Proceedings of the International Conference for Agricultural Machinery & Process Engineering
October 19-22, 1993, Seoul, Korea

STUDY ON SYSTEM COMPATIBILITY DEFORMATION MODEL OF ROTARY TILLER UNDER LATERAL LOADS

Hangxiang Hu, Changbing Wang, Zhengzhong Sang
P.O. Box 733, Jiangsu Institute of Technology
Zhenjiang, China 212013

ABSTRACT

The model of rotary tillers under side loads established here is a statically indeterminate system. By means of FEM method, the deformation of side gearbox and right side board are calculated. Therefore the side deformations of rotary tillers under different lateral loads are discussed systematically. The results show that the rotary tiller system would bear the loads and deform unequally. Authors' calculation also indicates that the lateral deforming values of right side board and side gearbox are almost the same, and more than 98% of the loads is borne by the side

Key words: rotary tiller, lateral load, lateral deformation, deforming coordination

INTRODUCTION

Rotary tiller is a kind of tillage implement widely used in the paddy and dry field of southeast Asia. When a rotary tiller is under operation, the left and left bent blades' alternating soil–cutting, slanting traction phenomemon, linearity problem, direction–turning in the field and even the vibration of the implement may lead the rotary tiller to bear lateral loads. Therefore, the lateral deformation is existed and the implement's strength is weakened and sometimes results failure This problem has not been investigated thoroughly for a long time. With the theory of systematics, the initial study of integral and individual part of a rotary tiller is made in this paper.

MODEL BUILDING OF ROTARY TILLER UNDER LATERAL LOADS

Model Analyses

A rotary tiller can be divided into six sections: headstock 1, central gearbox 2, left and right main beam 3, side gearbox 4, right side board 5 and cutter bar system 6 (see Fig. 1). The central gearbox and left and right main beam are connected firmly with the tractor. Their deformation is much less than that of the operation section or cutter bar system. It is assumed that they are fixed ends for simplicity.

It is generally accepted by previous studies that the lateral loads are borne by the side gearbox and right side board, and they have nothing to do with the cutter bar in resisting the lateral loads; the equal lateral deformation will engender under lateral loads. Obviously, these two viewpoints are obtained on the hypothesis of the cutter bar to be a two–force bar which eliminates the force between two ends and brings no elastic deformation. However, the real cutter bar is an elastic component and its mounted bent blades' alternating soil–cutting will also produce lateral loads. Thus, such results are not exact and therefore are worth of further investigations.

Model Building

It is assumed by the authors that a rotary tiller as an elastic system and the lateral loads are borne mainly but unequally by side gearbox, side board (i.e. right side board) and cutter bar. Each part has some deformation.

The two ends of the cutter bar are connected with other parts by roller bearings. It is assumed to be a hinge structure, neglecting the jointpoint's clearance. The system model under lateral loads is shown in Fig.2. Because many blades are distributed uniformly along the cutter bar, the cutter's axial force (lateral load) is supposed to be an uniform distribution. Forces and deformations are shown in Fig.3.

Model Calculation

There are altogether six unknown reacting forces at points A,B,C and D, three equilibrium equations and two conditional equations $M_B=0$, $M_C=0$ obtained from the hinges which means that the system is a statically indeterminate one to the first degree.[1]

We regard each part of the system as an isolated analysis unit (see Fig. 3). The side board undertakes external force P_1, cutter bar force F_{21}, reaction F_A at point A and moment M_A. It is easy to understand that point B's deformation Z_B corresponds to the external load P_1+F_{21} respectively, i.e.,

$$Z_B = f_1(P_1 + F_{21}). \tag{1}$$

Similiarly, we have $Z_C=f_3(P_1+F_{23})$ for the side gearbox. As for the cutter bar, it undertakes the side board's force F_{12}, uniformly distributed force pl and side gearbox's reaction F_{32}. Its deformation $Z_{BC}=f_2'(F_{12}, \text{pl}, F_{32})$. Known from the figure, we have: $F_{32}=-(F_{12}+\text{pl})$. Hence:

$$Z_C = f_3(P_3 + F_{12} + \text{pl}) \tag{2}$$

$$Z_{BC} = f_2(F_{12} + \text{pl}) \tag{3}$$

In the Eq. 1 to 3, $F_{12}$is unknown. Therefore, one deformation compatibility equation is needed. Because of the continuity of the system, we have:

$$Z_{BC} = Z_C - Z_B \quad \text{(positive for tensility)} \tag{4}$$

Substituting Eq. 1, 2 and 3 into Eq. 4 results in:

$$f_2(F_{12}, \text{pl}) = f_3(P_3 + F_{12} + \text{pl}) - f_1(P_1 + F_{21}) \tag{5}$$

From Eq. 5, we have:

$$F_{12} = f(P_1, P_2, p) \tag{6}$$

Thus, each point's deformation value is $Z_B=F_1(P_1, P_2, p)$ or $Z_C=F_2(P_1, P_2, p)$ and each component's distributed load can be calculated.

