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Feedback Control of Chaotic Systems

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Abstract

We study how to design conventional feedback controllers to drive chaotic trajectories of the well-known systems to their equilibrium points or any of their inherent periodic orbits. The well-known chaotic systems are Henon map and Duffing's equation, which are used as illustrative examples. The proposed feedback controller forces the chaotic trajectory to the stable manifold as OGY method does. Simulation results are presented to show the effectiveness of the proposed design method.

1. Introduction

Chaos is an ubiquitous and robust nonlinear phenomenon which permeates all fields of science. Roughly speaking, chaos is a more exotic form of steady-state response. A linear system cannot exhibit chaotic vibrations. It can be seen only in the deterministic nonlinear systems. When nonlinearity is present, there exists a wide range of parameters where the steady-state response is bounded, but not periodic. Instead, the response waveform becomes erratic with a broad continuous (rather than discrete as in the periodic case) frequency spectrum. Moreover, the response is so sensitivity to initial conditions that unless a computer of infinite word length is used in the simulation, no long-term prediction of the precise solution waveform is possible.

One of the inherent properties of the chaotic systems is so called the sensitivity to initial conditions(SIC). The two trajectories which are started from arbitrarily close initial conditions diverges exponentially. Lyapunov exponents are the measure of such an exponential diverging rates. So, they can serve as a criterion whether the system is chaotic or not. Another important property is the fractal structure. Many of the chaotic systems reveal the self-similarity. Hence, even the infinitesimal portion of the chaotic attractor has as much information as the whole system has.

It is often desired that chaos be avoided and/or that the system performance be improved or changed in some way. Thus, given a chaotic attractor we study how feedback controller can lead a chaotic trajectory to a desired attracting time-periodic motion and improved performance. Since the chaotic attractors are closures of the set of unstable orbits, stabilizing their equilibrium points and any of their inherent periodic orbits is interesting. If the stabilizing is easily achieved, one might use the chaotic system for multi-purposes. Therefore, the system having purposely built-in chaotic dynamics deserves the desired flexibility.

2. Previous Researches

In the past few years, there has been increasing interest in controlling the chaotic systems. E. Ott *et al.*[1] suggests an outstanding work called OGY method. Their method does not need any mathematical model. Introducing a carefully chosen small time-dependent perturbation of the acceptable parameter is all for the control of the system. Ute Dressler *et al.*[2] extend the range of applicability of the OGY method using time delay coordinates. Experimental control of chaotic system (a gravitationally buckled, amorphous magnetoelastic ribbon) is performed by W. L. Ditto *et al.* [3]. The numerical simulation of the kicked double rotor is the first attempt to control the chaotic system in engineering sense by F. J. Romeiras *et al.* [4]. But all of the previous researches are basically based on the OGY method, which shows the long time to achieve control. The average time to achieve control,

$\langle\tau\rangle$ is linearly decreasing with respect to the maximum perturbation p^* in log-log scale [1][4]. Averagingly, 10^3~10^4 order of iterations are needed to achieve control. This is the inherent ergodic property of the Chaotic attractor and the main drawback of the OGY method.

We propose the conventional feedback control method to control the chaotic systems. Our method has a disadvantage of requiring the mathematical model of the system but reveals fast convergence to the desired performance. The minimum time to achieve control and the minimal effort to control is a trade-off.

3. Numerical Examples

3.1 Henon Map

An extension of the quadratic map on the line to a map on the plane was proposed by the French astronomer Henon:

$$x_{n+1} = 1 + y_n - ax_n^2$$
$$y_{n+1} = bx_n \quad (1)$$

When $b = 0$, one obtains the logistic map studied by May and Feigenbaum. Values of a and b for which one will get a strange attractor include $a = 1.4$ and $b = 0.3$. This map is plotted on the $x-y$ plane with graph limits $-2 \le x \le 2$ and $-0.5 \le y \le 0.5$ as shown in figure 1. a). After obtaining the attractor, the graph is rescaled to focus on one small area of the attractor in which the fractal structure is shown in figure 1. b) and c). The reported Lyapunov exponent is $\lambda_1 = 0.2$ and the fractal dimension is $d_L = 1.264$.

