman problem [11] and the Hitchcock problem [12]. A major issue in defining the energy function for direction finding will be examined in Section III, and the rest of this section will address the neuron state transition scheme.

2. Asynchronous State Transition Mode

In Fig. 1, the transition of neuron states are shown to depend on the ways of operating the switch/sample unit. As noted by Takeda and Goodman [12], there are several possible ways of doing it. When we turn on all the switches synchronously at some discrete time n, we can simultaneously update all the neurons. This state transition scheme, referred to as the synchronous state transition, seems to be a 'normal' one. By contraries in the stability proof in [9,10], we have experienced 'unexpected' results of this scheme, i.e. 'oscillatory or wandering' behaviour of the neuron states around the minima of the energy function, and moreover have failed to implement the state transition in a stable manner. A clear understanding of the reason for these unfavorite features still remains as an open question in the neural network community. However, it should be noted that the unfavorite features can arise from the ways of updating all the neuron states, i.e. the way of operating the switches in Fig. 1. Takeda and Goodman noted that the reason can come from the self-feedback ($w_{ii} \neq 0$) or non-zero off-diagonal weight terms ($w_{ij} \neq 0$). Unfortunately, both terms are wholly dependent on each application under consideration.

We choose the asynchronous neuron state transition scheme [12] so as to reduce the above unfavorite features as small as possible. The switches in this asynchronous mode is turn on and off with random delay between each switch such that neurons change their states

$$u_i(n + \Delta t_i) = \sum_{j=1}^{N} w_{ij}\cdot v_j(n + \Delta t_i) + b_i$$

$$\text{and} \;\; v_i(n + \Delta t_i + \varepsilon) = N(u_i(n + \Delta t_i)) \qquad (4)$$

where Δt_i is random time delays and ε is a small positive constant. Given N positive random variables, one may decide the order of neuron state transition; one neuron corresponding to the smaller time delay updates its state earlier and the other corresponding to the larger delay does latter. This transition scheme means that only one neuron is updated at some instance and that it can use information about new states of other neurons that have already updated their states. Takeda and Goodman suggested an ascending order, $0 \leq \Delta t_1 \leq \Delta t_2 \leq \cdots \leq \Delta t_N$, which was very successful in solving the Hitchcock problem [12]. At the beginning of this study, we had examined this scheme for state transition and observed that the ascending order is not effective in solving the direction finding problem as what will follow. This point has allowed us to further find an important aspect occurring during neuron state transition.

Let us consider the specific state transition of a neuron from 0 to 1 under the condition that the rest of neurons are zero. The energy of zero-state neurons is readily seen to be zero as defined in (3) and that of one-state neuron is also obtained by calculating (3). The difference energy level between them gives a clue for judging what amount energy level is increases or decreased due to the specific state transition. By following this specific procedure for each neuron, we can obtain the energy difference levels for all the neurons and then sort them in an ascending order, i.e. the first for the lowest energy level and the last for the highest one. The state transition of one neuron corresponding to the lower energy level is obviously seen to minimise as large as possible. Thus, this paper will update each neuron state according to the ascending order of the energy level difference. This scheme at least provides the chance of decreasing the energy (3) more rapidly and safely by updating the neurons with the lower energy level earlier than those of the higher. We will further examine the effectiveness of the proposed state transition scheme for Hopfield neural network-based optimisation in Section IV.

III. Hopfield Model-Based Direction Finding

In the applications of Hopfield neural networks, another important question is how to draw logical clues from our practical problems and then link them to neuron states. First, let us consider K equi-spaced array sensors to monitor M plane wave sources located at angles $\{\theta_m: m = 1, \cdots, M\}$. Using the quadrature demodulation-modulation unit as in Fig. 2, monitored signals can be obtained in a analytic form of sampled narrowband signals for $n = 1, \ldots, N_t$

$$\mathbf{y}(n) = [y_1(n), y_2(n), \ldots, y_K(n)]^T = \mathbf{S}\cdot\mathbf{c}(n) + \mathbf{n}_p(n) \quad (5)$$

In (5), the matrix $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M]$ consists of the steering vectors $\mathbf{s}_m = [1, e^{-j\tau_m}, e^{-j2\tau_m}, \cdots, e^{-j(K-1)\tau_m}]^T$ ($\tau_m = \kappa\cdot d\,\sin(\theta_m)$ denotes aphase difference, κ and d are the wave number and the gap between equi-spaced array sensors), $\mathbf{c}(n) = [c_1(n), c_2(n), \cdots, c_M(n)]^T$ is the complex amplitude vector of M sources, and $\mathbf{n}_p(n) = [n_{p,1}(n), n_{p,2}(n), \cdots, n_{p,K}(n)]^T$ denotes the preprocessed complex noise vector.

