'93 한국자동제어학술회의논문집 (1993.10.20~22)

분절적이고 유연성 있는 우주 구조물의 동역학적 해석 및 자세제어

°백명진*, Harry G. Kwatny**
*한국항공우주연구소, **Drexel University, U.S.A.

Nonlinear Dynamics and Attitude Control of Articulated and Flexible Spacecraft

°M.-J. Baek*, Harry G. Kwatny**
*Korea Aerospace Research Institute, ** Drexel University, U.S.A.

Abstract

This paper extends the authors' prior work on the regulation of flexible space structures via partial feedback linearization (PFL) methods to articulated systems. Recursive relations introduced by Jain and Rodriguez are central to the efficient formulation of models via Poincaré's form of Lagrange's equations. Such models provide for easy construction of feedback linearizing control laws. Adaptation is shown to be an effective way of reducing sensitivity to uncertain parameters. An application to a flexible platform with mobile remote manipulator system is highlighted.

1 Introduction

Our goal is to demonstrate the application of recently developed innovations for modeling and control of articulated systems to a spacecraft configuration representative of Space Station Freedom with a Mobile Remote Manipulator System (SSF/MRMS). The problem considered is the attitude regulation of ths space station while the MRMS undergoes arbitrary prescribed maneuvers. The issue of attitude control for such a configuration has received attention in the literature, most notably in the papers of Modi and his coworkers [9] and Wie et al [14], in which linear controllers are applied and various stability problems are noted. Such systems are inherently nonlinear and this raises questions both with respect to modeling, especially when flexibility is present, and with respect to control system design.

The methods considered herein address the essential nonlinearity of these systems directly. A unified approach to modeling and nonlinear control system design is employed. The space station attitude control issues addressed herein are related to the attitude control problems defined by Mah et al [9] and Wie et al [14] except that we focus on the short time scale problem (time scale of minutes) associated with MRMS motion whereas in the aforementioned works MRMS induced disturbances are considered but primarily in terms of their affect on long term behavior (time scale of orbits).

As a matter of fact, we show that the stabilization issues are far more subtle and critical than suggested in either [9] or [14]. The nonlinear inertial cross-couplings, especially when platform flexibility is considered, severely limits the achievable performance with linear regulators. Thus, we consider control system design for decoupling and stabilization with respect to MRMS motion. Because the associated dynamics are nonlinear in an essential way, we consider nonlinear control design using partial feedback linearization. This method effectively cancels certain nonlinearities and, hence, there arise important robustness issues. As a result adaptation is considered to be an important adjunct to this class of controllers. Since the control problem of interest herein evolves on a short time scale, we do not include environment (orbital frequency) disturbance torques in our analysis.

2 Modeling of Articulated Spacecraft

In the following paragraphs we summarize the necessary concepts and explain how they are integrated into the Lagrangian framework [7,12]. The key issue is the formulation of the kinetic energy function and we focus on that construction.

We adopt the convention, by which any vector $a \in \mathbf{R}^3$ is converted into a skew-symmetric matrix $\tilde{a}(a)$:

$$\tilde{a}(a) := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \qquad (2.1)$$

just as commonly done for angular velocity. Rodriguez et al [7,12] define the *spatial velocity at point C* of any body-fixed reference frame with origin at point C as $V_c := [\omega, v_c]$ where v_c is the velocity of point C and ω is the angular velocity of the body. Let O be another point in the same body and let r_{co} denote the location of C in the body frame with origin at O. Then the spatial velocity at point C is related to that at O by the relation

$$V_c = \phi(r_{co})V_o \qquad (2.2)$$

where

$$\phi(r_{co}) := \begin{bmatrix} I & 0 \\ -\tilde{r}_{co} & I \end{bmatrix}, \text{ and its adjoint } \phi^*(r_{co}) := \begin{bmatrix} I & \tilde{r}_{co} \\ 0 & I \end{bmatrix}. \qquad (2.3)$$

Consider a serial chain composed of K+1 rigid bodies connected by joints as illustrated in Figure 1. The bodies are numbered 0 through K, with 0 denoting the base or reference body, which may represent any convenient inertial reference frame. The k^{th} joint connects body k-1 at the point C_{k-1} with body k at the point O_k.