3.2 Duffing's Equation

Duffing's equation is a nonlinear oscillator with a cubic stiffness term, to describe the hardening spring effect. A modified version of duffing's equation for a nonlinear inductor in an electrical circuit is also treated by Ueda. In this paper, we consider another modified Duffing's equation which is more general, studied by Moon and Holmes.

$$\dot{x} = y$$
$$\dot{y} = -px - x^3 - qy + r\cos(wt) \quad (2)$$

The magnitude of forcing term, r is the source of chaotic vibration. Some typical periodic and chaotic solutions are plotted on figure 2, where $p = 0.4$, $q = -1.1$, $w = 1.8$, and the values of r are cited therein.

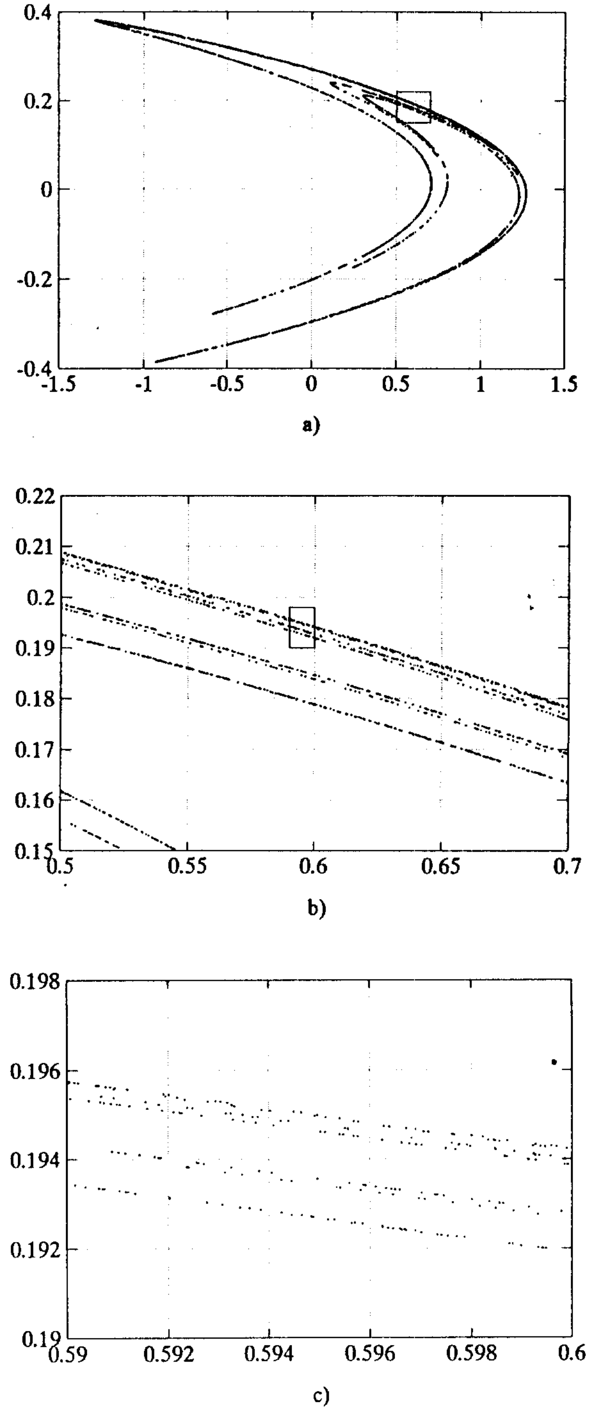


Figure 1. Illustration of Self-similarity of the structure of the Henon attractor at different scales : The small box on the figure is enlarged into the following figure. Note the change of scale in the axes.

