

## **Parameter Identification for the Tractor Dynamic Model by use of a Forced Vibration Experiment**

**R. Noguchi\*, O. Kinoshita\*\*, E. Inoue\*\* and K. Nakano\*\*\***

### **ABSTRACT**

Physical parameters in the forward direction of the tractor, which mainly affects the vibration characteristics of the tractor dynamic model, were able to be identified in a short time by using the Gauss-Newton method with extremum searching based on the data obtained from a forced vibration experiment. It was clarified that a period for the updating of the parameter estimates method has effects on the convergence accuracy of identification for the spring constant in the forward direction of the tractor.

Key Word : Tractor dynamic model, Parameter identification,  
Phase method, Gauss-Newton method

### **INTRODUCTION**

From the viewpoint of the human engineering "Goldman and Gierke (1960)", it is very important to design the optimal control for reduction of the vibration of the tractor by using the dynamic model "Collins (1991)" (Fig.1). On the other hand, parameters of the tractor tires of the dynamic model had sufficiently been studied as the function of frequency "Lines and Young (1989); Lines and Murphy (1991)". However, the parameters of the tire characteristics included the tractor body should be measured quickly. Because when the tractor is being controlled on the fields, these parameters have an important influence on the vibration characteristics of it.

This paper deals with the method for the quick identification of parameters of the tractor tires included the tractor body by use of the Gauss-Newton method with extremum searching which is now being established "Toyota et al. (1992)" based on the forced vibration data. Lastly the efficiency of the present identification method for estimating the values of the spring constant and the viscous damping coefficient in the forward

---

\* Institute of Agri. and Forest Eng., Univ. of Tsukuba, Tsukuba, Ibaraki, 305 Japan

\*\* Agri. Machinery Lab., Faculty of Agri., Kyushu Univ., Fukuoka, 812 Japan

\*\*\* Electrical Eng., Fukuoka Institute of Technology, Fukuoka, 811-02 Japan

direction of the tractor was discussed in comparison with those parameters calculated by the phase method.

## EXPERIMENT AND MEASUREMENT APPARATUS

An illustration of the forced vibration experiment of the riding tractor with rotary tiller is shown as Fig. 2. The small-size riding tractor (11.2 kW) was used in this experiment. The force shaker can give the artificial vibration to the tractor as the periodical forced vibration occurred from a working instrument as like rotary tiller.

5G (G; Gravitational constant) accelerometers set on the tractor to analyze the vibration characteristics of it. A three-dimension accelerometer was used on the center of gravity of the tractor to analyze the forward and vertical direction accelerations, and two accelerometers set on the front and rear of the tractor to calculate the pitching acceleration. The forced vibration data which were measured from the load-cells on the top of the shock absorbers and the acceleration data were recorded by multi-channel recorder.

## IDENTIFICATION METHOD

### Simplified dynamic model

The dynamic model (Fig. 1) was solved by the three degree of freedoms on the two dimensional plane. These degree of freedoms are the forward direction, the vertical direction, and the pitching direction of the tractor. If the tractor dynamic model is dealt with the linear model, the motion equations is able to solve by use of Lagrange's kinetic equation. The frequency response function is derived by the Laplace transformation of the equations.

Then the equation of the tractor dynamic model is expressed as the following state equation,

t : time  
"·": the derivative with respect to time  
X (t) : state vector(6×1):  $[\dot{x} \ x \ \dot{y} \ y \ \dot{\phi} \ \phi]^T$  (T: transpose)  
 $\tau$  (t) : input vector(1×1)  
A : system matrix(6×6)  
B : driving matrix(6×1)

$$\dot{X}(t) = A X(t) + B \tau(t). \quad (1)$$

However, when the vibration in the pitching direction occurs on the tractor, all vibration characteristics is effected from the spring constant and the viscous damping coefficient in the

forward and vertical direction of the tractor dynamic model. So it is very difficult to identify only a spring constant in the forward direction of the tractor included in A-matrix from the data of input  $\tau(t)$  and output  $X(t)$ . But when the tractor was vibrated by 5.3 Hz forced frequency on forced angle  $43^\circ$ , there were very small value of the pitching acceleration on the tractor, then mutual relation which were forward or vertical acceleration with pitching acceleration could be neglected. So the dynamic model can be simplified in the forward direction as follows,

