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ANALYSIS OF THE LATERAL MOTION OF A TRACTOR-TRAILER COMBINATION (II)

Operator/Vehicle System with Time Delay for Backward Maneuver

S. W. Mugucia, R. Torisu and J. Takeda
Department of Agricultural Engineering
Iwate University
Ueda 3-8-18 , Morioka 020, Japan

ABSTRACT

In order to analyze lateral control in the backward maneuver of a tractor-trailer combination, a kinematic vehicle model and a human operator model with time delay were utilized for the operator/vehicle system. The analysis was carried out using the frequency domain approach. The open-loop stability of the vehicle motion was analyzed through the transfer functions. The sensitivity of the stability of the vehicle motion, to a change in the steering angle, was also analyzed. A mathematical model of the closed-loop operator/vehicle system was then formulated. The closed-loop stability of the operator/vehicle system was then analyzed. The effect of the delay time on the system was also analyzed through computer simulation.

Key Word : Backward maneuver, Frequency domain analysis, Human operator describing function, Lane change maneuver, Time delay, Tractor-trailer combination

INTRODUCTION

Articulated vehicles, such as tractor-trailer combinations and trailed implements, are widely used in agriculture. To effectively utilize these type of vehicles, it is necessary to match the vehicle characteristics with the controller characteristics of the human operator. The study of the interaction between an operator and a vehicle requires an adequate model of the system that predicts the vehicle response to steering control. A kinematic vehicle model for the tractor-trailer combination proposed by Torisu et al. (1992, a) was adopted in this study. The open-loop control response of the vehicle, to a fixed steering angle input, was evaluated by computer simulation. As Kageyama (1986) and Torisu et al. (1993) showed, the backward motion of an articulated vehicle is also difficult to control. A simple human operator model, for a tractor-trailer combination, was proposed by Torisu et al. (1992, b). The model however, did not consider the time delay inherent in a human operator. In order to take care of this anomaly, a modified model was proposed in this study, which includes the time delay. The closed-loop operator/vehicle system stability for the vehicle was then analyzed using the model. The authors hope to utilize this model to carry out further work in this field.

Nomenclature

A	: center of the front axle of a tractor	$A(x_A, y_A)$
B	: center of rear axle of tractor	$B(x_B, y_B)$
C	: hitch point	$C(x_C, y_C)$
D	: center of trailer axle	$D(x_D, y_D)$
α	: steer angle of tractor front wheels	
θ	: tractor heading angle	
θ_G	: the desired tractor heading angle	
β	: semi-trailer heading angle relative to the tractor	
β_G	: the desired trailer heading angle	
$(\theta+\beta)$	: absolute trailer heading angle	
V	: forward velocity of tractor-trailer combination	
ℓ_1	: tractor wheelbase	
ℓ_2	: distance from the hitch point to trailer axle	
h	: distance from the tractor rear axle to the hitch point	
H	: lateral displacement of desired path	
y_A	: lateral displacement of point A	
y_B	: lateral displacement of point B	
y_D	: lateral displacement of point D	
K_1	: tractor heading feedback gain	
K_2	: lateral displacement feedback gain	
K_3	: trailer heading feedback gain	
HODF	: human operator describing function $\alpha=f(\theta, y, \beta)$	

II. THEORY

2.1 Vehicle Model and its Open-loop Stability

A kinematic vehicle model was adopted in this study. A schematic representation of the vehicle under consideration is shown in Fig. 1. The detailed derivation of the model, together with a discussion of the pertinent assumptions and restrictions, was presented by Torisu et al. (1992, a). For the sake of completeness, The assumptions made in writing the equations of motion are described again briefly. The vehicle is assumed to be moving at a constant forward speed over a smooth, hard and horizontal surface. Bounce, pitch and roll motions may thus be ignored. The bodies of the tractor and trailer are assumed to be rigid. The hitch point is assumed to be frictionless, and the trailer has freedom to rotate in yaw relative to the tractor. The same simplifications of linearity, small angles and small oscillations are used as in the previous work. The resulting linear vehicle equations of motion are shown in Table 1. By solving this system of differential equations, the open–loop characteristics of the vehicle can be determined, if the input steering angle α is specified. The analysis here deals only with the fixed control response, that is, stability and control characteristics associated with a locked steering wheel. Transfer functions for the trailer angle β and lateral displacement y_D, are