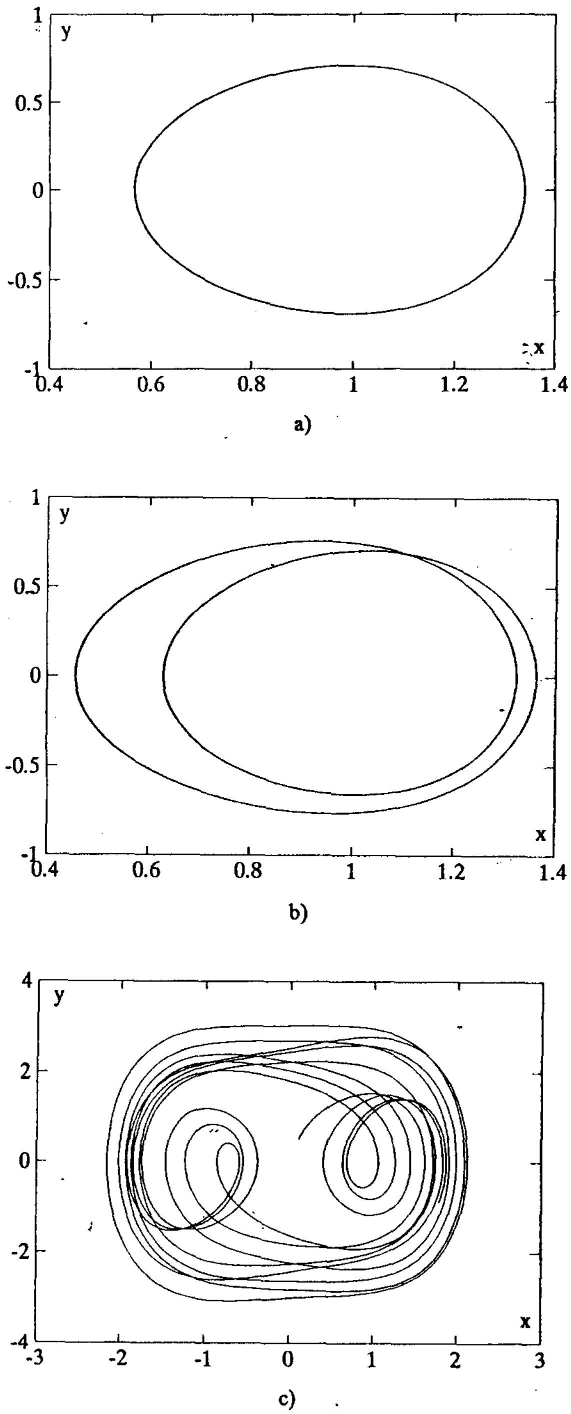


Figure 2. Some typical solutions of the Duffing's equation

a)	$r = 0.66$	: period - 1 solution
b)	$r = 0.71$	: period - 2 solution
c)	$r = r_c = 2.1$	: chaotic solution

4. Design of the Feedback Controller

4.1 Feedback control of the Henon map

We want to stabilize the Henon map on its equilibrium point. It has two equilibrium points, but the only one (x_e, y_e) will be considered, for near that point the whole system trajectory is wandering chaotically.

$$x_e = \frac{b-1+\sqrt{(b-1)^2+4a}}{2a} \qquad (3)$$
$$y_e = bx_e$$

After linearizing at the equilibrium point and adding control matrix following the canonical form, Henon map is represented as a linear system near that point as follows.

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = A \begin{bmatrix} x_n \\ y_n \end{bmatrix} + Bu_n \qquad (4)$$

where $A = \begin{bmatrix} -2ax_e & 1 \\ b & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $u_n = K \begin{bmatrix} x_n - x_e \\ y_n - y_e \end{bmatrix}$.

Since the linearized system is completely state controllable, the original Henon map is locally controllable near the equilibrium point.

To determine the feedback gain matrix K, Ackermann's formula can be used,

$$C = [B \quad AB],$$

$$W = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix},$$

$$T = CW,$$

$$K = [\alpha_2 - a_2 \quad \alpha_1 - a_1] T^{-1}$$

where C is the controllability matrix, a_i's are the coefficients of the characteristic polynomial of A and α_i's are the coefficients of the desired characteristic polynomial of $A + BK$.