DEFORMATION COMPUTATION OF 1G–175A ROTARY TILLER

Component Analysis of 1G–175A Rotary Tiller

Due to their complicated structure and loads system of the side gearbox and side board, the Finite Element Method (FEM) is used in authors' calculation. The established FEM network model is shown in Fig. 4. The resistant torque formed by the cultivating resistance and the driving moment of the transmission gear will be in equilibrium. These resistances will act as supporting reactions on the pin and roller support of the axle. Besides, the driving moment, the gear's meshing force also generates certain reaction and bending moment at the bearing. Based on the structural character, 3–dimensional shell element is adopted in FEM computation to form a discretization network model.[2]

Side Gearbox In order to determine the lateral deforming principleof the side gearbox uner different lateral loads, deformations under various lateral loads are calculated here. The maximum values of lateral deformation is obtained by calculation (refer to Fig. 5(a)). From Fig. 5(a), it is known that the deformation is obviously in linear relationship versus the load within elastic range. The fitted deforming equation:

$$Z_C = 3.098 \times 10^{-5}P - 2.222 \times 10^{-2} \quad \text{(mm)} \tag{7}$$

To simplify, we calculate its equivalent Flexra Rigidity $(EI)_3^*$ when it is viewed as a cantilever beam:[3]

$$(EI)_3^* = \frac{PL^3}{3Z_C} = 648118 \quad \text{MN} \cdot \text{mm}^2 \tag{8}$$

The initial deformation 2.222×10^{-2}mm is not considered in above calculation because it is caused by the force F_2 which is generated by the bending moment action originated from the cutting counter force of the cutter bar system and the meshing force of gearbox on the side gearbox and side board. If the resultant of the lateral loads is zero, Z_C should be reduced to zero, too.

Side Board Similiarly, the maximum lateral deformation of side board under a series of lateral loads is shown in Fig. 5(b). Because the force system of side board is simpler than that of side gearbox, the lateral deformation approximates to zero under no lateral loads. The deformation and load are also in linear relationship. The fitted deforming equation:

$$Z_B = 1.9728 \times 10^{-3} + 1.21 \times 10^{-2} \quad \text{(mm)} \tag{9}$$

The equivalent Flexra Rigidity $(EI)_1^*$ of side board as a cantilever beam:

$$\begin{aligned}(EI)_1^* &= PL^3 / 3Z_B = 10177.8 \\ &< (EI)_3^* = 648118 \quad \text{MN} \cdot \text{mm}^2,\end{aligned} \tag{10}$$

Hence, the rigidity of side board is much less than that of side gearbox. The load–bearing capacity is poorer, either. The reason of existing initial deformation is the same as that of side gearbox.

Cutter Bar The force system acting on the cutter bar system is shown in Fig. 6. It is a drag (or press) bar deforming model. The cutter bar is made of seamless steel tube of 76 × 7 (mm × mm) with the modulus of elasticity E 2.1×10^9 N / mm^2.

$$dZ_{BC} = \frac{N(z)}{EA} dz \tag{11}$$

where $N(z) = F_{12} + pz$, therefore, we obtain:

$$\begin{aligned} Z_{BC} &= -\int_0^l \frac{F_{12} + pz}{EA} dz + C \\ &= -4.805 \times 10^{-7} p - 5.491 \times 10^{-10} F_{12} + C \end{aligned} \tag{12}$$

The negative means the system is under pressure. C is the initial deformation under the action of the bending moment.

Deforming Analysis of 1G–175A Rotary Tiller

With the above individual analyses and calculation of 1G–175A rotary tiller's parts and from the Eq. 5, we obtain:

$$\begin{aligned} &-(4.805 \times 10^{-7} p + 5.491 \times 10^{-10}) + C = [3.098 \times 10^{-5}(F_{12} + pl - P_3) - 2.222 \times 10^{-2}] - \\ &\quad [1.9728 \times 10^{-3}(P_1 - F_{12}) + 1.21 \times 10^{-2}] \end{aligned} \tag{13}$$

Substituting the initial values into Eq. 13 yields $C = -0.02222 - 0.0121 = -0.0343$, so:

$$F_{12} = -27.0566p + 0.9845392P_1 + 0.0154607P_3 \tag{14}$$

Therefrom, we obtain:

side board deformation:

$$Z_B = 0.0533773p + 3.05 \times 10^{-5} P_1 - 3.05 \times 10^{-5} P_3 + 0.0121 \quad (\text{mm}) \tag{15}$$

cutter bar deformation:

$$Z_{BC} = -4.65643 \times 10^{-7} p - 5.4061 \times 10^{-10} P_1 - 8.4895 \times 10^{-12} P_3 - 0.03432 \quad (\text{mm}) \tag{16}$$

side gearbox deformation:

$$Z_C = 0.0533768p + 3.05 \times 10^{-5} P_1 - 3.05 \times 10^{-5} P_2 - 0.02222 \quad (\text{mm}) \tag{17}$$