Let a reference frame $\mathfrak{F}^k$, with origin at O_k, be so oriented that its z-axis passes through C_k in the undeformed configuration. We will use a coordinate specific notation in which vectors represented in $\mathfrak{F}^i$ (or its tangent space) will be identified with a superscript "i". Coordinate free relations carry no superscript. The k^{th} joint has n_k, $1 \le n_k \le 6$ degrees of freedom which can be characterized by n_k quasi-velocities $\beta(k)$ and a joint map matrix $H(k) \in \mathbb{R}^{6 \times n_k}$ so that $V_{o_k} - V_{c_{k-1}} = H(k)\beta(k)$.

Rodriguez and his coworkers establish the coordinate free recursive velocity relation

$$V(k) = \phi(r_{co}(k-1))V(k-1) + H(k)\beta(k) \qquad (2.4)$$

or in coordinate specific notation

$$V^i(k) = \phi(r^i_{co}(k\text{-}1))V^i(k\text{-}1) + H^i(k)\beta^i(k) \qquad (2.5)$$

Let us assume that H(k) and β(k) are specified in the frame $\mathfrak{F}^k$ and V(k-1) has been computed in the frame $\mathfrak{F}^{k-1}$. Then it is convenient to compute V(k) in the k^{th} frame

$$V^k(k) = diag(L_{k-1,k}, L_{k-1,k})\phi(r^{k-1}_{co}(k\text{-}1))V^{k-1}(k\text{-}1) + H^k(k)\beta^k(k) \qquad (2.6)$$

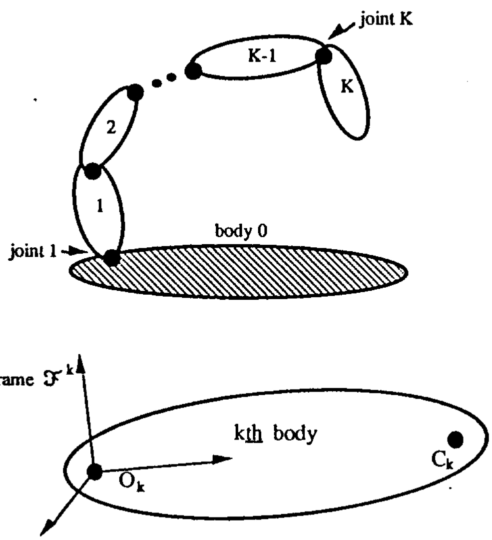

Figure 1. A serial chain composed of K+1 rigid bodies numbered 0 through K and K joints numbered 1 through K. On an arbitrary k^{th} link the inboard and outboard joint hinge points are designated O_k and C_k. The body fixed reference frame $\mathfrak{F}^k$ has its origin at O_k.

If $V^0(0)$ is given, then equation (2.6) allows us to compute recursively, for k=1,..,K, the linear velocity of the origin of $\mathfrak{F}^k$ and the angular velocity of $\mathfrak{F}^k$, both represented in the coordinates of $\mathfrak{F}^k$. In what follows we take $V^0(0)=0$. Abusing notation somewhat, it is convenient to define

$$\phi(k,k\text{-}1) := diag(L_{k-1,k}, L_{k-1,k})\phi(r^{k-1}_{co}(k\text{-}1)) \qquad (2.7)$$

so that (2.6) can be written

$$V^k(k) = \phi(k,k\text{-}1)V^{k-1}(k\text{-}1) + H^k(k)\beta^k(k), \; k = 1, \dots, K, \; V^0(0)=0 \qquad (2.8)$$

It is necessary to define a spatial inertia tensor as well. Consider the k^{th} rigid link and let $I_{cg}(k)$ denote the inertia tensor about the center of gravity in coordinates $\mathfrak{F}^k$, m(k) denote the mass, and a(k) denote the position vector from the center of gravity to an arbitrary point O. The spatial inertia about the center of gravity, M_{cg}, and about O, M_o, are

$$M_{cg}(k) = \begin{bmatrix} I_{cg} & 0 \\ 0 & mI \end{bmatrix}, \qquad (2.9a)$$

$$M_o(k) = \phi^*(a)M_{cg}\phi(a) = \begin{bmatrix} I_o & m\tilde{a} \\ -m\tilde{a} & mI \end{bmatrix} \qquad (2.9b)$$

where I_o is the inertia tensor about O.

