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Controller Design of Variable Structure System with an Integral-Augmented Sliding Surface for Uncertain MIMO Systems

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Abstract

In this paper, an variable structure system with an integral-augmented sliding surface is designed for the improved robust control of a uncertain multi-input multi-output(MIMO) system subject to the persistent disturbances. To effectively remove the reaching phase problems, the intergal augmented sliding surface is defined, then for its design, the eigenstructure assignment technique is introduced. To guarantee the designed performance against the persistent disturbance, the stabilizing control for multi-input system is also designed. The stability of the global system and performance robustness are investigated. The example will be given for showing the usefulness of algorithm.

1. Introduction

The *Variable Structure System*(VSS) can provide the effective means for controlling an uncertain dynamical system. The most distinct feature of the variable structure system is the presence of the sliding mode on the predetermined sliding surface[1-3]. The design stage of the the multi-input VSS are as follows: First, the sliding mode is designed to have some prescribed properties. Next, it is assufed that the sliding mode can exist at any point of the intersection $S = 0$ of the sub-sliding manifold. Finally, it is guaranteed that the representative point of the system reaches a sliding surface in finite time because of the stability in the reaching phase[3]. By proper design of the sliding surface, the desired output dynamics can be obtained. Several design methods including the eigenstructure assignment, geometric approach, and the optimal technique[5-8,12-14] are suggested, and well summarized in [15]. All these methods yield a linear dynamics in the sliding surface. Moreover, the nonlinear dynamics can be assigned for better transient dynamics than that of only linear methods[2,9], and the integral action can be augmented to improve the steady state performance[12-14], and called as an *integral VSS*(IVSS).

Unfortunately, since the sliding surfaces used in the previous VSS's are fixed, naturally, the reaching phase exists for the initial condition far from the sliding surface[4]. The reaching phase is defined as the trajectory from a given initial state to the first touching to the intersection of each m-sliding manifold, in which the robustness to the parameter variations and disturbance can not be guaranteed[4]. And it is difficult to find the designed performance of the sliding surface in the output. This problem is compounded, when the *hierarchical control* methodology[1] is applied[4]. Moreover, introducing the integrator without removing the reaching phase can inevitably results in the overshoot problem because the integral should be re-regulated to zero in steady state[12-14]. Thus no overshoot which is one advantage of the VSS is sacrificed. To slide from a given initial condition without any reaching phase, the sliding surface basically should be a function of the initial condition explicitly or implicitly.

Few researches deal with the problems of the *reaching phase* compared to the established works on the VSS. Only partial or restricted results on this subjects have been obtained[2,10,11]. The alleviation of these problems is the use of the high-gain feedback to reduce the reaching time[10]. This has the drawbacks related to the high-gain feedback sensitive to the unmodelled dynamics and actuator saturation[6]. In [2], Itkis proposes the adaptive changes of the sliding surface to reduce the reaching problem. This method is effectively improved by [11] for only a second order system, but the initial condition is limited to some degree in state space.

In this paper, an variable structure system with an integral-augmented sliding surface for the improved robust control of a multi-input multi-output(MIMO) systems. The reaching phase problems are isolated and the results from removing them are concentrated on, specially, the performance robustness in presence of the persistent disturbances. By the simulation studies, the effectiveness of the algorithm is compared with that of the VSS's with the linear sliding surface and previous integral-augemented one in [12].

2. Variable Structure System with an Integral-Augmented Sliding Surface

2.1 System Descriptions and Basic Backgrounds

The problem of the designing the VSS controller is considered for a multivariable system:

$$\dot{Y}(t) = (A + \Delta A(Y,t)) \cdot Y(t) + (B + \Delta B(Y,t)) \cdot U(Y,t) + D(Y,t) \quad Y^0 = Y(0) \qquad (1)$$

where $Y \in \Re^n$, $U \in \Re^m$, and $rank(B) = m$. The primary design goal of the VSS controller is to asymptotically stabilize this uncertain system with quality of guaranteeing the prescribed performance designed for the nominal of (1a). For simple formulations, the nonsigular coordinate transformation, T is introduced as[5]

$$X(t) = [X_1^T \; X_2^T]^T = T(Y,t) \cdot Y(t) \qquad (2)$$

such that $T(X,t)\cdot B(Y,t)=[\mathrm{O}\; B_2(X,t)^T]^T$

where $X_1\in\Re^{n-m}$ and $X_2\in\Re^m$ are the partitions of $X\in\Re^n$. By (2) the dynamics of (1) can be represented in *regular form*[4,5] in X space as

$$\dot{X}_1(t)=A_{11}\cdot X_1(t)+A_{12}\cdot X_2(t),\qquad X_1^0 \qquad (3a)$$

$$\dot{X}_2(t)=(A_{21}+\Delta A'_{21}(t))\cdot X_1(t)+(A_{22}+\Delta A'_{22}(t))\cdot X_2(t)$$
$$+(B_2+\Delta B'_2(t))\cdot U(t)+D'_2(X,t),\quad X_2^0 \qquad (3b)$$

where $rank(B_2)=m$; $A_{11}\in\Re^{(n-m)\times(n-m)}, A_{12}\in\Re^{(n-m)\times m}, A_{21}\in\Re^{m\times(n-m)}$, and $A_{22}\in\Re^{m\times m}$ are known constant matrices; and $X_1^0\,\&\,X_2^0$ are the initial conditions transformed from Y^0. It is well-known that the VSS design will exhibit the strong invariant property with respect to disturbance vector if only if the matching restriction is satisfied[1,2]. Thus, Assumption 1 is introduced as