- m : mass of the tractor [kg]
- k : spring constant in the forward direction of the tractor, ( $k = k_3 + k_4$ ) [N/m]
- c : viscous damping coefficient in the forward direction of the tractor, ( $c = c_3 + c_4$ ) [N·s/m]
- $x_1(t)$ : displacement in the forward direction at the center of gravity of the tractor ( $x_1(t) = x(t)$ )
- $x_2(t)$ : velocity in the forward direction at the center of gravity of the tractor ( $x_2(t) = \dot{x}(t)$ )

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \tau(t). \quad (2)$$

And measured position of the output data was the center of gravity of the tractor in our forced vibration experiment, so the output equation is expressed as follows,

- $y_1(t)$ : displacement of the output data
- $y_2(t)$ : velocity of the output data

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \quad (3)$$

### Phase method

This method is very popular for measuring the characteristics value of the rubber products something like the tractor tire or rubber crawler of the combine wheel, and this makes it possible to measure exactly the characteristics value of the spring constant and the viscous damping coefficient between small value and large value of it "Sekiguchi and Asami (1981)" by the following two equations,

- $\omega$  : angular velocity [rad/s]
- $\Phi$  : phase angle [rad]
- G : accelerance (frequency response value) [ $m/(N \cdot s^2)$ ]

$$c = \left| \frac{\omega \sin \Phi}{G(\omega)} \right|, \quad (4)$$

$$k = m \omega^2 \pm \frac{\omega^4}{G^2(\omega)} - c \omega^2. \quad (5)$$

### Gauss-Newton method with extremum searching

In general, the Gauss-Newton method is a kind of quasi-linearization "Marguardt (1963)". This identification method makes it possible to identify the parameters included in A-matrix by decreasing the error between the data of system output (plant output) and data of simulator model output corresponding to the same input data. When the data of the simulator model is equivalent to those of system output exactly, the parameters are already identified.

The parameters  $k$  and  $c$  are expressed in the vector form,

$$\Xi = [k \quad c]^T \quad (6)$$

and the correction value can be calculated by the following equation,

$i$  : number of correction

$T_s$ : period for the updating of the parameter estimates

$\Xi^i$ : parameter vector of the simulator model at  $(t - T_s)$

$h$  : output vector of the simulator model

$y$  : output vector obtained from the experiment

$$(y = [y_1(t), y_2(t)]^T),$$

$$\delta \Xi = \left[ \begin{array}{c} t \\ t - T_s \end{array} \left[ \begin{array}{c} \frac{\partial h(\Xi^i)}{\partial \Xi^T} \end{array} \right]^T \left[ \begin{array}{c} \frac{\partial h(\Xi^i)}{\partial \Xi^T} \end{array} \right] dt \right]^{-1} \left[ \begin{array}{c} t \\ t - T_s \end{array} \left[ \begin{array}{c} \frac{\partial h(\Xi^i)}{\partial \Xi^T} \end{array} \right]^T [y - h(\Xi^i)] dt \right]. \quad (7)$$

Finally, the parameter estimate  $\Xi$  can be updated at  $(t)$  in the form of the following equation,

$$\Xi = \Xi^i + \delta \Xi. \quad (8)$$

In the next stage, the new  $\Xi$  is replaced with  $\Xi^i$ . In the Gauss-Newton method with extremum searching, the following indices are introduced in order to the convergency for the true value,

n: computation times between t and t - T<sub>s</sub>, (T<sub>s</sub>/n: sampling time)

$$Q1 = \sum_{j=1}^n \left\| \mathbf{y} \left( T_s \times \frac{j}{n} \right) - \mathbf{h} \left( T_s \times \frac{j}{n} : \Xi + \delta \Xi \right) \right\|^2,$$

$$Q2 = \sum_{j=1}^n \left\| \mathbf{y} \left( T_s \times \frac{j}{n} \right) - \mathbf{h} \left( T_s \times \frac{j}{n} : \Xi \right) \right\|^2, \quad (9)$$

If Q1 > Q2, then the δΞ should be changed to the next stage as follows,

$$\delta \Xi_t = \frac{\delta \Xi_{t-T_s}}{2}. \quad (10)$$

Conversely, if Q1 ≤ Q2, then the δΞ should not be changed.

