

**ANALYSIS OF THE LATERAL MOTION OF A  
TRACTOR-TRAILER COMBINATION (I)  
Operator/ Vehicle System Model for Forward Maneuver**

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**ABSTRACT**

In order to analyze lateral control in the forward maneuver of a tractor-trailer combination, a human operator model and a kinematic vehicle model were utilized for the operator/vehicle system. By combining the vehicle and operator models, a mathematical model of the closed-loop operator/vehicle system was formulated. A computer program was developed so as to simulate the motion of the tractor-trailer combination. In order to verify the operator/vehicle system model, the results of the field trials were compared with the simulated results. There was found to be reasonably good agreement between the two.

**Key Word :** Forward maneuver, Human operator describing function, Lane change maneuver, Tractor-trailer combination

**INTRODUCTION**

The control of trailed implements is one of the basic operations in the field of Agricultural Engineering. A tractor-trailer combination is representative of such a system. The study of the operator/vehicle control system models, as shown by Hoffman (1975/76) and Reid (1981), has been primarily concerned with high speed single unit vehicles such as automobiles. The vehicle models thus utilized have been dynamic models, as discussed by Vlk (1982). A kinematic vehicle model for slow speed vehicles, proposed by Torisu et al. (1992, a) was used in this study. A human operator describing function for a tractor-trailer combination was also proposed by the authors and utilized in the study. An operator/vehicle model composed of these two elements was then utilized to analyze the lane change maneuver of a tractor-trailer combination. The closed-loop characteristics of both forward and backward maneuvers can be analyzed using this model. The model is however, limited since it does not include time delay. Analysis was carried through actual field experiments, and mathematical simulation.

## Nomenclature

<b>A</b>	: center of the front axle of a tractor	$A(x_A, y_A)$
<b>B</b>	: center of rear axle of tractor	$B(x_B, y_B)$
<b>C</b>	: hitch point	$C(x_C, y_C)$
<b>D</b>	: center of trailer axle	$D(x_D, y_D)$
$\alpha$	: steer angle of tractor front wheels	
$\theta$	: tractor heading angle	
$\theta_G$	: the desired tractor heading angle	
$\beta$	: semi-trailer heading angle relative to the tractor	
$\beta_G$	: the desired trailer heading angle	
$(\theta+\beta)$	: absolute trailer heading angle	
<b>V</b>	: forward velocity of tractor-trailer combination	
$l_1$	: tractor wheelbase	
$l_2$	: distance between the hitchpoint and trailer axle	
<b>h</b>	: distance from the tractor rear axle to the hitch point	
<b>H</b>	: lateral displacement of desired path	
$y_A$	: lateral displacement of point A	
$y_B$	: lateral displacement of point B	
$y_D$	: lateral displacement of point D	
$K_1$	: tractor heading feedback gain	
$K_2$	: lateral displacement feedback gain	
$K_3$	: trailer heading feedback gain	
<b>HODF</b>	: human operator describing function $\alpha=f(\theta, y, \beta)$	

## II. THEORY

### 2.1 Vehicle Modeling

A tractor-trailer combination is considered a slow-moving vehicle. A kinematic vehicle model proposed by Torisu et al. (1992, a) was thus adopted, in which the effects of all forces were neglected. In the model, the vehicle is assumed to be moving at a constant forward speed over a smooth, hard and horizontal surface. Bounce, pitch and roll motions may thus be ignored. The bodies of the tractor and trailer are assumed to be rigid. The hitch point is assumed to be frictionless, and the trailer has freedom to rotate in yaw relative to the tractor. Small angles and small amplitudes are also assumed. A schematic representation of the vehicle under consideration is shown in Fig. 1. If consideration of a single unit vehicle is needed, the trailer part in the figure above is simply ignored.

A fixed plane coordinate system is adopted with the x-axis in a longitudinal direction and the y-axis in a lateral direction. For the small angles assumed, the forward velocity along the x-axis is approximately equal to the constant forward velocity **V** of the vehicle. The tractor as a single unit vehicle has two degrees of freedom corresponding to the lateral displacement **y** and heading (yaw) angle  $\theta$ .