Assumption 1:(*Matching Condition*)

The uncertainties $\Delta A(\cdot,\cdot)$, $\Delta B(\cdot,\cdot)$ and disturbance $D(\cdot,\cdot)$ satisfy the matching condition and are bounded as the following

$$\Delta A'_{21}=B_2\cdot\Delta A_{21},\quad |\Delta A_{21_{ik}}(t)|<\alpha_{21_{ik}} \qquad (4a)$$

$$\Delta A'_{22}=B_2\cdot\Delta A_{22},\quad |\Delta A_{22_{ij}}(t)|<\alpha_{22_{ij}} \qquad (4b)$$

$$\Delta B'_2=B_2\cdot\Delta B_2,\quad |\Delta B_{2_{ii}}(t)|<\beta_{2_i}<1(diagonal) \qquad (4c)$$

$$\Delta D'_2=B_2\cdot\Delta D_2,\quad |D_{2_{ij}}(t)|<\gamma_{2_{ij}} \qquad (4d)$$

for $i,j=1,2,\ldots,m,\ \&\ k=1,2,\ldots,(n-m)$.

The nominal system of (3) can be described as

$$\dot{\bar{X}}=\Lambda\cdot\bar{X}+\Gamma\cdot v(t) \qquad (5)$$

where

$$\Lambda=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},\qquad \Gamma=\begin{bmatrix}0\\ B_2\end{bmatrix}.$$

which will be used in the output performance design. Using (4) and (5), the system of (3) can be rewritten in neat form as

$$\dot{\bar{X}}=\Lambda\cdot\bar{X}+\Gamma\cdot[U(t)+E(X,t)] \qquad (6a)$$

where $E(X,t)$ signifies the *lumped uncertainties* or *persistent disturbances* as

$$E(X,t)=\Delta A_{21}\cdot X_1+\Delta A_{22}\cdot X_2+\Delta B_2\cdot U(t)+\Delta D(t) \qquad (6b)$$

For (6), the VSS controller will be designed by two stages, i.e., the design of the sliding manifold and the switching control design.

For the use later, the integral terms, $X_0\in\Re^r, r=<n$ are augmented to the system (6) as

$$\dot{X}_0(t)=A_0\cdot X(t)=A_{01}\cdot X_1(t)+A_{02}\cdot X_2(t),\qquad X_0^0 \qquad (7)$$

where X_0^0 and $A_0\equiv[A_{01}:A_{02}]\in\Re^{r\times n}$ are its initial condition and the coefficient matrix for matching of the dimension, respectively, both will be determined in the sliding surface design.

To obtain the design goal, the reaching phase problems are to be solved, as the demerits of the conventional VSS's. The reason for the existence of the reaching phase will be reviewed for the conventional sliding manifold $S:\Re^n\to\Re^m$ composed of the set of the sub-manifold as[1,2]

$$S(X)\equiv C^T\cdot X=C_1\cdot X_1+C_2\cdot X_2,\qquad (=\mathrm{O}) \qquad (8)$$

where $rank(C_2)=m$. Since the sliding surface of (8) defines geometrically only the fixed states satisfying $S(\cdot)=\mathrm{O}$ for a given $X^0\notin S(\cdot)\neq\mathrm{O}$, the reaching phase exists. During this phase the robustness is not guaranteed, thus the designed performance is not preserved. To perfectly remove the reaching phase, it is required that (i) the sliding surface should be defined from a given initial condition, and (ii) the control input should be enable to establish the sliding mode at every point on the sliding mode.

2.2 Integral-Augmented Sliding Surface

To get rid of the reaching problems, as the first stage, an integral-augmented sliding manifold, $S(\cdot):\Re^{n+r}\to\Re^m$ is proposed for (7) by composing as

$$S(\hat{X})\equiv S_L(\hat{X})+S_I(\hat{X})\quad(=\mathrm{O}) \qquad (9a)$$

where $S_L(\cdot):\Re^n\to\Re^m, S_I(\cdot):\Re^r\to\Re^m$ are the modified conventional linear and intentionally integral-augmented terms, respectively as

$$S_L(\hat{X})=C_L^T\cdot\hat{X}=C_{L1}\cdot\hat{X}_1+C_{L2}\cdot\hat{X}_2=\sum_{i=1}^{n}c_{Li}\cdot\hat{x}_i \qquad (9b)$$

$$S_I(\hat{X})=C_I^T\cdot\hat{X}_0=C_{I1}\cdot X_{01}+C_{I2}\cdot X_{02}=\sum_{i=1}^{r}c_{Ii}\cdot\hat{x}_{0i} \qquad (9c)$$

$$\hat{x}_i=x_i-x_i^0,\qquad for\quad i=1,2,\ldots,n$$

$$\hat{x}_{0i}=\int_0^t x_i(\tau)d\tau-x_{0i}^0,\qquad for\quad i=1,2,\ldots,n$$

$$C_i=constant,\qquad for\quad i=L\,\&\,I.$$

This sliding surface geometrically defines all the states satisfying

$$X(t)\in\{X(t)\in\Re^n\,|\,S(X)=\mathrm{O}\text{ and }\dot{S}(X)=\mathrm{O}\}. \qquad (10)$$