The spatial velocity and spatial inertia matrix and, hence, the kinetic energy function for the entire chain can now be conveniently constructed. Let us define the chain spatial velocity and joint psuedo-velocity

$$V := [V^t(1), \dots, V^t(K)]^t, \quad \beta := [\beta^t(1), \dots, \beta^t(K)]^t \qquad (2.10)$$

so that we can write

$$V = \Phi H p, \qquad (2.11)$$

where

$$\Phi := \begin{bmatrix} I & 0 & \dots & 0 \\ \phi(2,1) & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \phi(K,1) & \phi(K,2) & \dots & I \end{bmatrix}, \quad H := \begin{bmatrix} H(1) & 0 & \dots & 0 \\ 0 & H(2) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & H(K) \end{bmatrix}$$

$$\phi(i,j) := \phi(i,i\text{-}1)\dots\phi(j\text{+}1,j), \quad i=2,..,K \text{ and } j=1,..,K\text{-}1$$

The kinetic energy function for the chain consisting of links 1 through K is

$$K.E._{chain} = \frac{1}{2}p^t \mathfrak{M} p \qquad (2.12)$$

where the *chain inertia matrix* is

$$\mathfrak{M} := H^*\Phi^*M\Phi H, \quad M := diag(M_o(1), \dots, M_o(K)) \qquad (2.13)$$

Remarks:

(1) Finite element reduction: One approach to finite element reduction is based on collocation by splines. Our implementation of this method is described in [2]. It is simple and convenient for the class of models of interest herein.

(2) Poincaré's Equations: The above definitions and constructions provide the kinetic energy function in the form $\mathfrak{T}(q,p) = p^t\mathfrak{M}(q)p$, i.e., in terms of quasi-velociety p [1,4,10]. Hence, we have the form:

$$\mathfrak{M}(q)\dot{p} + \mathfrak{C}(q,p)p + \mathfrak{F}(q) = Q_p \qquad (2.14)$$

where

$$\mathfrak{C}(q,p) := -\left[\frac{\partial(\mathfrak{M}p)}{\partial q} V\right]p + \frac{1}{2}\left[\frac{\partial(\mathfrak{M}p)}{\partial q} V\right]^t p + \sum_{j=1}^{m} p_j X^t_j U^t \mathfrak{M}$$

$$\mathfrak{F}(q) := V^t(q)\frac{\partial \mathcal{V}(q)}{\partial q^t}, \quad Q_p := V^t(q)Q$$

Notice that Q_p denotes the generalized forces represented in the p-coordinate frame whereas Q denotes the generalized forces in the $\dot{q}$-coordinate frame (aligned with q). Q_p is actually more convenient because the quasi-velocities are usually represented in appropriate body frames.

(3) Taylor Linearization: A straight forward computation shows that the Taylor linearized dynamics are

$$\dot{q} = V(0)p \qquad (2.15a)$$

$$\mathfrak{M}(0)\dot{p} + \mathfrak{C}(0,0)p + \frac{\partial \mathfrak{F}}{\partial q}(0)q = \Delta Q_p \qquad (2.15b)$$

3 Nonlinear Attitude Control via PFL

The approach to attitude control design considered herein derives from a now well established theoretical basis for control design by *feedback linearization* [6]. In recent work, including [2,3], we have tailored this technique to take advantage of the special structure of Lagrangian dynamics either in the form of classical Lagrange's equations or Poincaré's equations.