On the other hand, the tractor-trailer combination has three degrees of freedom. These degrees of freedom correspond to the lateral displacement  $y$  of the tractor, the heading angle  $\theta$  of the tractor, and the heading angle  $\beta$  of the trailer. The resulting linear vehicle equations of motion are shown in Table 1.

## 2.2 Open-Loop Stability of Tractor-Trailer Combination

By solving the system of differential equations in Table 1, the open-loop characteristics of the vehicle can be determined, if the input steering angle  $\alpha$  is specified. The open-loop block diagram for the vehicle is shown in Fig. 2. Transfer functions for the trailer angle  $\beta$  and lateral displacement  $y_D$ , are

$$\beta(s) = -\frac{V(\ell_2 + h)}{\ell_1(\ell_2 s + V)} \alpha(s) \quad (1)$$

$$y_D(s) = -\frac{V^2(hs - V)}{\ell_1 s^2(\ell_2 s + V)} \alpha(s) \quad (2)$$

To check for open-loop stability, then

$$\ell_2 s + V = 0 \quad (3)$$

or

$$s = -\frac{V}{\ell_2} \quad (4)$$

Since  $\ell_1$  and  $V$  are positive,  $s$  is negative. The transfer functions therefore contain negative poles. This indicates that the motion of the tractor-trailer combination is stable. A physical interpretation of this concept can be furnished by considering the forward straight line motion, of a tractor-trailer combination, with an initial trailer angle offset. The lateral displacement of point D from the straight line, and the trailer heading angle gradually decrease to zero

## 2.3 Operator Modeling

An operator model proposed by Torisu et al. (1992, b), and called "cognition model," was adopted for this study. In this model, there are three control cues. The position control cue is the lateral deviation of one point on the vehicle from the desired course. The attitudes of the tractor and the trailer are the other two control cues. Fig. 3 shows the desired motion of the vehicle. In the forward maneuver, the operator usually controls the tractor-trailer combination while watching the tractor. The center of the tractor rear axle B is thus chosen as the guide point, the

point on the vehicle that is guided over the reference or desired path. The lateral deviation ( $y_B(t)-H$ ) of the center of the tractor rear axle is then the lateral position control cue. The deviations  $(\theta(t)-\theta_G)$  and  $(\beta(t)-\beta_G)$  in the tractor heading angle  $\theta$  and the trailer heading angle  $\beta$  respectively, are taken to be the two attitude control cues. The operator is assumed to be constantly noting these deviations with respect to the desired course. He strives to keep the tractor-trailer combination in the desired lane of the course by minimizing these deviations. He accomplishes this by generating controlled adjustments of the steering wheel. The HODF, represented by the steering angle  $\alpha$ , is

$$\alpha(t) = K_1(\theta(t) - \theta_G) + K_2(y_D(t) - H) + K_3(\beta(t) - \beta_G) \quad (5)$$

For simplicity,  $\theta_G$  and  $\beta_G$  are assumed to be zero.

#### 2.4 Closed-loop Characteristics of Operator/vehicle System

Closed-loop characteristics, of the operator/vehicle system in a forward lane change maneuver, are shown in Fig. 4. These characteristics incorporate the vehicle equations of motion and the operator describing function. The characteristics are defined by the control loop, which involves the feedback of vehicle motion quantities to attain the desired state of motion. This feedback loop is set up by and through the operator and is closed about the vehicle characteristics. The control action of the operator is characterized by the three gains  $K_1$ ,  $K_2$  and  $K_3$  of the HODF. The control problem is resolved by the simultaneous solution of the operator/vehicle system of differential equations. Table 2 shows the resulting third order characteristic equation. With proper selection of gains and vehicle parameters, the roots of the characteristic equation can be determined using Cardan's formula for the roots of a characteristic equation.