Obviously $S(X^0)=0$ for any initial $X^0\in\Re^n$, so the sliding surface of (10) is defined from any given initial $X^0\in\Re^n$. Thus, one requirement to remove the reaching phase is satisfied. The full reduced-order ideal sliding mode dynamics(ISMD) with n-order of (9) can be obtained as

$$\dot{X}_1=A_{11}\cdot X_1(t)+A_{12}\cdot X_2(t) \qquad (11a)$$

$$\dot{X}_2=-C_{L2}^{-1}[(C_{L1}A_{11}+C_{I1})\cdot X_1(t)+(C_{L1}A_{12}+C_{I2})\cdot X_2(t)] \qquad (11b)$$

which is the dynamic interpretation of (9a). Thus the solution of (11) for a given X^0 generates the integral manifold coinciding with (9), and the stability of (11) is equal to that of the sliding surface itself. Therefore, the stable design for the sliding surface will be carried out in order to yield the desirable performances using the eigenstrucuture assignment to (11). Manipulating (11), it leads to

$$\dot{X}(t)=\Lambda\cdot X(t)+\Gamma\cdot v(t),\qquad X^0 \qquad (12a)$$

where $v\in\Re^m$

$$v=-K_1\cdot X_1(t)-K_2\cdot X_2(t) \qquad (12b)$$

and

$$K_1=B_2^{-1}[A_{21}+C_{L2}^{-1}(C_{L1}A_{11}+C_{I1})] \qquad (12c)$$

$$K_2=B_2^{-1}[A_{22}+C_{L2}^{-1}(C_{L1}A_{12}+C_{I2})]. \qquad (12d)$$

Since, as can been in (12a), it equals to the nominal dynamics of (5), the design of the sliding surface is the performance design to the nominal systems of (3), and the reverse argument also holds.

To determine the feedback gains in (12), i.e., the coefficient of (9), it is assumed that the desired closed-loop poles and those right eigenvectors are given, the procedures of the eigenstructure assignment are as follows[16]:

1) Compute the maximal rank matrix

$$N_i=[N_{1i}^T\,N_{2i}^T]^T,\qquad S_i=[S_{1i}^T\,S_{2i}^T]^T \qquad (13)$$

for $i=1,\ldots,s$ satisfying the following relation:

$$[\Lambda-\lambda_i I_n,\,\Gamma]\cdot N_i=\mathrm{O}$$

where $N_i\in C^{(n+m)\times m},\qquad S_i\in C^{(n+m)\times n},\quad\&\quad \sum_i^s d_i=n.$

2) Form the generalized right eigenvectors for $i = 1,\ldots,s$ as follows:

$$v_{ij} = S_{1i}v_{ij-1} + N_{1i}p_{ij}, \qquad j = 1,\ldots,d_i \tag{14}$$

where $v_{i0} = 0$ and $pij\ (i = 1,\ldots,s; j = 1,\ldots,d_i)$ are selected to satisfy such that V is full rank and $\lambda_i = \bar{\lambda}_k$ implies $v_{ij} = \bar{v}_{kj}$ where

$$[\Lambda - \lambda_i I_n, \Gamma][v_{ij}^T\ w_{ij}^T]^T = v_{ij-1}. \tag{15}$$

3) Calculate vector chains as follows:

$$w_{ij} = s_{2i}v_{ij-1} + N_{2i}p_{ij} \tag{16}$$

for $i = 1,\ldots,p; j = 1,\ldots,d_i$.

4) Calculate feedback gain

$$K = [K_1\ K_2] = WV^{-1}. \tag{17}$$

For the more detailed procedures, one may refer to [16]. Then, the coefficients of the sliding surface (9) can be obtained using (12c-d) and (17) as

$$C_{L2}^{-1}(C_{L1}A_{11} + C_{I1}) = B_2K_1 - A_{21} \tag{17a}$$

$$C_{L2}^{-1}(C_{L1}A_{12} + C_{I2}) = B_2K_2 - A_{22} \tag{17b}$$

This may be not unique, but gives the practical design guideline with much degree of freedom. The choice of C_{I1} and C_{I2} determines the necessary order of the integration and A_0. Specially, if $C_{L2} = I_m$ without loss of generality, then

$$C_{L1}A_{11} + C_{I1} = B_2K_1 - A_{21} \tag{18a}$$

$$C_{L1}A_{12} + C_{I2} = B_2K_2 - A_{22} \tag{18b}$$

Finally, to complete the design of the sliding surface, the initial value of the integral action is found as

$$X_I^0 = -C_I^- \cdot (C_1 \cdot X_1^0 + C_2X_2^0) \tag{19}$$

where C_0^- is the reft-pseudo inverse of C_I as $C_I^- = [C_I^TC_I]^{-1}C_I^T$. Thus, the integral states converge zero from the this finite values of (19) and its rate convergence depends the relationship between C_is for a given x^o.