$$\beta(s) = \frac{V(\ell_2 + h)}{\ell_1(\ell_2 s - V)}\,\alpha(s) \tag{1}$$

$$y_D(s) = \frac{V^2(hs + V)}{\ell_1 s^2(\ell_2 s - V)}\,\alpha(s) \tag{2}$$

To check for open–loop stability, then

$$\ell_2 s - V = 0 \tag{3}$$

or

$$s = \frac{V}{\ell_2} \tag{4}$$

Since ℓ_1 and V are positive, s is also positive. The transfer functions therefore contain positive poles. This indicates that the motion of the tractor–trailer combination is unstable. In order to further investigate this instability and its sensitivity to a change in the steering angle, a program in BASIC was designed to simulate the backward motion of a tractor–trailer combination. The analysis here was carried out in the time domain. The parameters of the simulated Vehicle are shown in Table 2. The motion of the vehicle for various values of fixed steering angle α is shown in Fig. 3. The initial position of the vehicle was kept constant throughout all the simulation runs. The initial trailer heading angle β_0 was also held constant at 20 degrees. The trajectories of point D were plotted until the trailer heading angle, relative to the tractor, reached sixty degrees. Jackknifing was assumed to occur at this value and the plotting was terminated. At a steering angle of 10.258 degrees, a perfect and stable steady state circular motion can theoretically be attained. It is evident from these results that the stability of the motion is very sensitive to any change in this critical value of the steering angle. Even a small change, such as a tenth of a degree, has a very big effect on the stability. It is practically impossible to distinguish such small changes. It is therefore practically impossible to maintain an open-loop, steady state, circular, backward motion.

2.2 Operator Modeling

An operator model proposed by Torisu et al. (1992, b), and called "cognition model," was adapted for this study and expanded to include time delay. In the original model, the operator is assumed to have three control cues. In the backward maneuver, the operator usually controls the tractor-trailer combination while watching the trailer. The center of the trailer axle D is thus chosen as the guide point, the point on the vehicle that is guided over the reference or desired path. The lateral deviation (y_D(t)−H) of the center of the trailer axle is then the lateral position control cue. The deviations (θ(t)−θ_G) and (β(t)−β_G) in the tractor heading angle θ and the trailer heading angle β respectively, are taken to be the two attitude control cues. The HODF, is thus

$$\alpha(t)=K_1[R(t)-y_D(t)]+K_2[\theta(t)-\theta_G]+K_3[\beta(t)-\beta_G] \tag{5}$$

Where R is the reference signal. When using frequency domain analysis, it is simpler to use a HODF with one input and one output. The original HODF was therefore first modified to the following form

$$\alpha(t)=K_1[R(t)-y_D(t)]+K_2\theta_1(t) \tag{6}$$

where

$$\theta_1=\theta+\beta=\frac{\dot{y}_D}{V}$$

The modified human operator describing function with time delay is

$$\alpha(t)=K_1[R(t-\tau)-y_D(t-\tau)]+K_2\dot{y}_D(t-\tau) \tag{7}$$

2.3 Closed-loop Characteristics of Operator/vehicle System

Closed-loop characteristics, of the operator/vehicle system incorporate the vehicle equations of motion and the operator describing function. The block diagram for the closed-loop control of the vehicle is shown in Fig. 2. After Laplace transformation, the resulting open-loop operator transfer function is

$$P(s)=\frac{\alpha(s)}{\epsilon(s)}=(K_1+K_2s)e^{-\tau s} \tag{8}$$

The delay time can be approximated by Taylor's expansion as

$$e^{-\tau s}=(1-\tau s) \tag{9}$$

or by Pade's first order approximation as

$$e^{-\tau s}=\frac{(2-\tau s)}{(2+\tau s)} \tag{10}$$

The simpler Taylor approximation was used in this study. The open–loop vehicle transfer function is

$$G(s)=\frac{y_D(s)}{\alpha\,(s)}=\frac{V^2(hs+V)}{\ell_1 s^2(\ell_2 s-V)} \tag{11}$$