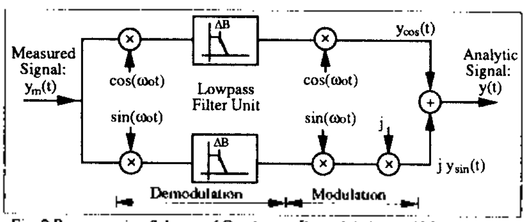

Fig. 2 Preprocessing Scheme of Quadrature Demodulation and Modulation. $\omega_0 = 2\pi(f_0 - \Delta B/2)$, ΔB: the bandwidth of lowpass filter, f_0: the considered frequency, j: complex number, $y_{cos}(t)$ and $y_{sin}(t)$: the sine and cosine quadrature components, and the analytic signal $y(t) = y_{cos}(t) + j\,y_{sin}(t)$.

A major problem of interest in this paper is to decide whether a source at some direction θ is present or not. We may divide the range of a limited steering angle to some desired resolution. For example, given the range between the starting and final directions θ_s and θ_f ($\theta_f > \theta_s$), our interest is concerned with $N_\theta+1$ directions $\{\theta_i = \theta_s + i\cdot\Delta\theta: i = 0, 1, \cdots, N_\theta\}$ with direction resolution $\Delta\theta = (\theta_f - \theta_s) / N_\theta$ (N_θ = the direction index). If a source at the direction θ_i is present, then the decision result may be described as a value of 1 and, if not, it may be a value of 0. One bit may be sufficient to describe the decision state at direction θ_i. Thus each neuron state v_i is related to the decision state of direction θ_i. For $N_\theta+1$ directions, $(N_\theta+1)$ neurons participate to solve the direction finding problem.

To approach the direction finding problem, we should define an adequate cost function that is of the quadratic form similar to (3). As introduced by Rastogi et al. [13], the orthogonal projection matrix $\mathbf{P}_i = \mathbf{u}_i\cdot\mathbf{u}_i^H$ (the super script H denotes the Hermitian operator), which is constructed by the unit steering vector $\mathbf{u}_i^T = [1, e^{-j\tau_i}, e^{-j2\tau_i}, \cdots, e^{-j(K-1)\tau_i}]/\sqrt{K}$ ($\tau_i = \kappa\cdot d\,\sin(\theta_i)$) of direction θ_i, can be exploited to examine the presence of a source at the direction. By projecting time series $\mathbf{y}(n)$ onto the orthogonal matrices $\{\mathbf{P}_i\}$, we obtain $(N_\theta+1)$ direction components

$$\mathbf{d}_i(n) = [\mathbf{u}_i\cdot\mathbf{u}_i^H]\cdot\mathbf{y}(n) = a_i(n)\,\mathbf{u}_i, \quad (6)$$

which include useful information to judge existence of source at direction θ_i so that they may be related to the decision result described as the neuron states $\{v_i\}$. It is readily seen that if v_i is close to 1 then much 'weighting' value is assigned to the unit vector $\mathbf{u}_i(n)$ while in case of $v_i = 0$ no significance is given to it. Let the projected direction matrix $\mathbf{D}(n) = [\mathbf{d}_1(n), \mathbf{d}_2(n), \cdots, \mathbf{d}_{N_\theta+1}(n)]$ $(= [\mathbf{P}_1\cdot\mathbf{y}(n), \mathbf{P}_2\cdot\mathbf{y}(n), \cdots, \mathbf{P}_{N_\theta+1}\cdot\mathbf{y}(n)])$ at time index n and the decision state $\mathbf{v} = [v_1, v_2, \ldots, v_{N_\theta+1}]^T$. Then we can construct a K-dimensional signal $\mathbf{y}_s(n) = \mathbf{D}(n)\cdot\mathbf{v}$ and compare it to the sampled signal $\mathbf{y}(n)$. Here we can define the cost function as the mean squared errors between the sampled and constructed signals for the N_t samples