### RESULTS AND DISCUSSIONS

After forced vibration experiment, we analyzed the data by using the FFT analyzer (A&D, AD-3525 FFT ANALYZER) to measure the angular velocity ω, phase angle Φ, and accelerance G. Especially the right and the left vibration forces showed the same phase and same amplifier value of spectral each other. So, the total vibration force was able to considered as twice of one side vibration force.

At first, the parameters k and c were identified by using the phase method. Secondly, these parameters were identified by using the Gauss-Newton method with extremum searching proposed here. When this method was applied, velocity and displacement data were calculated respectively by integration and double integration of the acceleration data of the period [t - T<sub>s</sub>] in the forced vibration experiment.

Finally, identification experiments were performed with respectively 2 ms and 6 ms period for the updating. The results of parameter identification by those two method are shown as Fig. 3~Fig. 6. Reliability of the parameter identified by phase method is generally high, so the values estimated by the phase method were dealt as the true values in this identification for discussion in the next conclusions.

Results of identification of the parameter k (2 ms period for the updating) were excellent from 0 ms to 10 ms as shown in Fig. 3. In spite of stable identification until 80 ms, the identified parameter was diverging after 80 ms. However after 120 ms, the parameter was recovered its stability. This phenomena shows the fact that the accuracy of identification for the parameter k

depends on the information amount of input and output data, which give an influence on the convergency of identification. Hence, if the displacement data indicate the zero level, it is difficult to identify the parameter  $k$ . On the other hand, results of identification of the parameter  $c$  (2 ms period for the updating) were excellent from first 0 ms to 40 ms as shown in Fig. 4. But after 40 ms, identification for the parameter  $c$  is more fluctuating than that for the parameter  $k$ . Especially, when the velocity data indicated near the zero level, it was difficult to identify the parameter  $c$ .

If the parameter is not able to be identified, the period for the updating of the parameter estimates  $T_s$  in Eq. 7 should be change to give the guarantee of the convergency of identification, when the Gauss-Newton method with extremum searching is using. Because when a period for the updating is small, the information of data may not enough to identify. Then, as in the next stage, the period for the updating was changed to 6 ms from 2 ms.

Results of identification of the parameter  $k$  (6 ms period for the updating) is shown as Fig. 5. Convergence time was taken more need than that in the identification with 2 ms period for the updating. But the identified parameter was not diverging in the total evaluation on the period from start time to end time as shown in Fig.5. So it is concluded that the longer the period for the updating, the better identification for the parameter  $k$ , however, the more convergence time is taken in this experiment. On the other hand, identification of the parameter  $c$  (6 ms period for the updating) (Fig. 6) could not change in convergence speed of identification.

As a result, the identification by the Gauss-Newton method with extremum searching for the parameters  $k$  and  $c$  makes it possible to converge to the truth value of the parameter more quickly than that by phase method in the time ranges which gives the sufficiently information of the input and output data. However, the problems of the initial conditions and parameter divergence must be discussed more carefully to in application of the Gauss-Newton method with extremum searching.

## CONCLUSIONS

By using the data of input forces and output accelerations obtained from a forced vibration experiment for the riding tractor and the Gauss-Newton method with extremum searching, the spring constant  $k$  and viscous damping coefficient  $c$  in the forward direction of the tractor were identified.

As a result, it was clarified that :

(1) Identification by the Gauss-Newton method with extremum searching makes it possible to converge to the truth value of the parameter  $k$  more quickly than that by the phase method.

(2) The period for the updating has effects on the convergence accuracy of identification by the Gauss-Newton method with extremum searching for the parameter  $k$ , but not the parameter  $c$  in this experiment.

#### ACKNOWLEDGEMENTS

The authors acknowledge Assoc. Prof. P. Chen of Fukuoka Junior College of Technology, and Prof. J. Sakai of Kyushu University for their useful advice and development of the experimental apparatus used in this research at Kyushu University.