2.3 Stabilizing Control

As the second phase, the control input will be designed. For the reason that it is difficult for the control to directly establish the sliding mode on the pre-determined sliding surface, the sliding surface is transformed to s^* space by $H_s(X,t) = [C_{L2}B_2]^{-1}$

$$S^*(\hat{X}) = [C_{L2}B_2]^{-1} \cdot S(\hat{X}) \tag{20}$$

based on Theorem in [1] as

Theorem 2: The equation of the sliding mode is invariant with respect to the nonlinear transformations

$$S^*(X) = H_s(X,t) \cdot S(X), \quad U^*(X) = H_u(X,t) \cdot U(X)$$

$$\text{for} \quad detH_s \neq 0\ \&\ detH_u \neq 0 \tag{21}$$

where '*det*' denote determinate of a matrix.

Proof: See [1].

This theorem means that the sliding mode equation is governed by the original (10) if the components of the controlled vector undergo discontinuity on the new surface $s^*(\hat{X}) = 0$ or the components of the new control vector $U^*(\cdot)$ undergo discontinuity on the already chosen surface $S(\hat{X}) = 0$, that is $s^*(\hat{X}) = 0 \Leftrightarrow S(\hat{X}) = 0$. Thus the performances designed in (10) can be guaranteed by the sliding mode on the new surface $s^*(\hat{X}) = 0$. To generate the sliding mode on $s^*(\hat{X})$, the following class of feedback control is employed as

$$U(X_1,X_2) = \nu_{eq}(X_1,X_2) + \Delta\nu(X_1,X_2) \tag{22a}$$

where ν_{eq} is the equivalent control for the nominal system of (1) determined according to the design of the sliding manifold as

$$\nu_{eq}(X_1,X_2) = -[(C_{L1}A_{11} + C_{L2}A_{21} + C_{L1}) \cdot X_1 + (C_{L1}A_{12} + C_{L2}A_{22} + C_{L2}) \cdot X_2] \tag{22b}$$

which governs the desired main sliding dynamics for (10), and $\Delta\nu$ cancels out the uncertainties and external disturbances to maintain the sliding mode on pre-specified manifold from x^o

$$\Delta\nu(X_1,X_2) = -[\Psi_0 \cdot X_0 + \Psi_1 \cdot X_1 + \Psi_2 \cdot X_2 + \delta \cdot sgn(S^*) + \kappa \cdot S^*] \tag{22c}$$

and the switched gain matrices $\Psi_0 \in \Re^{m\times r}, \Psi_1 \in \Re^{m\times(n-m)}, \Psi_2 \in \Re^{m\times m}$, and $\delta\ \&\ \kappa \in dia[\Re^{m\times m}]$ can be selected by the inequalities as follows:

$$\Psi_{0ih} \begin{cases} > 0 & for\ \left(s_i^* \cdot x_{0_h}\right) > 0 \\ < 0 & for\ \left(s_i^* \cdot x_{0_h}\right) < 0 \end{cases}$$

$$\Psi_{1ik} \begin{cases} > (\alpha_{21ik} + \beta_{2i} \cdot K_{1ik})/(1-\beta_{2i}) & for\ (s_i^* \cdot X_{1k}) > 0 \\ < -(\alpha_{21ik} + \beta_{2i} \cdot K_{1ik})/(1-\beta_{2i}) & for\ (s_i^* \cdot x_{1k}) < 0 \end{cases}$$

$$\Psi_{2ij} \begin{cases} > (\alpha_{22ij} + \beta_{2i} \cdot K_{1ij})/(1-\beta_{2i}) & for\ (s_i^* \cdot x_{2j}) > 0 \\ < -(\alpha_{22ij} + \beta_{2i} \cdot K_{1ij})/(1-\beta_{2i}) & for\ (s_i^* \cdot x_{0j}) < 2 \end{cases}$$

$$\delta_i \begin{cases} > \left(\gamma_{2i}\right)/(1-\beta_{2i}) & for\ (s_i^*) > 0 \\ < -\left(\gamma_{2i}\right)/(1-\beta_{2i}) & for\ (s_i^*) < 0 \end{cases} \tag{22d}$$

$$\kappa_i > 0.$$

for $h = 1,\ldots,r,\ \ i,j = 1,2,\ldots,m,\ \ k = 1,2,\ldots,(n-m),\ \&\ l = 1,2,\ldots,n.$
For this control, the existence of the sliding mode on every point of $s^*(\hat{X}) = 0$ and stability will be investigated in next Theorem.

Theorem 3: The closed loop system, (1) with (42), is totally asymptotically stable with respect to $S(\hat{X}) = 0$, eventually to the origin of $(2n+r)$-th order state space provided that $s^*(\hat{X}) = 0$ is asymptotically stable.

Proof: Take Lyapunov candidate function as

$$V = 1/2S^{*T} \cdot S^*. \tag{23}$$

From (3) and (40c), the derivative of $s^*(X)$ becomes

$$\dot{S}^*(t) = (C_{L1}A_{11} + C_{L2}A_{21} + C_{I1}) \cdot X_1 + (C_{L1}A_{12} + C_{L2}A_{22} + C_{I1}) \cdot X_2 + \nu_{eq}(X_1,X_2) + \{\Delta A_{21} \cdot X_1 + \Delta A_{22} \cdot X_2 + D_2(X,t) - \Delta B_2(\nu_{eq}(X_1,X_2) + \Delta\nu(X_1,X_2))\}. \tag{24}$$