#### 2.5 Simulation of the Forward Maneuver of a Tractor-trailer Combination

It is evident from Fig. 4 that, if the control loop of the trailer part of the tractor trailer combination is ignored, the control of the tractor-trailer combination is equivalent to that of a single unit vehicle. A computer simulation program in BASIC was developed to simulate the forward maneuver of a tractor-trailer combination. Simulation was first carried out using the HODF of a single vehicle, and then repeated with the HODF of a tractor-trailer combination. The parameters of the simulated tractor-trailer combination are as shown in Table 3. These parameters are the same as for the actual tractor-trailer combination used for field experiments. From the results of the simulation, it was found that the motion of the tractor-trailer combination for both cases was the identical. From the foregoing, it was concluded that, in the forward maneuver of a tractor-trailer combination, the control of the trailer part can be ignored.

In the analysis of a lane change maneuver, the initial values of state variables  $\theta(0)$  and  $\beta(0)$  are assumed to be zero. From Equ. (5), the initial steering angle  $\alpha(0)$  then becomes

$$\alpha(0) = K_2(y_D(0) - H) \quad (6)$$

In previous work carried out by Torisu et al. (1992, a), the initial steering angle  $\alpha(0)$  was set to zero in the field experiments. Multi-stage simulation of these experimental results was introduced. In the first stage of simulation, gains were set to zero to make  $\alpha(0)$  zero. The gains were then gradually changed as the simulation program went on. This process suggests an adaptive operator behavior. This simulation process is a bit cumbersome especially if it is applied to the analysis of the backward maneuver. To use a simpler simulation process, the initial steering angle  $\alpha(0)$  in the field experiments was set according to Equ. (4).

### III. RESULTS AND DISCUSSION

#### 3.1 Field Experiments

Field experiments were carried out with a tractor-trailer combination instrumented as shown in Fig. 5. Drivers with different levels of skill were instructed to drive the tractor-trailer combination forwards, in a lane change maneuver between two parallel lines marked on a tarmac surface. The trajectories of the centers of the tractor front axle, tractor rear axle and trailer axle, of a tractor-trailer combination, were plotted on a flat tarmac surface. This was achieved by using colored inks dripping from suspended intravenous drip containers. These trajectories were then measured manually and recorded. Potentiometers, connected through gears to the front wheel and hitch point kingpins, were used to measure the steering and trailer heading angles respectively. The two potentiometers were connected to a data recorder. At the beginning of each run, the initial steering angle was set.

#### 3.2 Comparison of Simulation and Field Experiments

Computer simulation of several runs of forward maneuvers was carried out. Results of an actual forward lane change maneuver and its simulation are shown in Fig. 6 and Fig. 7 respectively. The trajectories for both the real and simulated results were in good agreement. It was therefore concluded that the proposed operator/vehicle model can adequately describe the forward maneuver of a tractor-trailer combination when the initial steering angle is not zero.

## CONCLUSIONS

- 1) The kinematic model was found suitable for a tractor-trailer combination since it is a slow-moving vehicle.
- 2) The closed-loop operator/vehicle model proposed, can adequately describe the forward maneuver of a tractor-trailer combination.
- 3) The model was found adequate in simulating the maneuver when the initial steering angle was not zero.
- 4) The forward maneuver of a tractor-trailer combination was equivalent to that of a single unit vehicle.

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Table 1 Vehicle equations of motion

$$\begin{aligned}
 \ell_1 \dot{\theta} - V\alpha &= 0 \\
 \dot{y}_B - V\theta &= 0 \\
 \dot{y}_A - V(\alpha + \theta) &= 0 \\
 \ell_2 \dot{\beta} + V\beta + (\ell_2 + h)\dot{\theta} &= 0 \\
 \dot{y}_D - V(\theta + \beta) &= 0
 \end{aligned}$$