Applying the Jury's stability test to the linearized system, the ranges for the elements, K_i $i = 1,2$ of the feedback matrix, K are found to be the following as shown in figure 3.

i) $|K_1 + 2ax_e K_2 + b| < 1$

ii) $K_2 < \left(1 - \frac{b}{1+2ax_e}\right) - \frac{1}{a+2ax_e} K_1$ (5)

iii) $K_2 < -\left(1 + \frac{b}{2ax_e - 1}\right) - \frac{1}{2ax_e - 1} K_1$

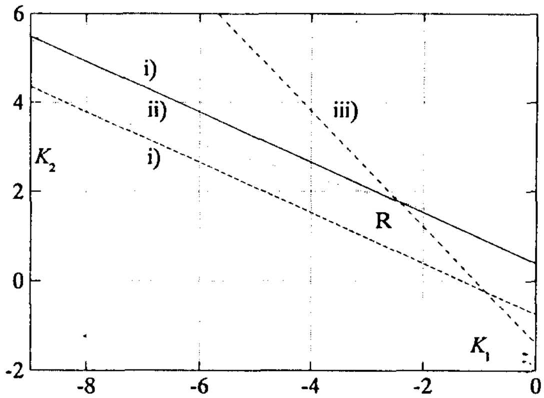


Figure 3. Stable region of the feedback gain (K_1, K_2) for the control of Henon map : Triangular region, R is the stable region. The curve i), ii) and iii) denote the conditions obtained from Jury's stability test (eq.(5)).

Since the above range is calculated form the linearized system, eq. (5) isn't the precise representation.

The state space trajectories of the controlled Henon map are plotted on figure 4. a) and c) where $K_1 = 1.2$, $K_2 = 0$ and $K_1 = 3$, $K_2 = -1.5$ are used respectively. The control of the higher order orbits are straightforward.

4.2 Feedback control of the Duffing's equation

Our objective in this section is to control the chaotic trajectories to the equilibrium point and any of their inherent periodic orbits. Let (x^*, y^*) be one of the inherent orbits, which acts as a reference input. Then, (x^*, y^*) is also the solution of the eq. (2). Subtracting eq. (2) with (x, y) being replaced by (x^*, y^*) results in

$$\begin{aligned} \Delta\dot{x} &= \Delta y \\ \Delta\dot{y} &= -p\Delta x - q\Delta y - (x^3 - x^{*3}) + (r_c - r^*)\cos(wt) + u(t) \end{aligned} \tag{6}$$

where $\Delta x = x - x^*$, $\Delta y = y - y^*$, r_c is the magnitude of forcing term that causing chaotic response, r^* is for the reference, and $u(t)$ is the nonlinear control input.

Based on eq. (5), we choose the control law, u(t) as follows.

$$u(t) = -K_x(x - x^*) - K_y(y - y^*) + 3x^2x^* - 3xx^{*2} - (r_c - r^*)\cos(wt) \tag{7}$$

Finally, the eq. (6) becomes

$$\begin{aligned} \Delta\dot{x} &= \Delta y \\ \Delta\dot{y} &= -(p + K_x)\Delta x - (q + K_y)\Delta y - \Delta x^3 \end{aligned} \tag{8}$$

Consider the Lyapunov Candidate

$$V = \frac{p + K_x}{2}\Delta x^2 + \frac{1}{4}\Delta x^4 + \frac{1}{2}\Delta y^2 \tag{9}$$

Differentiating (9) shows the negative semi-definiteness of V.

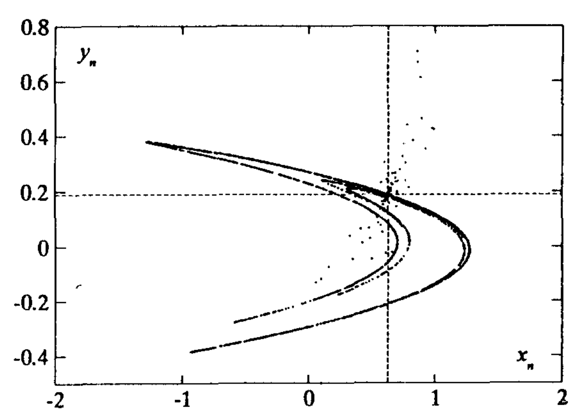
$$\begin{aligned} \dot{V} &= [(p + K_x)\Delta x + \Delta x^3]\Delta\dot{x} + \Delta y\Delta\dot{y} \\ &= [(p + K_x)\Delta x + \Delta x^3]\Delta y + \Delta y[-(p + K_x)\Delta x - (q + K_y)\Delta y - \Delta x^3] \\ &= -(q + K_y)\Delta y^2 \le 0 \end{aligned}$$