3.1 Partial Feedback Linearizing Control

The spacecraft models formulated above are of the from:

$$\dot{q} = V(q)p \qquad (3.1a)$$

$$\mathcal{M}(q,t)\dot{p} + \mathcal{C}(q,p,t)p + \mathcal{F}(q,t) = G\tau \qquad (3.1b)$$

The class of attitude control problems we invesigate is best characterized by partitioning the coordinate vector, and correspondingly the quasi-velocity vector, into two parts

$$q = \begin{bmatrix} \xi \\ u \end{bmatrix}, p = \begin{bmatrix} \omega \\ v \end{bmatrix} \qquad (3.2)$$

where ξ represents the controlled body attitude parameters and ω the corresponding body angular velocity, whereas u,v represent the remaining coordinates and velocities, respectively. Then in partitioned form, the equations are:

$$\dot{\xi} = \Gamma(\xi)\omega \qquad (3.3a)$$

$$\dot{u} = \Sigma(\xi,u)v \qquad (3.3b)$$

$$M_\omega\dot{\omega} + N\dot{v} + F_\omega = G_\omega\tau \qquad (3.3c)$$

$$N^T\dot{\omega} + M_v\dot{v} + F_v = G_v\tau \qquad (3.3d)$$

Our goal is to regulate the outputs y = ξ. The concept of partial feedback linearization (PFL) is a general approach to the design of nonlinear control systems for a general class of systems with smooth nonlinearities [6]. Attitude control of spacecraft using feedback linearization was first used by Dwyer [5]. A PFL compensation for the system (3.3) is a nonlinear feedback law of the form

$$\tau = \mathcal{A}(\xi,\omega,u,v,t) + \mathcal{B}(\xi,\omega,u,v,t)\alpha \qquad (3.4)$$

which provides a closed loop attitude response in the linear, decoupled form

$$\ddot{\xi} = \alpha \qquad (3.5)$$

Specific conditions for the existence and construction of such controllers are given in Isidori [6]. Herein we describe the construction of PFL controllers for spacecraft modeled by Poincaré's equations.

The main constructive result is summarized in the following proposition:

Proposition 3.1: The PFL control for regulation of the outputs y = ξ for the system defined by (3.3) takes the form of (3.4) with

$$\mathcal{A} = [G_\omega - NM_v^{-1}G_v]^{-1}\{F_\omega - NM_v^{-1}F_v + [NM_v^{-1}N^T - M_\omega]\Gamma^{-1}\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\} \qquad (3.6)$$

$$\mathcal{B} = [G_\omega - NM_v^{-1}G_v]^{-1}[M_\omega - NM_v^{-1}N^T]\Gamma^{-1}$$

proof: We prove the proposition by direct construction, in two steps. First, we use linearizing feedback to reduce (3.3c) to the form $\dot{\omega} = \beta$ which we then reduce to (3.5) by a second linearizing feedback. The composition of these to control laws gives the desired result. Equation (3.3d) can be solved for $\dot{v}$:

$$\dot{v} = -M_v^{-1}N^T\dot{\omega} - M_v^{-1}F_v + M_v^{-1}G_v\tau$$

which allows it elimination from (3.3c):

$$[M_\omega - NM_v^{-1}N^T]\dot{\omega} + F_\omega - NM_v^{-1}F_v = [G_\omega - NM_v^{-1}G_v]\tau$$

Now we choose the feedback control law:

$$\tau = [G_\omega - NM_v^{-1}G_v]^{-1}\left\{F_\omega - NM_v^{-1}F_v + [M_\omega - NM_v^{-1}N^T]\beta\right\}$$

which yields:

$$\dot{\omega} = \beta$$

Now, differentiation of (3.3a) provides:

$$\ddot{\xi} = \frac{\partial\Gamma\omega}{\partial\xi}\dot{\xi} + \Gamma(\xi)\dot{\omega} = \frac{\partial\Gamma\omega}{\partial\xi}\Gamma(\xi)\omega + \Gamma(\xi)\beta$$

Choose, the control law:

$$\beta = \Gamma^{-1}(\xi)\left\{\alpha - \frac{\partial\Gamma\omega}{\partial\xi}\Gamma(\xi)\omega\right\}$$

to obtain:

$$\ddot{\xi} = \alpha$$

and the desired composite linearizing control law is:

$$\tau = [G_\omega - NM_v^{-1}G_v]^{-1}\left\{F_\omega - NM_v^{-1}F_v + [M_\omega - NM_v^{-1}N^T]\Gamma^{-1}\left[-\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\right]\right\}$$

which is the stated result. §

Remarks:

(1) The linearizing control law is local if the parameterization of the angular configuration is local. However, there is some flexibility here because one may choose alternate parameterizations (e.g. Gibbs or Euler parameters), as appropriate to the problem. In either case, Γ has known singular points which limit the range of linearizability.