$$\begin{aligned}\mathcal{E} &= \frac{1}{N_t}\sum_{n=1}^{N_t}\|\mathbf{y}(n)-\mathbf{y}_s(n)\|^2 - \frac{1}{N_t}\sum_{n=1}^{N_t}\|\mathbf{y}(n)\|^2 \\ &= \sum_{i=1}^{N_\theta+1}\sum_{j=1}^{N_\theta+1}\mathrm{Re}\{\frac{1}{N_t}\sum_{n=1}^{N_t}a_i(n)^*\cdot a_j(n)\cdot\delta_{ij}\}\cdot v_i v_j \\ &\quad - \sum_{i=1}^{N_\theta+1}\frac{2}{N_t}\sum_{n=1}^{N_t}\|a_i(n)\|^2\cdot v_i \end{aligned} \quad (7)$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of complex number, the symbol * is the complex conjugate, and a scalar value $\delta_{ij} = \mathbf{u}_i^H\cdot\mathbf{u}_j$ is the direction cosine between two unit vectors. As explained in [13], the cost function is of the quadratic form with respect to the decision states $\{v_i\}$ and is also similar to the Hopfield energy function (3). This implies the possibility of solving the direction finding problem using the Hopfield neural network-based optimisation technique.

To remove the instability arising from the non-zero diagonal terms $w_{ii} \neq 0$ of the coefficients of $v_i v_i$ in (7), we may add to the cost function (7) another cost term

$$\sum_{i=1}^{N_\theta+1}\{\frac{1}{N_t}\sum_{n=1}^{N_t}\|a_i(n)\|^2\}\cdot v_i(1-v_i) \quad (8)$$

whose minimisation constraints the decision results v_i to lie in 0 or 1. Therefore, it is straightforward to obtain the notations of the interconnection weights and the bias terms as in (3) by computing the negative gradient of the added cost function of (7) and (8) with respect to the decision states $\{v_i\}$

$$w_{ij} = -\frac{2}{N_t}\mathrm{Re}\{\frac{1}{N_t}\sum_{n=1}^{N_t}a_i(n)^*\cdot a_j(n)\cdot\delta_{ij}\} \quad \text{for } i \neq j,\ w_{ii} = 0, \quad (9)$$

$$b_i = \frac{1}{N_t}\sum_{n=1}^{N_t}\|a_i(n)\|^2\}. \quad (10)$$

The interconnection weights w_{ij} are shown to be computed from the mean of real parts of inner product of two direction vectors $a_i(n)\mathbf{u}_i$ and $a_j(n)\mathbf{u}_j$, and the bias terms to be the mean squared value of direction vector $a_i(n)\mathbf{u}_i$. The weights and the bias terms in (9) and (10) appear similar to an time averaged version of the corresponding terms in [13-15]. They are substituted into those of the state transition scheme (4) in Section II.2 Therefore, we are ready to investigate the effectiveness of the Hopfield network-based optimisation technique for direction finding in the array sensor systems.

IV. Simulation Results and Discussions

Computer simulations were carried out to examine the decision results by applying the Hopfield model-based optimisation to the direction finding problem in the array sensor system that consists of 16 equi-spaced sensors (K=16).

Other simulation parameters were as follows: the space d was chosen to satisfy $\kappa \cdot d = \pi$ (κ = wave number $2\pi f_o/c$, f_o = considered frequency and c = wave speed), a sampling time $\Delta T = f_o/128$ (128 words per period), and the normalised bandwidth of low-pass filter is $\Delta B = 0.1$ (pass-bandwidth × $2 \cdot \Delta T$). We considered four sources (M = 4) located at {6.0°, 15.0°, 31.0°, 44.0°} whose relative amplitudes were $\{|C_{s,i}|\}$ = {1.0, 0.71, 1.1, 1.0}. Given the signal-to-noise ratio (SNR), the Gaussian random variables with zero mean and variance that is where ratio:

$$\sigma^2 = \{ \sum_{i=1}^{4} |C_{s,i}|^2 / 2 \cdot 10^{SNR/10} \} \times \Delta B^{-2} \tag{11}$$

were added to the original source signals and then the preprocessing unit shown in Fig. 2 was employed to generate 'noisy' analytic signals as given in (5). The initial neuron potentials $\{u_{i,0}\}$ of neurons were set to 0.0 and the outputs $\{v_{i,0}\}$ were initialised by the small random variables uniformly distributed in $10^{-3}\times$[0,1]. The asynchronous state transition scheme proposed in Section II.2 were used to update the neuron outputs.

In this simulation, the range of direction-of-arrival (DOA) from 0.0° to 50.0° degrees was discretised at intervals of 0.5° degrees such that a Hopfield network model of 101 neurons was considered to examine the decision theoretic problem of source direction finding. For SNR = -20 [dB] (equivalent SNR = 0 [dB] after quadrature demodulation-modulation-based preprocessing), we first computed the weights and bias terms in (10) and (11), evaluated the difference energy levels of 101 neurons by changing each neuron state from 0 to 1, and then sort them in the ascending order. Fig. 3 shows a typical example for the evaluated energy difference levels of 101 neurons. According to the ascending order of the difference energy levels, neuron states were updated one by one.