Equality holds if and only if $\Delta y \equiv 0$, so that the Lyapunov function, V is negative semi-definite. Applying the invariant set theorem, eq. (8) is asymptotically stable. The plot of controlled Duffing's equation is shown in figure 5.

5. Simulation Results

Figure 4. a) shows the result of the feedback control of Henon map with feedback gains, $K_1 = 1.2$ and $K_2 = 0$. No control is applied before $n \ge 3000$. If $n \ge 3000$ and the trajectory seems to falls near the equilibrium point(say, basin of attraction), then the control is activated. The chaotic trajectory is driven to the equilibrium point very quickly. If we use the OGY method, it will take 10^3~10^4 iterations that the chaotic trajectory settles down to its equilibrium point.

After the control is just applied, the trajectory seems to move away from the attractor. But it does not take a long time that the transient trajectory which looks like wandering about a straight line, $y_n \approx 2x_n$ finally goes to the equilibrium point. From eq. (4), the stable and unstable directions which can be easily calculated are $e_s = [0.4612 \quad 0.8878]^T$ and $e_u = [-0.9881 \quad 0.1541]^T$, respectively. The stable manifold



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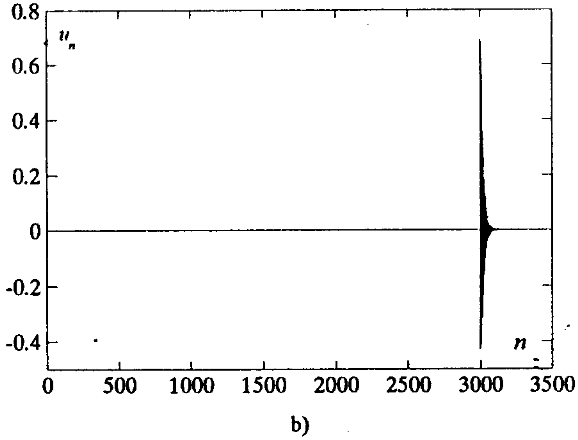
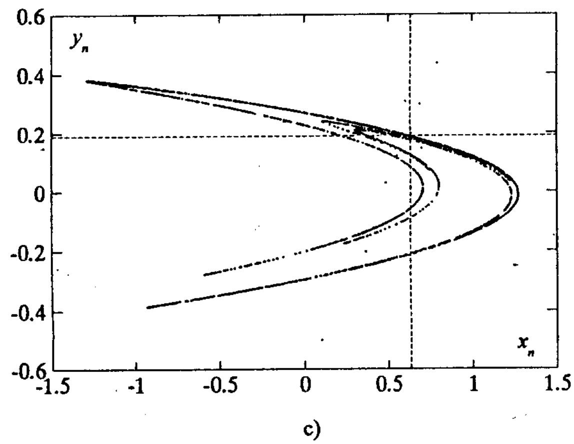