The above disscussion considers the general situation that all the parts are under the action of external lateral loads. However, in the real condition, there are usually only one or two kinds of loads which play as an leading role. See below:

1. Assuming that, at $p = 0$, $P_1 = 0$, only lateral load P_3 at the side gearbox acts on the rotary tiller, it is easy to know that P_3 usually comes from outside. We have:

$$Z_B = -3.05 \times 10^{-5} P_3 + 0.0121, \tag{18}$$

$$Z_{BC} = -8.48947 \times 10^{-12} P_3 - 0.03432, \tag{19}$$

$$Z_C = -3.05 P_3 - 0.02222, \tag{20}$$

and load bearing by the side board is $F_{21} = 0.0154607 P_3$.

Suppose that external load is $P_3 = 10^5$ N, we have:

$$Z_B = -0.2929\text{mm}, \ Z_{BC} = -0.03432\text{mm}, \ Z_C = -0.32722\text{mm}, \ F_{12} = 154.61\text{N}, \tag{21}$$

i.e., the side board only bears $\frac{154.61}{10000} \times 100\% \approx 1.55\%$ of the external load. So the left 98.45% will be borne by the side gearbox.

2. Assuming $p=0$, $P_3=0$ and only lateral load P_1 at the side board acts on the rotary tiller, we have:

$$Z_B = 3.05 \times 10^{-5}P_1 + 0.0121, \tag{22}$$

$$Z_{BC} = -5.40610 \times 10^{-10}P_1 - 0.03432, \tag{23}$$

$$Z_C = 3.05P_1 - 0.02222, \tag{24}$$

and load bearing by the side board is $F_{23}=0.9845392P_1$.

If we take P_1 to be 10^5 N, we have:

$$Z_B = 0.3171 \text{ mm}, \ Z_{BC} = -0.034325 \text{ mm}, \ Z_C = 0.28278 \text{ mm}, \ F_{23} = 9845.4 \text{ N}, \tag{25}$$

i.e., the side grarbox bears $\frac{9845.4}{10000} \times 100\% \approx 98.45\%$ of the external load and only 1.55 is borne by the side board.

3. Assuming $P_1=0$, $P_3=0$, we know that only additional lateral load pl caused by the blades' soil–cutting acts on the rotary tiller. Thus, we have:

$$Z_B = 0.0533773p + 0.0121, \tag{26}$$

$$Z_{BC} = -4.65643 \times 10^{-7}p - 0.03432, \tag{27}$$

$$Z_C = 0.0533768p - 0.02222, \tag{28}$$

load bearing by the side board $F_{21} = -27.0566p,$ (29)

and load bearing by the side gearbox $F_{23} = F_{21} + pl = 1722.9434p.$ (30)

Thus, at $p=1$ N / mm, we have:

$$Z_B = 0.0654773 \text{ mm}, \quad Z_{BC} = 0.0343205 \text{ mm}, \quad Z_C = 0.0311568 \text{ mm},$$

$$F_{12} = -27.0566 \text{ N}, \quad F_{23} = 1722.94 \text{ N} \tag{31}$$

i.e., $\frac{1722.94}{1750} \times 100\% \approx 98.45\%$ external lateral load is borne by the side gearbox. The side board bears only 1.546%.

According to the above analyses, we know that the load–bearing distribution rate of the side board and side gearbox is basically consistent. From Eq. 14, however, it is easy to see that the distribution rate will change provided that the external lateral loads act combinationally. The varible range is limited.

As for the lateral forces of the cutter bar originated from the left–and–right bent blades' alternating soil–cutting, the lateral forces should be alternating forces. But from the above analyses and calculation, we know that the lateral loads of the cutter bar have little effects on the overall lateral bearing and deformation. Hence, it brings no evident errors if they are not taken into consideration.

CONCLUSIONS

1. The lateral loads of the rotary tiller are borne unevenly by the total system and the relevant deformations are also unequal.

2. From the model computation by the FEM, it is known that the lateral deformations of side board and side gearbox are in linear relationship to the lateral loads within the elastic range. Their equivalent Flexra Rigidities are 10177.8 MN • mm^2 and 648118 MN • mm^2, respectively.

3. The deformation values of the side board and side gearbox under lateral loads are not equal, but they are almost the same, and the the cutter bar's deformation is quite minute.

4. When calculating the lateral deformation, it will bring no evident errors if neglecting the effects of the lateral loads of cutter bar on alternating forces.