The closed–loop operator/vehicle transfer function is

$$\frac{y_D(s)}{R(s)}=\frac{G(s)P(s)}{1+G(s)P(s)} \tag{12}$$

where

$$R(s)=\frac{H}{s}$$

The characteristic equation is obtained by equating the polynomial 1 + G(s)P(s) to zero. The roots of the characteristic equation must satisfy

$$1+G(s)P(s)=1+\left(K_1+K_2\,s\right)\frac{V^2(hs+V)}{\ell_1 s^2(\ell_2 s-V)}(1-\tau s)=0 \tag{14}$$

III. EFFECT OF DELAY TIME ON STABILITY

It was shown earlier that the backward motion of a tractor–trailer combination is very unstable. To stabilize the motion. the gains of the HODF must be properly selected. This model can be used to interpret the skill of the operator. A skilful operator would be interpreted as one who is able to select the gains properly. A poor operator on the other hand would be interpreted as one who is unable to make a good selection of the gains. By using the Hurwitz stability criterion, the range of gain values for which the system is stable can be determined for a given delay time. By using different values of delay time, the effect of delay time on stability range is analyzed. A computer program in BASIC was designed to determine the stability of the operator/vehicle system. Stability without time delay is shown in Fig. 4. The effect of delay time on stability is shown in Fig. 5. The results indicate that the stability range decreases as the delay time increases.

CONCLUSIONS

1) The backward open–loop vehicle motion of a tractor–trailer combination was shown to be unstable. The stability was also shown to be very sensitive to a change in the steering angle.
2) A modified operator model, with time delay, for a tractor–trailer combination was introduced .
3) The closed–loop stability of the operator/vehicle system was analyzed using the proposed HODF with time delay. An increase in the delay time led to a decrease in stability.

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Table 1 Vehicle equations of motion

$\ell_1\dot{\theta}+V\alpha=0$
$\dot{y}_B-V\theta=0$
$\dot{y}_A-V(\alpha+\theta)=0$
$\ell_2\dot{\beta}-V\beta+(\ell_2+h)\dot{\theta}=0$
$\dot{y}_D-V(\theta+\beta)=0$

Table 2 Solution of the operator/vehicle system model for backward motion

Variable	characteristic equation
y_D	$\Delta_{\theta D\beta}(s)y_D=V^3K_2H$
y_B	$\Delta_{\theta D\beta}(s)y_B=V^3K_2H$
y_A	$\Delta_{\theta D\beta}(s)y_A=V^3K_2H$
α	$\Delta_{\theta D\beta}(s)\alpha=0$
θ	$\Delta_{\theta D\beta}(s)\theta=0$
β	$\Delta_{\theta D\beta}(s)\beta=0$

Kernel polynomial operator:

$$\Delta_{\theta D\beta}(s)=\ell_1\ell_2s^3-V(\ell_1-\ell_2K_1+\ell_2K_3+hK_3)s^2-V^2(K_1+hK_2)s-V^3K_2$$

Human operator describing function:

$$\alpha=K_1\theta+K_2(y_D-H)+K_3\beta$$

Note s:Laplace operator

Table 3 Vehicle parameters

Parameter		Value
tractor wheelbase	ℓ_1	1.32 (m)
hitch distance	h	0.42 (m)
trailer length	ℓ_2	1.60 (m)
travel velocity	V	0.22 (m/s)
lane change width	H	6.0 (m)