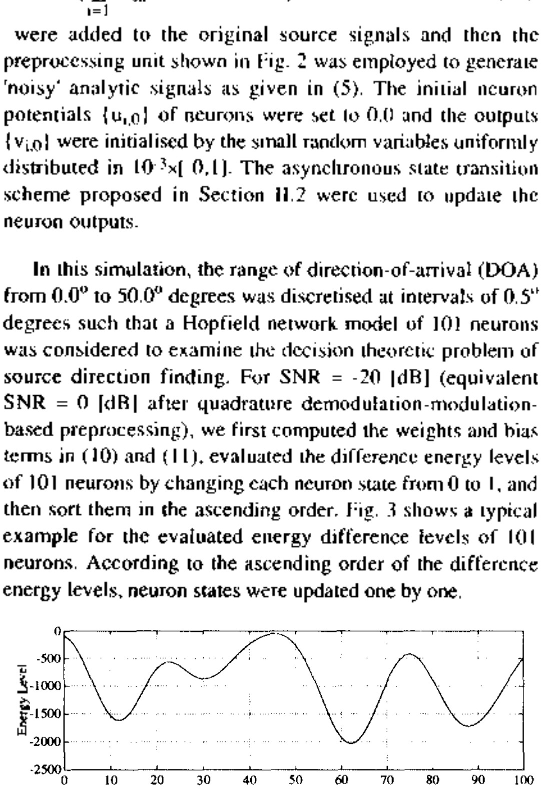

Fig. 3. Energy difference level for state transition of 101 neurons from 0 to 1.

Fig. 4 (a) illustrates the decision state (solid line), that is the neuron outputs, after updating 101 neurons. Note that the thick dashed line are the mean squared value of projected direction vectors $\{|b_i|\}$ in (10) and the thin vertical lines denote the position of four sources. Fig. 4 (b) shows the trend of the network energy defined in (7) during 101 neuron state updates. Specifically, the energy level is shown to remain constant after first 47 neurons were updated. This means that the decision state are settled down to the final steady solution. We further examined seven independent samples to see their final decision results. Table 1 shows the final results, their mean and variance.

To compare the relative performance, we chose the state transition scheme in previous work [12], referred to as

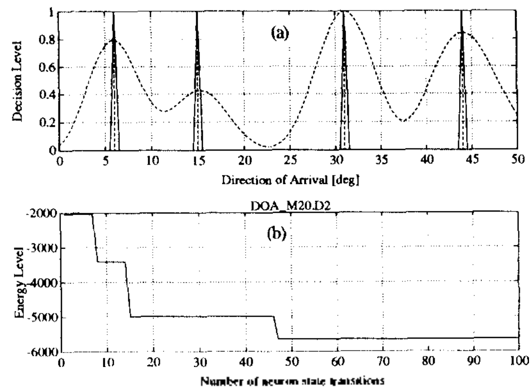

Fig. 4. Decision result and trend of energy level after 101 neuron state updates.

TABLE 1. Decision results for seven independent samples.

Samples	Source Locations [deg]:				Energy E
	θ_1	θ_2	θ_3	θ_4	
#1	6.0	15.5	31.0	44.0	- 5662.68
#2	6.0	15.0	31.0	44.0	- 5651.28
#3	6.0	15.0	31.0	44.0	- 5665.23
#4	6.0	15.5	31.0	44.0	- 5673.64
#5	6.0	15.5	31.0	44.0	- 5629.15
#6	6.0	15.5	31.0	44.0	- 5659.52
#7	6.0	15.0	31.0	44.0	- 5626.94
Mean	6.0	15.3	31.0	44.0	- 5652.64
Variance	0.00	0.14	0.00	0.00	1962.15

the 'sequential' state transition scheme, which is based on the order of time delays as $0 \le \Delta t_1 \le \Delta t_2 \le \cdots \le \Delta t_N$. Fig. 5 shows the final state and the trend of the network energy respectively, which is the best results among seven independent runs. This state update scheme is found to be ineffective in solving the direction finding problem in this paper.