Rearranging, it follows

$$\begin{aligned} \dot{S}^*(t) = &-\Psi_0 \cdot X_0 \\ &-[\Delta B_2(C_{L2}B_2)^{-1}(C_{L1}A_{11} + C_{L2}A_{21} + C_{I1}) - \Delta A_{21} + (I_m - \Delta B_2) \cdot \Psi_1] \cdot X_1 \\ &-[\Delta B_2(C_{L2}B_2)^{-1}(C_{L1}A_{12} + C_{L2}A_{22} + C_{I2}) - \Delta A_{22} + (I_m - \Delta B_2) \cdot \Psi_2] \cdot X_2 \\ &-[\delta sgn(S^*) - D_2(X,t)] \\ &-\kappa \cdot S^*. \end{aligned} \tag{25}$$

Finally, using (22d), the following equation can be derived

$$s_i^* \cdot \dot{s}_i^* < -\kappa_i \cdot s_i^{*2}, \qquad i = 1,2,\ldots,m \tag{26}$$

which implies that the proposed algorithm can guarantee the sliding mode at the every point on the new sliding surface $s^*(\hat{X}) = 0$. Therefore, based on Theorem 2, the motion equations in the sliding mode on the proposed sliding surface is invariant, and the controlled system is asymptotically stable to $s^*(\hat{X})=0$ naturally including the origin.

By Theorem 3, the reaching phase can be removed so that the designed performance for the nominal system of (10) is guaranteed, and the reachability of the controlled system does not need to be considered.

The original control $U(Y)$ for (1) can be found from (22) as

$$U_Y(Y)=U_X(TY). \tag{27}$$

Fig.1 shows the overall diagram of the algorithms, which can gives rises to the original design goal by the sliding mode on $s^*(\hat{X})=0$ with the predetermined performances in $S(\hat{X})=0$ without any reaching phase problems, whereas the conventional multivariable VSS and IVSS suggested by Chern and Wu[12] do not give the performance robustness due to the reaching phase problems.

To show the effectiveness of the algorithm, an example will be presented.

3. Illustrative Example

The control of an uncertain MIMO system is presented for the purpose of performance comparison between the VSS with a linear sliding surface, integral augmented VSS of Chern and Wu[12] and the proposed algorithms using the following 4-Th. order plant as

$$\dot{Y}=\begin{bmatrix}0 & 1 & 0 & 0\\ -a_{11}(t) & 0 & -a_{12}(t) & 0\\ 0 & 0 & 0 & 1\\ -a_{21}(t) & 0 & -a_{22}(t) & 0\end{bmatrix}\cdot Y+\begin{bmatrix}0 & 0\\ b_1(t) & 0\\ 0 & 0\\ 0 & b_2(t)\end{bmatrix}\cdot U$$

$$+[0 \quad h_1(t) \quad 0 \quad h_2(t)]^T,\quad Y^0=[4 \quad 0 \quad 2 \quad 0]^T \tag{28}$$

where the system parameters $a_{ij}(t)$, gains $b_i(t)$, and disturbances $h_i(t)$ of the plant are assumed such that

$$a_{ij}(t)=-1+\Delta a_{ij}(t),\quad -0.5<\Delta a_{ij}(t)<0.5$$

$$b_i(t)=1+\Delta b_i(t),\quad -0.5<b_i(t)<0.5$$

$$|h_i(t)|<4. \tag{29}$$

By the simple transformation, the system (28) can be transformed to

$$\dot{X}_1=X_2,\qquad X_1^0=[4\ 2]^T \tag{30a}$$

$$\dot{X}_2=A_{21}^0\cdot X_1+B_2^0\cdot[U+E(X,t)],\qquad X_2^0=O \tag{30b}$$

where $X_1=[x_1 x_2]^T$, $X_2=[x_3 x_4]^T$, A_{21}^0 and B_2^0

$$A_{21}^0=\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix},\quad \& \quad B_2^0=I_2 \tag{30c}$$

Letting $E(X,t)=0$, (30) becomes its nominal system. For (30), the three algorithms will be designed. To design the proposed algorithms, first, the proposed sliding surface is designed to have the double poles at each -3 and-4 with corresponding right eigenvectors as

$$[0.3162 \quad 0 \quad -0.9371 \quad 0]^T,\quad [0 \quad 0.2425 \quad 0 \quad -0.9701]^T \tag{31}$$

using the eigenstructure assignment to the nominal system of (30). By the computation algorithm, (14)-(17), the feedback gain of (12b) is founded as follows:

$$K=[K_1\ K_2]=\begin{bmatrix}8.0 & -1.0 & 6.0 & 0.0\\ -1 & 15.0 & 0.0 & 8\end{bmatrix}. \tag{32}$$

Then, using (18a) and (18b), the coefficients of the sliding surface can be determined. Eventually, the integral augmented sliding surface has the form of

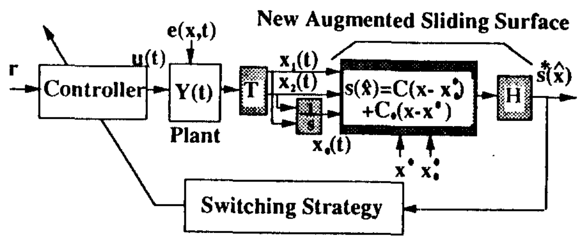


Fig. 1 Overall block diagram of the proposed lagorithm

$$S_p(t)=\begin{bmatrix}6 & 0\\ 0 & 8\end{bmatrix}\cdot\left(X_1(t)-\begin{bmatrix}4\\ 2\end{bmatrix}\right)+X_2(t)+\begin{bmatrix}9 & 0\\ 0 & 16\end{bmatrix}\cdot\left(X_{10}(t)-\begin{bmatrix}2.556\\ 1\end{bmatrix}\right). \tag{33}$$