Table 2 Solution of the operator/vehicle system model for forward motion

Variable	characteristic equation
$y_B$	$\Delta_{\theta B \beta}(s)y_B = -V^3 K_2 H$
$y_D$	$\Delta_{\theta B \beta}(s)y_D = -V^3 K_2 H$
$y_A$	$\Delta_{\theta B \beta}(s)y_A = -V^3 K_2 H$
$\alpha$	$\Delta_{\theta B \beta}(s)\alpha = 0$
$\theta$	$\Delta_{\theta B \beta}(s)\theta = 0$
$\beta$	$\Delta_{\theta B \beta}(s)\beta = 0$

Kernel polynomial operator:

$$\begin{aligned}
 \Delta_{\theta B \beta}(s) &= \ell_1 \ell_2 s^3 + V(\ell_1 - \ell_2 K_1 + \ell_2 K_3 + h K_3) s^2 \\
 &\quad - V^2 (K_1 + \ell_2 K_2) s - V^3 K_2
 \end{aligned}$$

Human operator describing function:

$$\alpha = K_1 \theta + K_2 (y_B - H) + K_3 \beta$$

Note s: Laplace operator

Table 3 Vehicle parameters

Parameter	Value
tractor wheelbase $\ell_1$	1.32 (m)
hitch distance $h$	0.42 (m)
trailer length $\ell_2$	1.60 (m)
travel velocity $V$	0.22 (m/s)
lane change width $H$	6.0 (m)