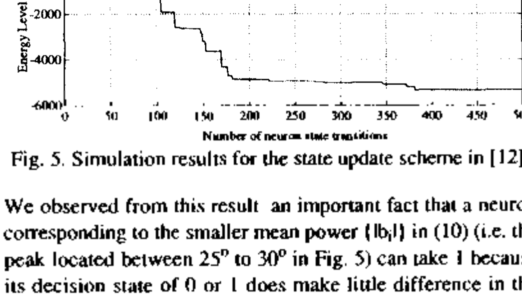

Fig. 5. Simulation results for the state update scheme in [12].

We observed from this result an important fact that a neuron corresponding to the smaller mean power $\{|b_i|\}$ in (10) (i.e. the peak located between 25° to 30° in Fig. 5) can take 1 because its decision state of 0 or 1 does make little difference in the energy level. Intuitively, this decision state may ignore due to no significance in the energy minimisation, i.e. the decision problem. This fact has allowed us to develop the proposed update scheme in Section II.2, that is the earlier update for the higher significance in the network energy, and, futhermore, to achieve the better performance of the Hopfield neural network-based decision-theoretic approach to the direction finding problem.

V. Concluding Remarks

We introduced another aspect for source direction finding in array sensor systems and fundamentals to approach that problem in a sense of classical decision theory. The mapping of decision states over the considered DOA range onto the outputs of the discrete Hopfield neural network are found to play a central role in this paper. A new state transition scheme is proposed and the simulation results at least may indicate that the proposed scheme is more successful in source direction finding than the previous one. Further decision-theoretic study on two issues - the closely located sources direction finding problem and the wideband sources direction finding problem - is in progress.

References

[1] R. O. Schmidt, *A Signal Subspace Approach to Multiple Source Location and Spectral Estimation*, Ph.D. Thesis, Stanford University, Stanford:CA, 1981.

[2] D. H. Johnson and S. R. DeGraff, "Improving the Resolution of Bearing in Passive Sonar Arrays By Eigenvalue Analysis," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-30, No. 4, pp. 638-647, 1982.

[3] R. Kumaresan and D. W. Tuffs, "Estimating the Angle of Arrival of Multiple Plane Waves," *IEEE Trans. Aerosp. Elect. Syst.*, Vol. AES-19, pp. 143-139, 1983.

[4] R. Roy and T. Kailath, "ESPRIT - Estimation of Signal Parameters via Rotational Invariant Techniques," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-37, No. 7, pp. 984-995, 1989.

[5] H. J. Eun et al., "Development of Acoustic Antenna Technology for Noise Source Detection," *KRISS Reports: KSRI-89-92-IR (198), KSRI-90-131-IR (1990) and KRISS-91-178-IR (1991)*, Korea Research Institute of Standards and Science, TAEJON, KOREA.

[6] W. Y. Hong, B. C. Kim, J. S. Lee, and W. C. Oh, "Alternative View of the Eigen Decomposition Based Method for Direction Finding," *Proceedings of the 1993 Korean Signal Processing Conference*, KAIST, TAEJON, Vol. 6, No. 1, pp 156-159, 1993.

[7] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing*, Cambridge University Press, Cambridge, 1986.

[8] H. L. Van Trees, *Detection, Estimation, and Modulation Theory: Part I*, John Wiley and Sons, New York:NY, 1968.

[9] J. J. Hopfield, "Neural Networks and Physical Systems with Emergent Collective Computational Ability," *Proceedings of the National Academy of Science*, Vol. 79, pp. 2554-2558, 1982.

[10] J. J. Hopfield, "Neurons with Graded Response Have Collective Computational Properties like Those Two-State Neurons," *Proceedings of the National Academy of Science*, Vol. 79, pp. 3088-3092, 1984.

[11] J. J. Hopfield and D. W. Tank, "Neural Computation of Decision in Optimization Problems," *Biological Cybernetics*, Vol. 52, pp. 141-152, 1985.

[12] M. Takeda and J. W. Goodman, "Neural Networks for Computation: Number Representations and Programming Complexity," Applied Physics, Vol. 25, No. 18, pp. 3033-3046, 1986.

[13] R. Rastogi, P. K. Gupta and R. Kumaresan, "Array Signal Processing with Inter-Connected Neuron-Like Elements," *Proceedings of IEEE ICASSP '87*, pp. 54.8.1-54.8.4, 1987.

[14] D. Goryn and M. Kaveh, "Neural Networks for Narrowband and Wideband Direction Finding," *Proceedings of IEEE ICASSP '88*, pp. 2164-2167, 1988.

[15] S. K. Jha and T. S. Durrani, "Bearing Estimation Using Neural Optimisation Methods," *Proceedings of IEEE ICASSP '89*, pp. 889-892, 1989.