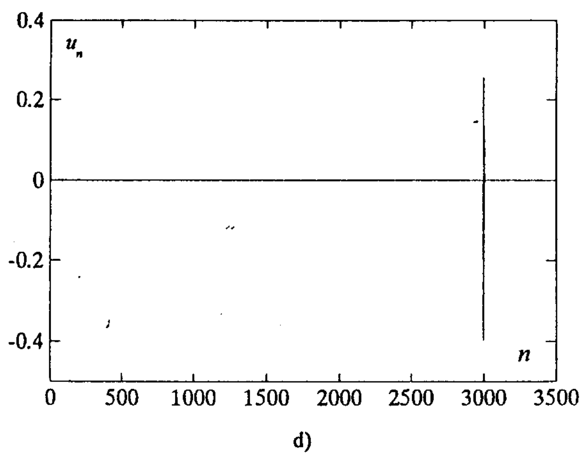


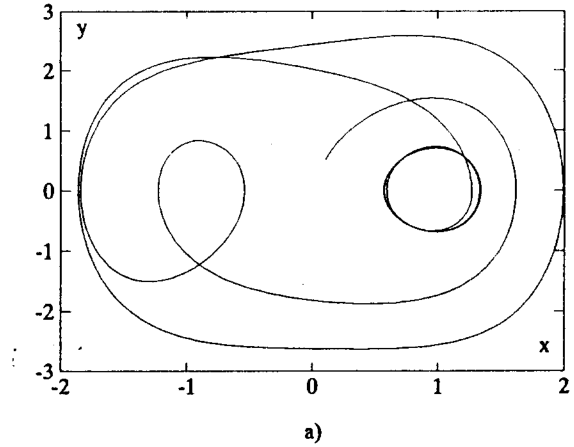
Figure 4. Controlled Henon map

a) $K_1 = 1.2$ and $K_2 = 0$: The cross point of the two dashed line is the equilibrium point.

b) $K_1 = 1.2$ and $K_2 = 0$: Control input, u_n

c) $K_1 = 3$ and $K_2 = -1.5$: Phase space trajectory

d) $K_1 = 3$ and $K_2 = -1.5$: Control input, u_n



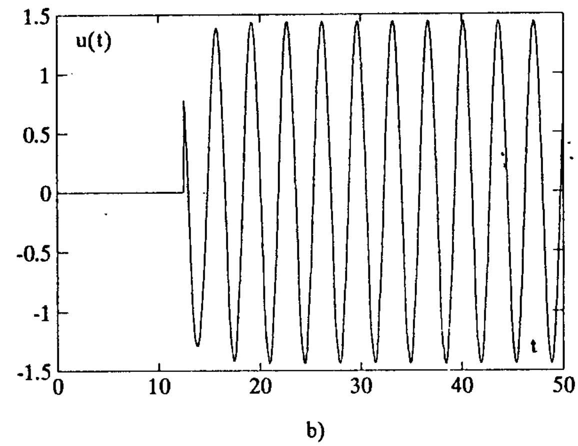


Figure 5. Controlled Duffing's Equation

a) The chaotic state space trajectory finally settles down to its inherent period - 1 orbit.

b) Control input, $u(t)$

near the equilibrium point lies on the following direction.

$$\theta_s = \tan^{-1}\left(\frac{0.8878}{0.4612}\right) = 62.55^{\circ}$$

$$\text{and} \qquad \frac{0.8878}{0.4612} = 1.925 \approx 2$$

Hence, it can be understood from the above fact that the main control effort of the proposed feedback controller is to force the next iterate onto the stable manifold of the equilibrium point as the OGY method does.

Comparing the figure 4. a) and c), the transient iterates are considerably reduced as the magnitude of feedback gains are increased.

The simulation results of Duffing's equation are plotted in figure 5. Total simulation time is 50 second, while control is started right after a quarter of that time is past. The everlasting oscillatory feature of control input is in order to compensate the exciting term, $r\cos(wt)$ in eq. (2). As shown in the Henon system, it also reveals fast convergence characteristic to the desired performance. Stabilizing more high periodic orbits is just a tedious work.

6. Conclusions and Future Works

We propose the feedback controller to control the chaotic system to its equilibrium point or any of its inherent periodic orbits. The main control effort of the proposed method for Henon map is to force the next iterate onto the stable manifold of the equilibrium point. For a continuous-time system, nonlinear feedback controller is used. The linear feedback controller is a more useful but more difficult one to obtain. So, it is left to the future work.

When compared with the OGY method, fast convergence to a desired performance is another advantage of our method while the control effort is not so much. But we have assumed that a mathematical model of the system is available. Thus, to overcome the model imperfection and the lack of model equation will be an interesting research issue.

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