5. Owing to the poor rigidity of the side board comparing to the side gearbox, the lateral loads are borne mainly by the side gearbox. According to the computation, the side gearbox bears more than 98% of the external load while the side board bears generally only less than 2%. To strengthen the rigidity of the side gearbox and weaken the rigidity of the side board properly may benefit the enhancement of the entirety strength. Thus , the equal-strength design principle can be approached.

REFERENCES

1. Liu, H.W. 1984. Mechanism of Materials (2nd volume). Higher Education Press, Beijng: 305-307.

2. Zhao, L.J. 1990. FEM Analyses of Mechanical Structure. Xian Jiao Tong University Press, Xian: 143-167, 207-222.

3. Ling, Z.H. and W.F. Chen. 1991. Mechanics of Materials (2nd volume). China National Science and Technology Share Ltd. Co., Taiwan: 242-248.

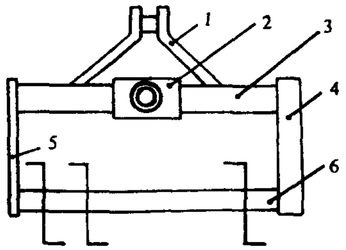

Fig. 1 Structural diagram of rotary tiller

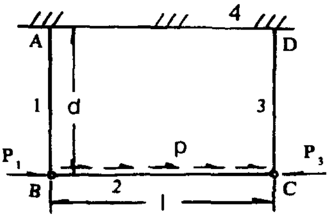

Fig. 2 System model under lateral loads

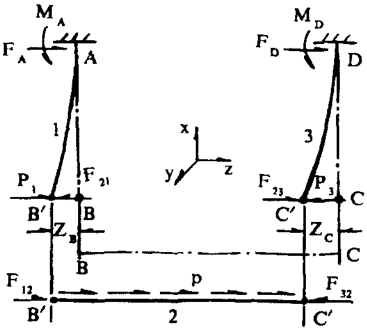

Fig. 3 System elements' analysis

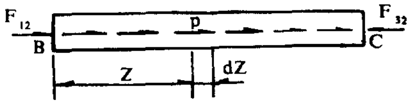

Fig. 6 Deformation model of 1G–175A cutter bar

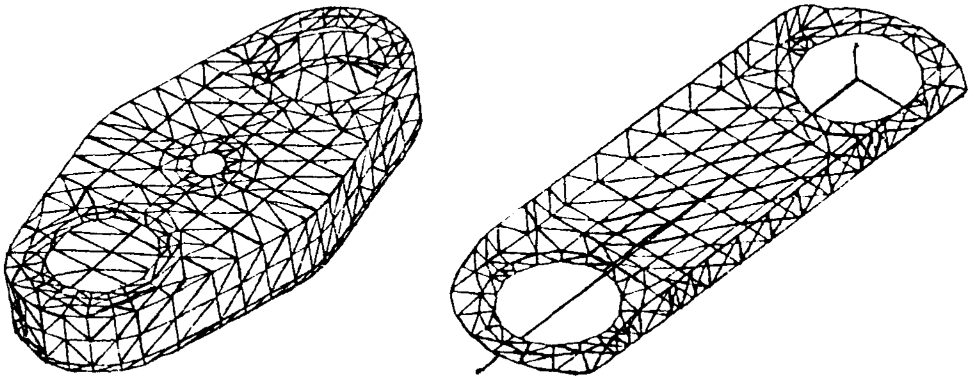

Fig. 4 FEM models of side gearbox and right side board

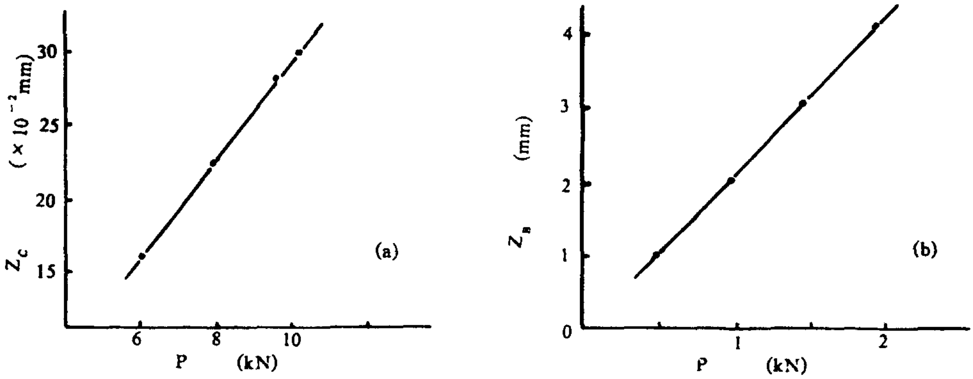

Fig. 5 Lateral deformation curves of 1G–175A side gearbox and right side board