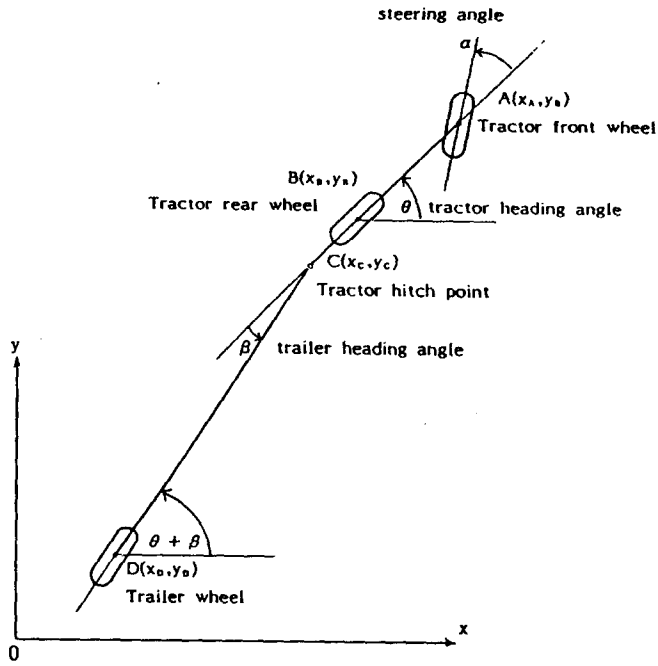


Fig. 1 Layout of a Tractor-trailer Combination

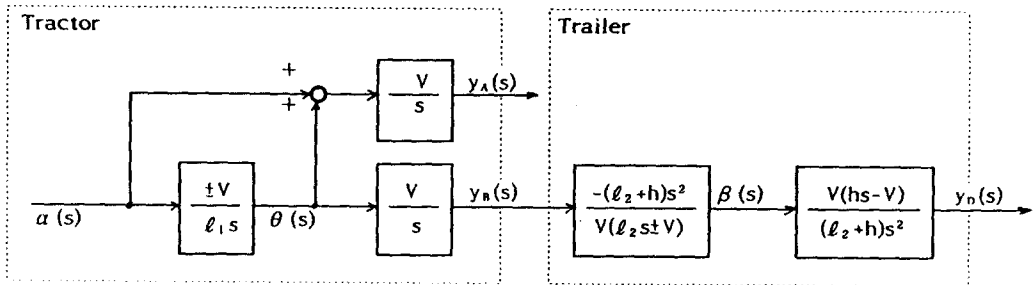


Fig. 2 Open-loop characteristics of a Tractor-trailer Combination

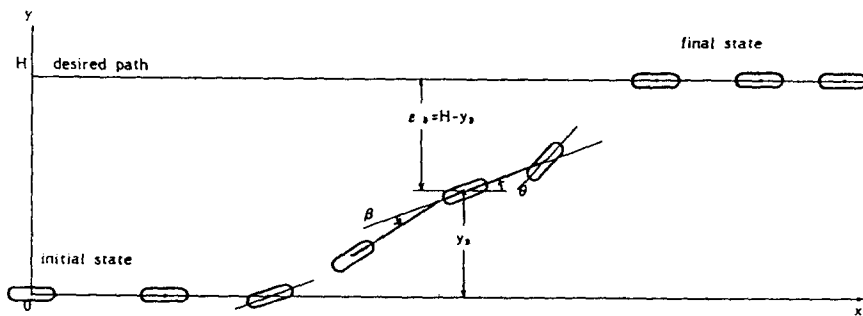


Fig. 3 Forward Lane Change Maneuver of a Tractor-trailer Combination



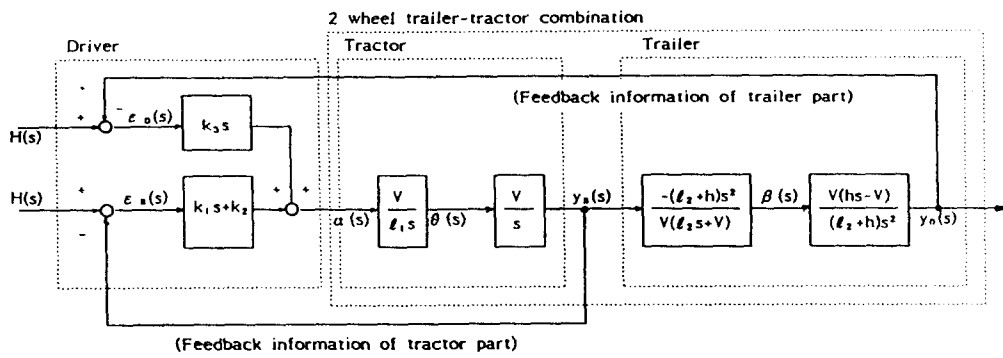


Fig. 4 Closed-loop characteristics of a Tractor-Trailer Combination