#### REFERENCES

1. Collins, T. S. 1991. Loads in tractor linkage when transporting rear-mounted implements, development of modelling and measurement techniques. *Journal of agricultural Engineering Research* 49:165-188.
2. Goldman, D. E. and H. E. V. Gierke. 1960. The effects of shock and vibration on man. *Lecture and Review Series (Naval Medical Research Institute, Bethesda, Maryland 8)(USA):60(3)*.
3. Lines, J. A. and N. A. Young. 1989. A machine for measuring the suspension characteristics of agricultural tyres. *Journal of Terramechanics* 26(3/4):201-210.
4. Lines, J. A. and K. Murphy. 1991. The radial stiffness of agricultural tractor tyres. *Journal of Terramechanics* 28(1):49-64.
5. Lines, J. A. and K. Murphy. 1991. The radial damping of agricultural tractor tyres. *Journal of Terramechanics* 28(2/3):229-241.
6. Marguardt, D. W. 1963. An algorithm for least squares estimation of nonlinear parameters. *Journal of SIAM* 11: 431-441
7. Sekiguchi, H. and T. Asami. 1981. Measurement method for large viscous damping. *Transactions of JSME* 47(422)c:1317-1326.
8. Toyota Y., K. Nakano, Y. Akiyama, M. Eguchi, R. Noguchi and M. Uchida. 1992. Computing-predictor network-based identification and exact model matching for a class of nonlinear mechanical systems. *IECON'92 (International Conference on Industrial Electronics, Control, Instrumentation, and Automation Nov.9-13 San Diego, CA USA) Proceedings:1246-1251*.
9. Noguchi, R. 1993. Study of the Theory and Dynamic Characteristics for Tractor & Rotary Tillage Systems. PhD theses.