Consequently, the necessary integral action with the initial condition is augmented as follows:

$$X_{I10}=\int_0^t X_1(\tau)d\tau,\qquad X_{I10}^0=[2.556 \quad 1]^T \tag{34}$$

On the other hand, the linear sliding surface is defined as

$$S_L(x)=\begin{bmatrix}2.3 & 0\\ 0 & 2.5\end{bmatrix}\cdot X_1+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\cdot X_2 \tag{34}$$

designed to have the simple pole at -2.3 and -2.5 of its sliding dynamics, and the sliding surface proposed by Chern & Wu becomes

$$S_C(t)=\begin{bmatrix}1 & 0\\ 0 & 1.2\end{bmatrix}\cdot X_0(t)+\begin{bmatrix}2 & 0\\ 0 & 2.3\end{bmatrix}\cdot X_1(t)+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\cdot X_2(t) \tag{35}$$

also designed to locate the poles of sliding dynamics at -1 and 1.15+j0.835. As a result of the selection of the proposed sliding surface, its ISMD becomes

$$\dot{X}_1=X_2,\qquad X_1^0=[4\ 2]^T \tag{36a}$$

$$\dot{X}_2=-\begin{bmatrix}9 & 0\\ 0 & 16\end{bmatrix}\cdot X_1(t)-\begin{bmatrix}6 & 0\\ 0 & 8\end{bmatrix}\cdot X_2(t),\quad X_2^0=O,\quad t\geq 0. \tag{36b}$$

As the second design stage, fortunately $[C_2B_2]=I_2$, the control input for the suggested sliding surface becomes

$$U_p=v_{eq,p}+\Delta v_p \tag{37a}$$

where $U_{eq,p}$, called equivalent control[1-3] of (33)

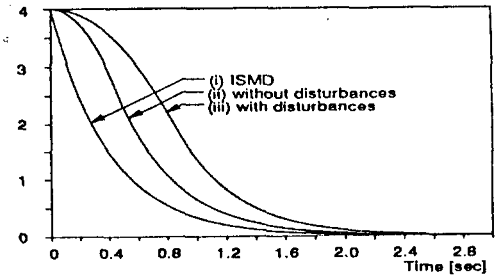
$$v_{eq,p}=-\left\{\begin{bmatrix}8 & 1\\ 1 & 15\end{bmatrix}\cdot X_1+\begin{bmatrix}6 & 0\\ 0 & 8\end{bmatrix}\cdot X_2\right\} \tag{37b}$$

which is determined in previous design stage for the sliding surface and the discontinuous control term is

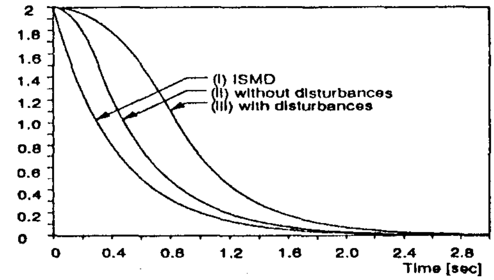
$$\Delta v_p=-\Psi^0\cdot X_0-\Psi^1\cdot X_1-\Psi^2\cdot X_2-\delta\cdot sgn(S_p) \tag{37c}$$

where Ψ^0, Ψ^1, and $\Psi^2\in R^{2\times 2}$ are the switching gain matrices selected by (22d) as in Table 1. For the other algorithms, the switching gain matrices are are also summarized in Table 1.

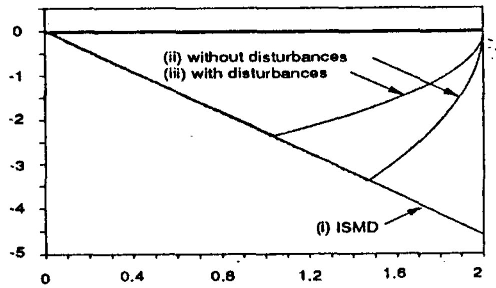
All the simulations are carried out for the $2[msec]$ sampling and on the conditions without or with the persistent disturbances for (30b). The results of the simulations for the three schemes are shown in Fig. 2 through Fig. 4. Fig.2 shows the results by the conventional VSS with the linear sliding surface. Fig. 3 shows the results of the IVSS by Chern and Wu. And for the proposed algorithm, the results are shown in Fig. 4. In each figure, (a) shows the three outputs of x_1 for the ISMD defined by the each sliding surface(i), for without disturbances(ii), and for with disturbances(ii). The outputs of x_2 for the three cases are depicted in each (b). Each (c) and (d) show the phase trajectories of x_1 and x_2, respectively for the