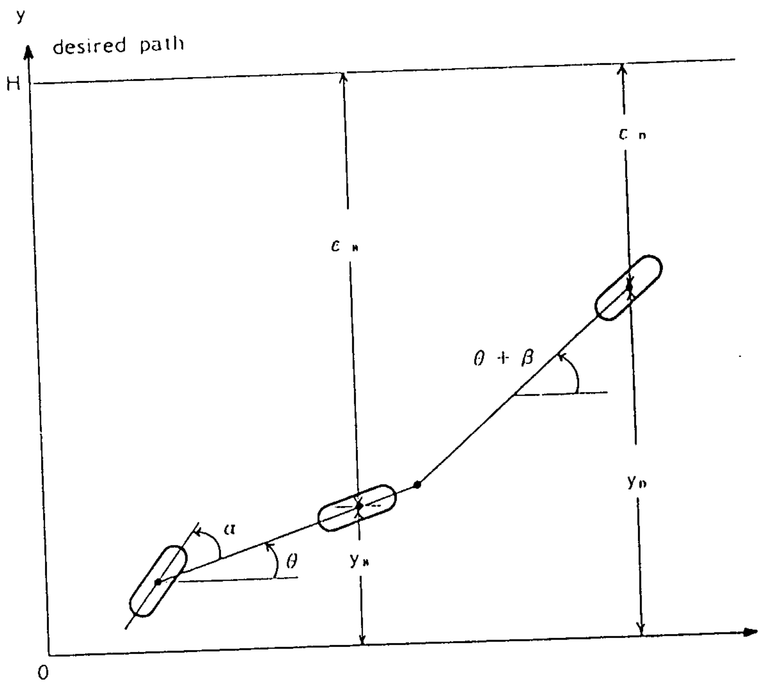

Fig. 1 Vehicle model in backward motion

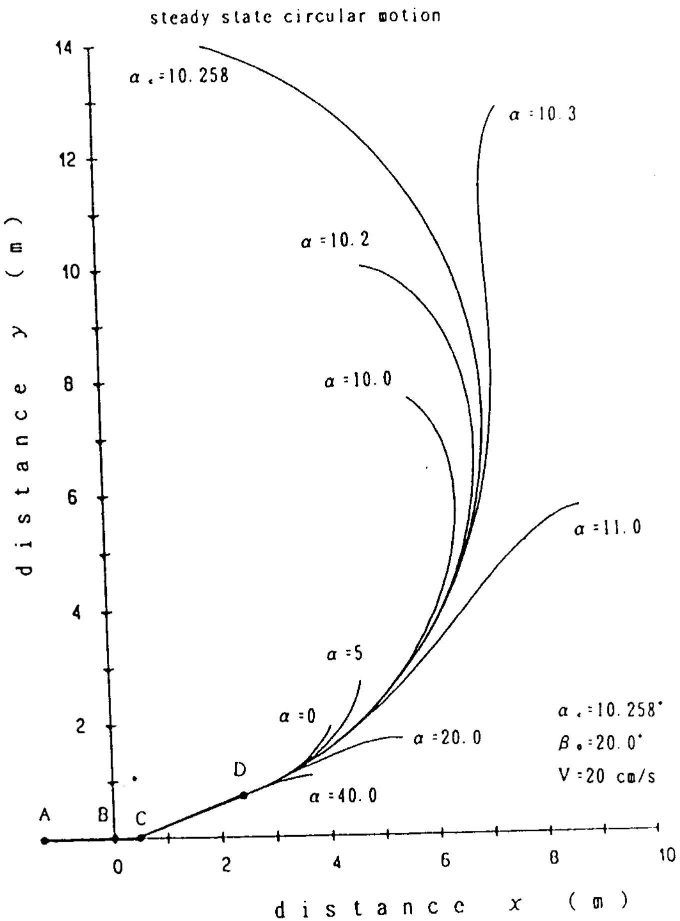

Fig. 2 Simulation of the response of the vehicle motion to various fixed steering angle inputs