(2) In the specific problem of interest herein we have $G_\omega = I_3$ and $G_v = 0$, so that (3.6) simplifies somewhat to:

$$\mathcal{A} = \{F_\omega - NM_v^{-1}F_v + [NM_v^{-1}N^T - M_\omega]\Gamma^{-1}\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\} \qquad (3.7a)$$

$$\mathcal{B} = [M_\omega - NM_v^{-1}N^T]\Gamma^{-1} \qquad (3.7b)$$

(3) The invertibility of M_v is assured because it is an inertia matrix for a physical subsystem which is consequently a positive definite matrix.

(4) Equation (3.5) may be rewritten

$$\dot{z} = Az + B\alpha,\ A := \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix}, B := \begin{bmatrix} 0 \\ I_3 \end{bmatrix} \qquad (3.8a)$$

we may easily choose a stabilizing control for (3.5)

$$\alpha = K_p\xi + K_r\dot{\xi} = Kz \qquad (3.8b)$$

3.2 Adaptive PFL Control

Because feedback linearization is a model based approach to control system design, it is necessary to anticipate some sensitivity to model uncertainty. In the present case, it is reasonable to assume that the kinematics are precisely known but that the dynamics are not. Thus, we consider the situation where the model contains uncertain parameters, denoted ϑ, which belong to a bounded set $\mathfrak{J}$. Equations (3.3), may be rewritten with these parameters explicitly shown

$$M_\omega(\vartheta)\dot{\omega} + N(\vartheta)\dot{v} + F(\vartheta)_\omega = G_\omega\tau \qquad (3.9a)$$

$$N(\vartheta)^T\dot{\omega} + M_v(\vartheta)\dot{v} + F_v(\vartheta) = G_v\tau \qquad (3.9b)$$

Because of its physical meaning, the invertibility of $M_v(\vartheta)$ is preserved for all values of $\vartheta \in \mathfrak{J}$. Consequently, a feedback linearizing control exists for all parameter values. Indeed, the control (3.4) as constructed via Proposition 4.1 is a parameter dependent control, which we rewrite in the form

$$\tau(\vartheta) = \mathcal{A}_\vartheta(\xi,\omega,u,v,t) + \mathcal{B}_\vartheta(\xi,\omega,u,v,t)\alpha \qquad (3.10)$$

The idea is to implement (3.9) with ϑ replaced by an estimate $\hat{\vartheta}$. When the estimated control $\tau(\hat{\vartheta})$ is applied, the system is not exactly feedback linearized and a simple computation shows that (3.5) is replaced by

$$\ddot{\xi} = \alpha + \Delta(\hat{\vartheta},\vartheta,\xi,\omega,u,v,t) \qquad (3.11)$$

The following proposition provides a parameter adaptive feedback linearizing control law.

Proposition 3.2: Consider the system defined by (3.3a&b) and (3.9) with control $\tau(\hat{\vartheta})$ where $\tau(\cdot)$ is given by (3.10) and α by (3.8b). Suppose that the residual Δ defined in (3.11) has the form

$$\Delta(\hat{\vartheta},\vartheta,\xi,\omega,u,v,t) = \Psi(\xi,\omega,u,v,t)(\vartheta-\hat{\vartheta}) \qquad (3.12)$$

Then an asymptotically stable controller is achieved with the parameter estimator

$$\dot{\vartheta} = Q\Psi^t(\xi,\omega,u,v,t)B^tPz \qquad (3.13)$$

where P is a symmetric, positive definite solution of

$$(A+BK)^tP + P(A+BK) = -I \qquad (3.14)$$

and Q is any symmetric, positive definite marix.

proof: Various forms of this result are well known, e.g. [13].

4 Summary of Simulation Results

In the following paragraphs we describe simulation results which compare linear and nonlinear (PFL) controllers for attitude control of a prototype space station. Prior to consideration of flexible platform studies were conducted with a rigid platform.