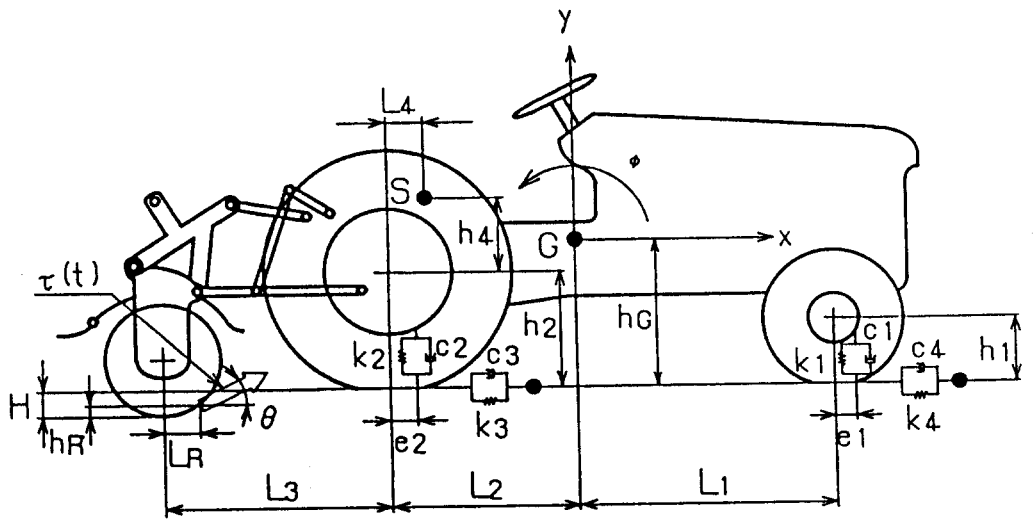


Fig. 1 Tractor dynamic model with rotary tiller

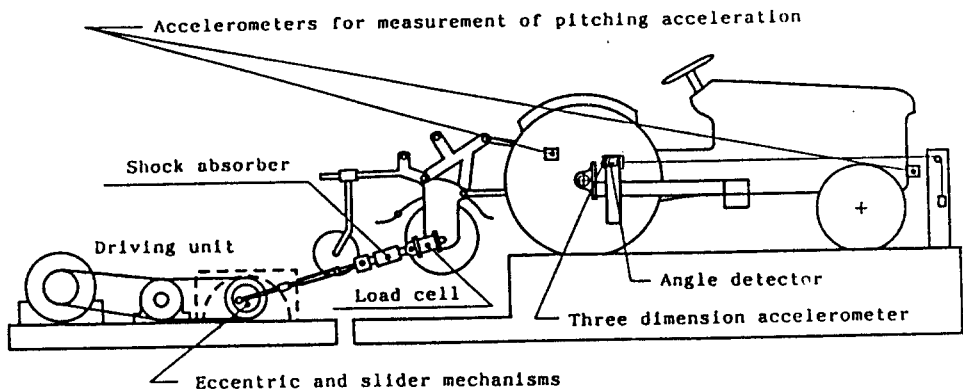


Fig. 2 Explanation of the force shaker and the forced vibration experiment of the riding tractor



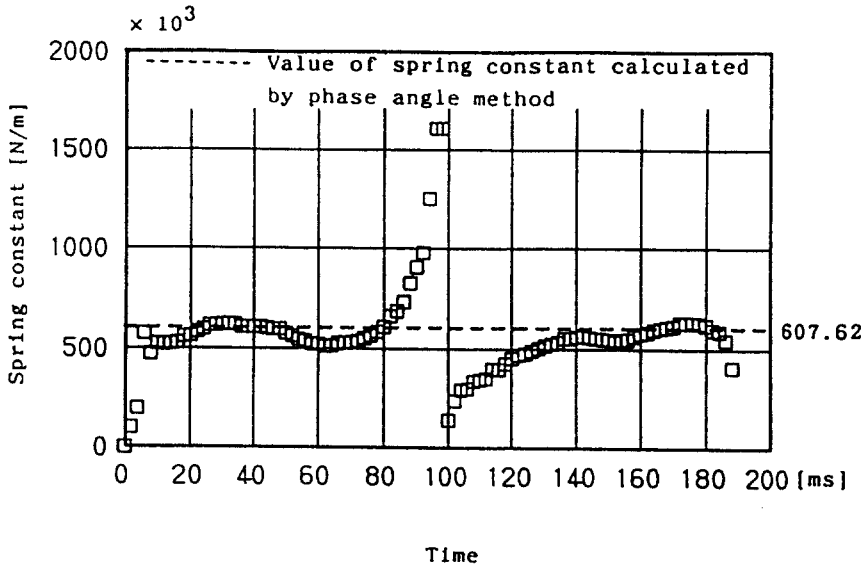


Fig. 3 Identification for the spring constant  $k$  in the forward direction of the tractor (2 ms period for the updating)

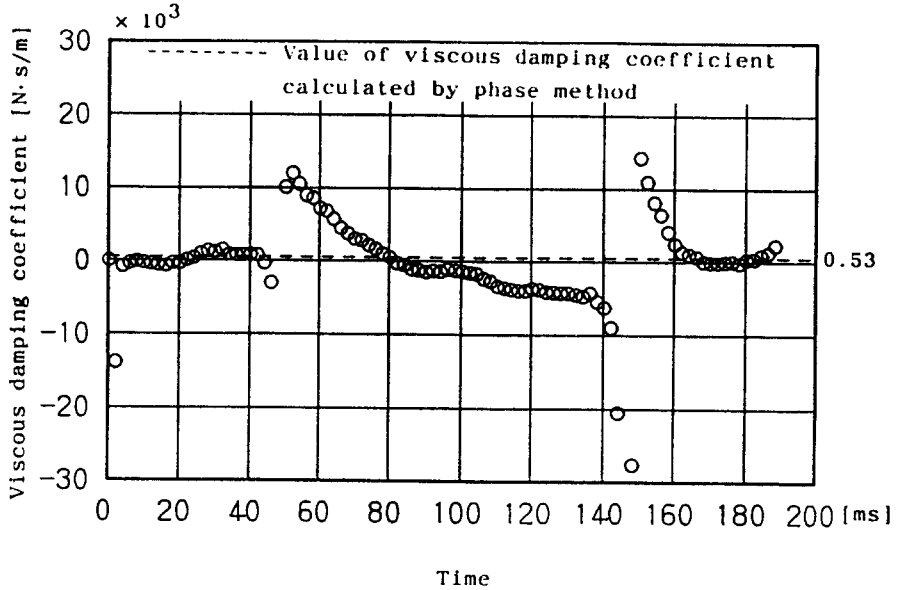


Fig. 4 Identification for the viscous damping coefficient  $c$  in the forward direction of the tractor (2 ms period for the updating)