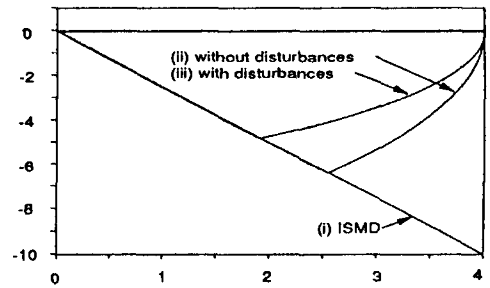
(a) outputs, x_1



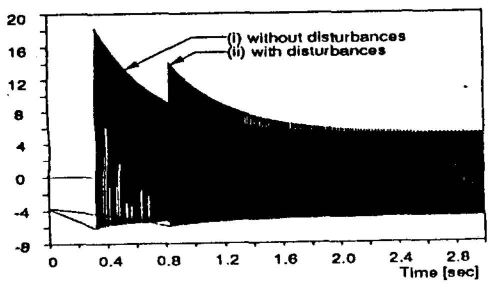
(b) outputs, x_2



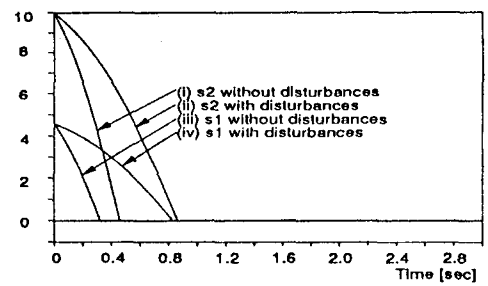
(c) phase trajectories of x_1



(d) phase trajectories of x_2

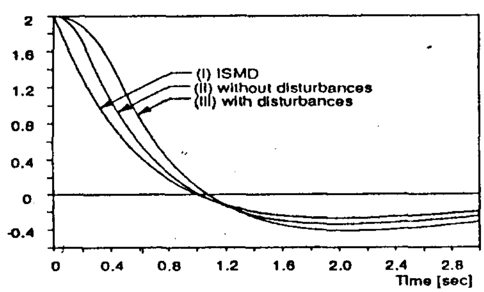


(e) control inputs, U_1

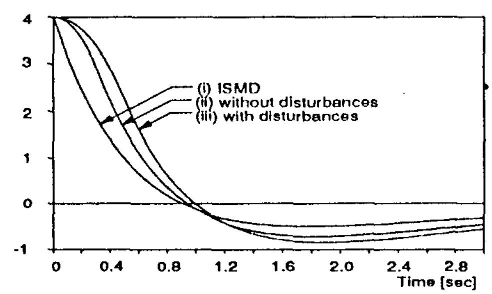


(f) sliding surface time trajectories

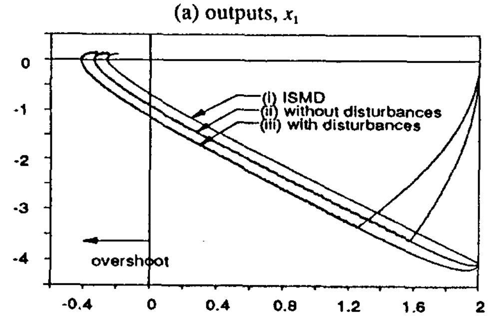
Fig. 2 Results of the conventional VSS with a linear sliding surface