4.1 System Configuration

The space station with MRMS is idealized to be composed of four articulated elements: the space station main body (body 1), the MRMS base (body 2), the upper (inner) MRMS arm (body 3), and the lower (outer) MRMS arm (body 4). It is asumed that the MRMS base, body 2, can move along a fixed path on the space station, body 1, while body 3 is joined to body 2 and body 4 to body 3 via joints with up to three rotational degrees of freedom. The setup is illustrated in figure 2. We consider the case where the MRMS joints are each resticted to one degree of freedom: joint 3 admits only rotations about the z-axis in the $\mathfrak{F}^3$ frame and joint 4 about the x-axis in the $\mathfrak{F}^4$ frame.

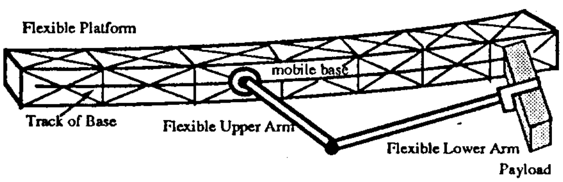

Figure 2. The system considered is composed of a flexible platform, a mobile base, and the flexible upper and lower arms.

The platform is treated as a flexible beam for which a model is developed in accordance with the finite element method described in [5], using collocation by splines as applied to a Timoshenko formulation of beam dynamics. Even with only two elements, the resultant system is excessively stiff. Thus, we reduce the system to retain 4 flexure degrees of freedom (8 modes) by retaining the so-called long wavelength dynamics, so that angular deformation coordinates are eliminated. Of these, 4 modes are near the control bandwidth (natural frequencies of about 3 rad/s) and the others are outside the bandwidth (approximately 10 rad/s). The result is 13 degrees of freedom with configuration variables:

- $R\in\mathbb{R}^3$, the location of point O_1 on body 1 relative to inertial space.
- $L_1\in SO(3)$, the relative angular orientation of $\mathfrak{F}^1$ with respect to inertial space.
- $\bar{\eta}_i\in\mathbb{R}^2$, $i=1,..,N(=2)$ platform deformation coordinates
- $\zeta\in\mathbb{R}$, the location of the MRMS base along undeformed track in the frame $\mathfrak{F}^1$.
- $\psi_{32}\in\mathbb{R}$, the relative angular orientation of $\mathfrak{F}^3$ with respect to $\mathfrak{F}^2$.
- $\phi_{43}\in\mathbb{R}$, the relative angular orientation of $\mathfrak{F}^4$ with respect to $\mathfrak{F}^3$.

The joint quasi-velocities are $\beta(1)=(\omega_1,v_1)$ the linear velocity v_1 and the angular velocity ω_1 of $\mathfrak{F}^1$, the linear velocity $\beta(2)=v_{2z}$ for joint 2, and the relative angular velocities $\beta(3)=\omega_{32}$ and $\beta(4)=\omega_{43}$ for joints 3 and 4.

We assume that the beam is a uniform, square boxbeam with outside dimension of 5 m. A material dissipation model of the type described in [3] is assumed. In addition, we assume some form of active or passive vibration suppression provides additional damping. Even so, the dominant modes of the structure are very lightly damped as will be seen in the simulation results.

4.2 System Equations

The dynamical equations of motion for the composite system including the space station with MRMS have been derived in terms of Poincaré's equations and take the form

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{R} \\ \dot{\eta} \\ \dot{\zeta} \\ \dot{\psi}_{32} \\ \dot{\phi}_{43} \end{bmatrix} = \begin{bmatrix} \Gamma(\xi_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1(\xi_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{4\times4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ v_1 \\ \upsilon \\ v_{2z} \\ \omega_{32z} \\ \omega_{43x} \end{bmatrix} \qquad (4.1a)$$

$$\mathcal{M}_p\dot{p} - \left[\frac{\partial(\mathcal{M}_p p)}{\partial q}V\right]p + \frac{1}{2}\left[\frac{\partial(\mathcal{M}_p p)}{\partial q}V\right]^t p + \begin{bmatrix} \tilde{\omega}_1 & \tilde{v}_1 & 0 \\ 0 & \tilde{\omega}_1 & 0 \\ 0 & 0 & 0_{7\times7} \end{bmatrix}\mathcal{M}_p p$$
$$- \begin{bmatrix} 0_6 & 0 & 0 \\ 0 & B_s & 0 \\ 0 & 0 & 0_3 \end{bmatrix} p - \begin{bmatrix} 0_6 & 0 & 0 \\ 0 & K_s & 0 \\ 0 & 0 & 0_3 \end{bmatrix} q = Q_p \qquad (4.1b)$$