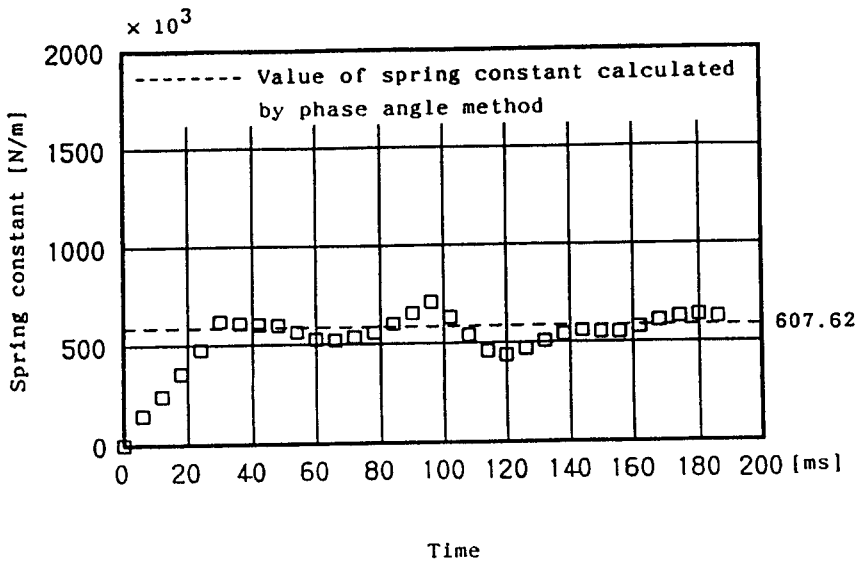


Fig. 5 Identification for the spring constant  $k$  in the forward direction of the tractor (6 ms period for the updating)

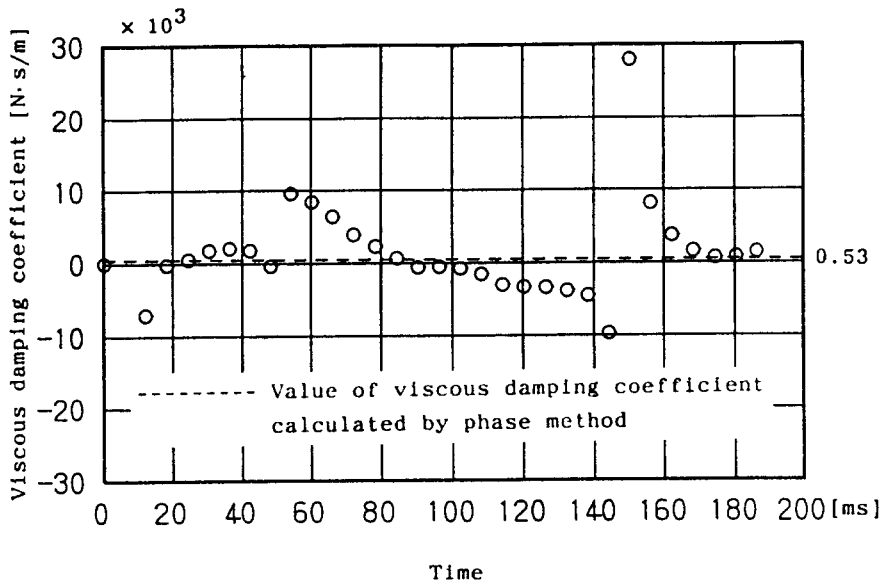


Fig. 6 Identification for the viscous damping coefficient  $c$  in the forward direction of the tractor (6 ms period for the updating)