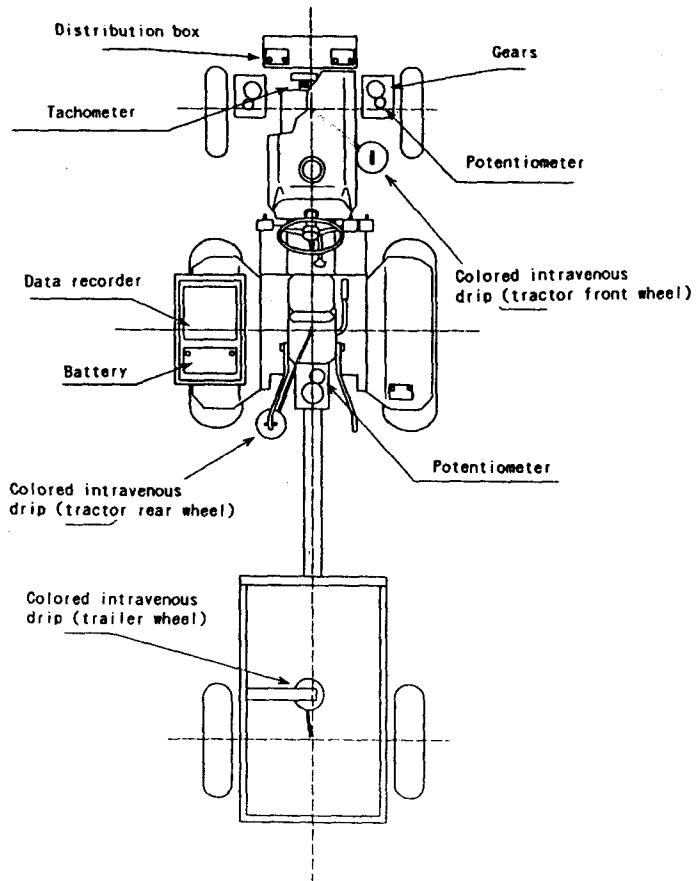


Fig. 5 Schematic diagram of experimental apparatus

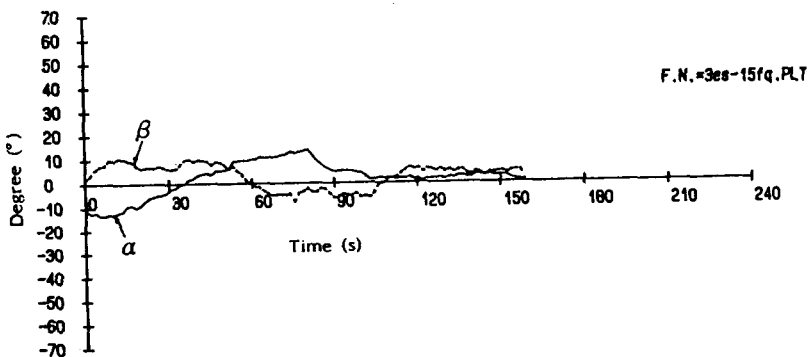
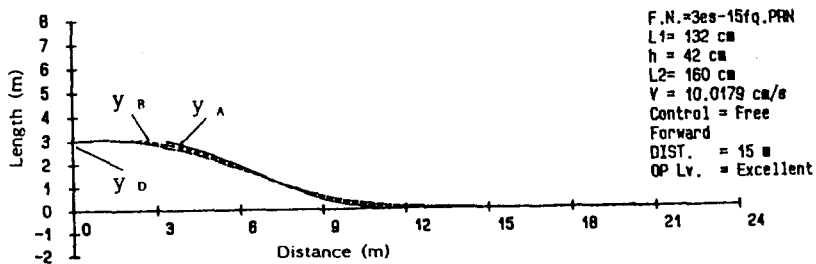


Fig.6 Result of an actual forward maneuver

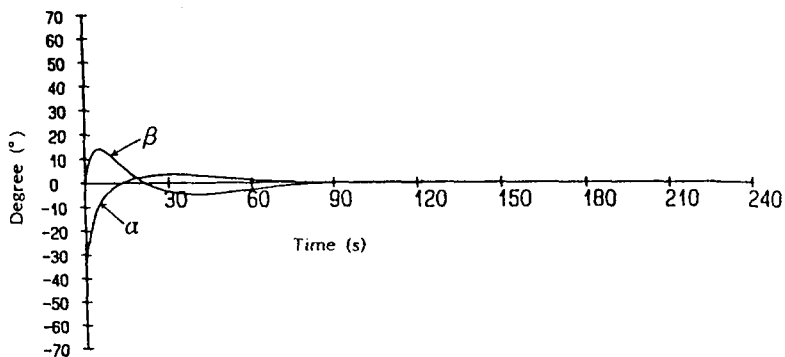
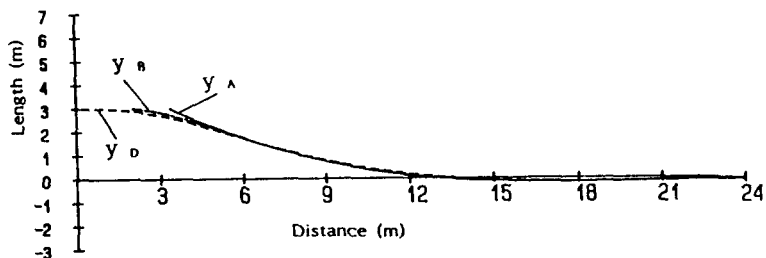


Fig.7 Computer simulation of the actual forward maneuver