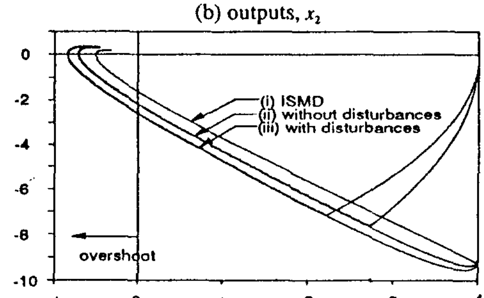
(a) outputs, x_1



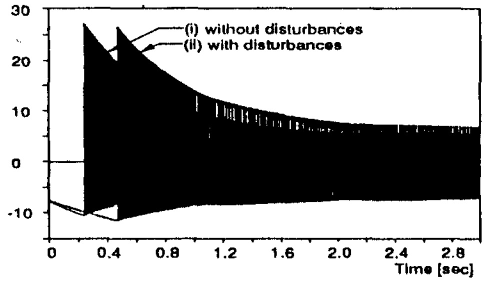
(b) outputs, x_2



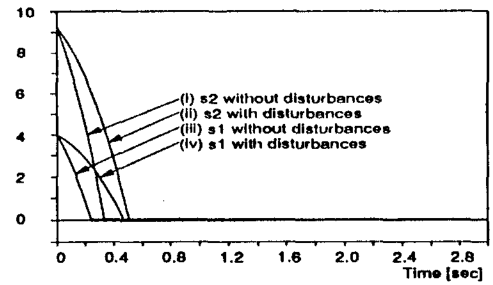
(c) phase trajectories of x_1



(d) phase trajectories of x_2

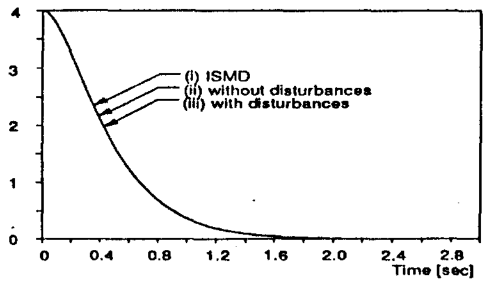


(e) control inputs, U_1

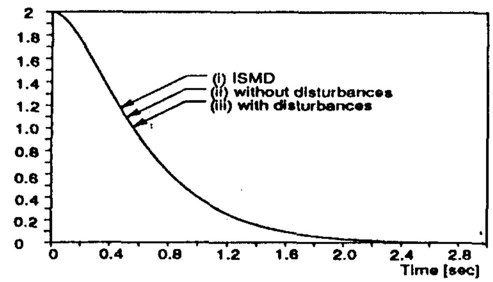


(f) sliding surface time trajectories

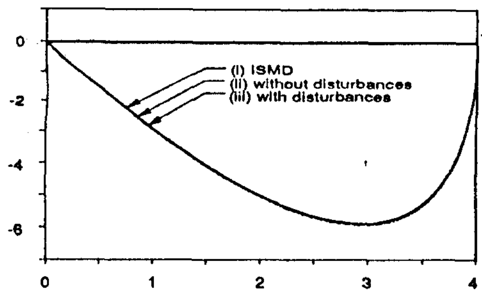
Fig. 3 Results of the IVSS suggested by Chern and Wu



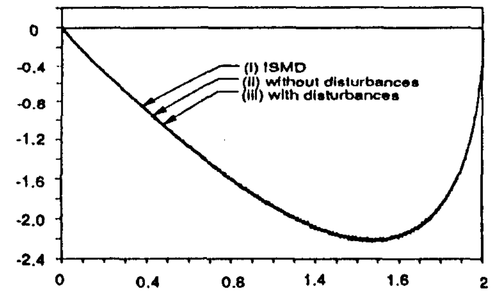
(a) outputs, x_1



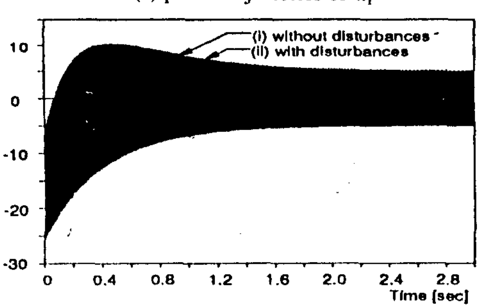
(b) outputs, x_2



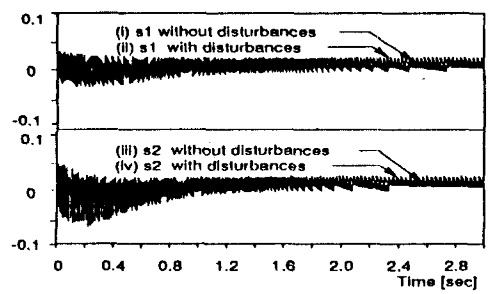
(c) phase trajectories of x_1



(d) phase trajectories of x_2



(e) control inputs, U_1



(f) sliding surface time trajectories

Fig. 4 Results of the proposed algorithm

three cases. The control and the time trajectories of the each sliding surface are presented in (e) and (f), respectively. As can be seen, the three case outputs of the conventional VSS and IVSS are different from each other because the effect of the disturbances during the reaching phase. The reaching phase of the trajectory from the initial state to the first touching of the sliding surface can be founded in the each trajectory, since the sliding surface does not deifned from the given initial condition. Thus, the controlled system is disturbed by the persistent disturbances. The exact predetermination of the desired output dynamics is not possible. Moreover, in the outputs by the IVSS, the overshoot occurs as the expected due to mere introducing of the integral actions.

On the other hand, in case of the proposed, the three outputs of x_1 and x_2 are identical without any reaching phase and with no overshoot as the designed. Thus, with the

Algorithm		Ψ^0	Ψ^1	Ψ^2	δ
proposed	+	$\begin{bmatrix} .01 & 0 \\ 0 & .01 \end{bmatrix}$	$\begin{bmatrix} 4 & 0.2 \\ 0.2 & 4 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
	–	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -.2 \\ -.2 & 0 \end{bmatrix}$	$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$	$\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$
linear	+	O	$\begin{bmatrix} 2 & 0.2 \\ 0.2 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
	–	O	$\begin{bmatrix} 0 & -.2 \\ -.2 & 2 \end{bmatrix}$	$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$	$\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$
Chern's	+	$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 3 & 0.2 \\ 0.2 & 3 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
	–	$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$	$\begin{bmatrix} 0 & -.2 \\ -.2 & 0 \end{bmatrix}$	$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$	$\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

Table 1 Selected gain constants satisfying the gain inequalities

and- with no overshoot as the designed. Thus, with the proposed technique, the performance robustness against the persistent disturbances is obtained, also, the prediction of the outputs is feasible.

From the above comparative simulation studies, the proposed algorithm has superior performance over the previous methods in view of the reaching phase problems, predetermined output dynamics, and robustness.

4. Conclusions

The VSS with an integral-augmented sliding surface are proposed for the improved robust control of a linear multivariable uncertain systems by removing the reaching phase problems. To deal with the problems of the reaching phase, an integral augmented sliding surface is defined, and for its effective design, the traditional eigenstructure assignment theory is employed. Using the transformation technique as a diagonalization method, the stabilizing control is designed to generate the sliding mode on the new transformed surface, while the designed performance is still conserved. This property is shown through the stability analysis. Using the suggested algorithm, the perfect robustness for whole trajectory can be effectively obtained under the bounded parameter variations and disturbances, while the conventional VSS's suffer from the reaching phase problems. The advantages of the algorithm can be pointed out as perfect performance robustness, predetermined output dynamics, and prediction of output.

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