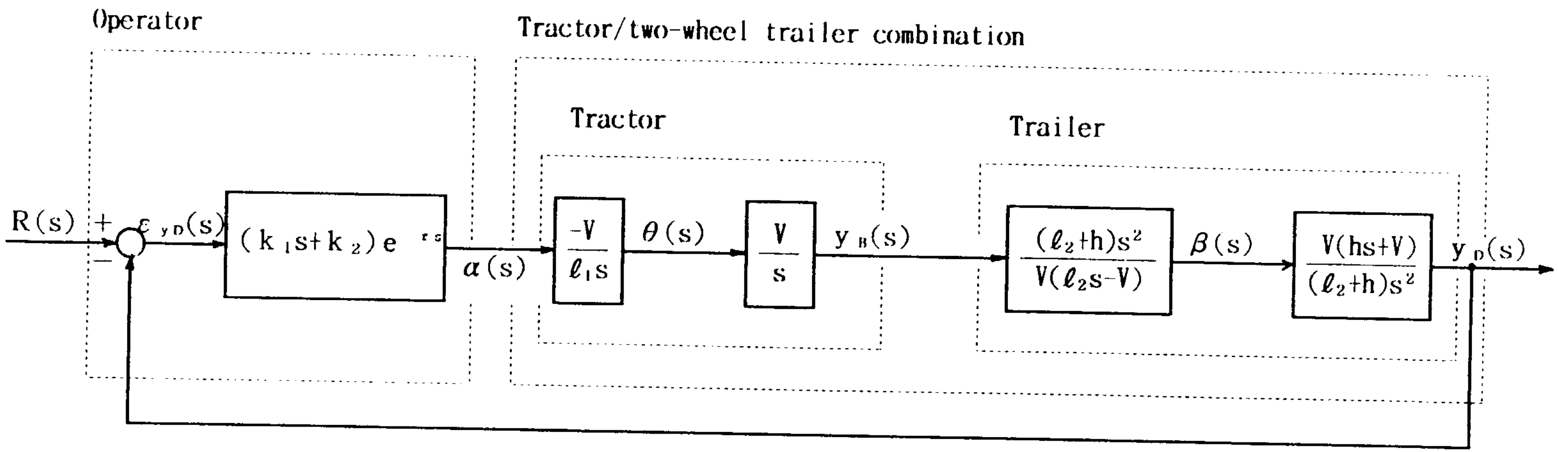

Fig. 3 Block diagram of the closed–loop control of the vehicle

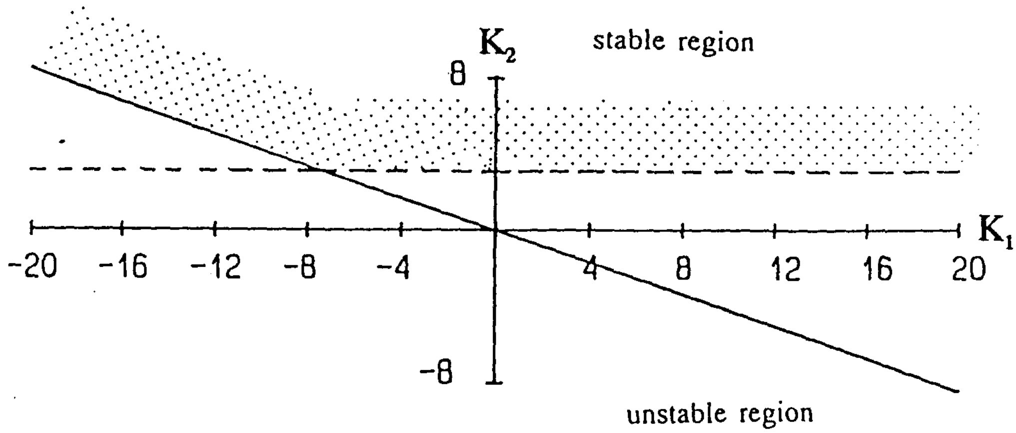

Fig. 4 Stability range for the gains of the operator/vehicle model without time delay

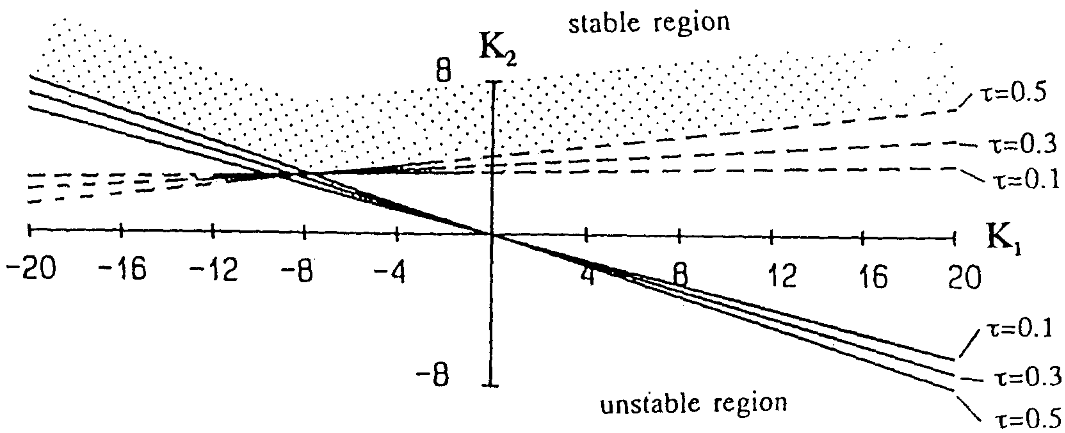

Fig. 5 Effect of delay time on the stability range for the gains of the operator/vehicle model