In this study we prescribe the MRMS motion and determine the corresponding SSF response. The MRMS motion is defined by prescribing the MRMS acceleration and computing the resultant motion using the kinematics (4.1a). Thus, we have

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\psi}_{32} \\ \dot{\phi}_{43} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{2z} \\ \omega_{32z} \\ \omega_{43x} \end{bmatrix} \qquad (4.2a)$$

$$\begin{bmatrix} \dot{v}_{2z} \\ \dot{\omega}_{32z} \\ \dot{\omega}_{43x} \end{bmatrix} = \begin{bmatrix} a_{2z} \\ a_{32z} \\ a_{43x} \end{bmatrix} \qquad (4.2b)$$

In all of the subsequent simulations we use the above MRMS motion model with the accelerations a_{2z}, a_{32z}, a_{43x} prescribed as constants.

4.3 Simulation Results

Simulation studies were conducted for both rigid and flexible platform models using linear, PFL and adaptive PFL control laws. The linear controls were obtained by applying the PFL construction to the linearized model, i.e., they are standard linear decoupling controls. Thus, allowing meaningful comparison of the linear and PFL controls. All simulation results will be presented in a similar fashion as two groups of curve: 1) a group of four sets of curves illustrating the MRMS motion and the important principle reference frame coordinates and/or velocities; and 2) a group of four sets of curves illustrating the platform flexure dynamics.

Table 1
Open and Closed Loop Eigenvalues

Open Loop	Nominal Closed Loop (k)	Detuned Closed Loop (k/8)
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	-0.1763 + 3.3205i	-0.1763 + 3.3205i
0	-0.1763 - 3.3205i	-0.1763 - 3.3205i
0	-0.1763 + 3.3205i	-0.1763 + 3.3205i
0	-0.1763 - 3.3205i	-0.1763 - 3.3205i
0	-0.1364 + 1.6505i	-0.1364 + 1.6505i
0	-0.1364 - 1.6505i	-0.1364 - 1.6505i
-10.4212 +10.5963i	-0.1364 + 1.6505i	-0.1364 + 1.6505i
-10.4212 -10.5963i	-0.1364 - 1.6505i	-0.1364 - 1.6505i
-10.8762 +10.5876i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-10.8762 -10.5876i	-0.2000 - 0.2040i	-0.0250 - 0.0979i
-0.2053 + 3.3267i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-0.2053 - 3.3267i	-0.2000 - 0.2040i	-0.0250 - 0.0979i
-0.2053 + 3.3290i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-0.2053 - 3.3290i	-0.2000 - 0.2040i	-0.0250 - 0.0979i

Stabilization with linear feedback

The linear attitude regulator was designed as a decoupling controller so that meaningful comparisons can be made with the PFL designs. Table 1 lists the open and closed loop eigenvalues for several different feedback gain values. The open loop set consists of 12 zero eigenvalues corresponding to the rigid body dynamics and an additional 8 corresponding to the platform flexure dynamics. The second column lists the eigenvalues resulting from a design intended to achieve the same attitude response as had been achieved in a study of the rigid body case. Notice that the first 14 eigenvalues correspond to the "zero dynamics" and remain fixed as the attitude gain is "detuned" in columns three and four. The zero dynamics modes include the 3 rigid body translation modes and 4 cantilevered beam modes of the platform. Although the nominal closed loop linear system is stable, application of the linear regulator to the nonlinear simulation with .1 rad error in each Euler angle yields a divergent trajectory. This is due to destabilizing inertial crosscoupling between the flexible and rigid body dynamics. Detuning of the closed loop appeared appropriate in order to reduce slewing rates and hence platform flexure. Moreover, it is clear that MRMS motion and attitude regulation performance will not in practice approach the levels demanded herein. For example, we impose an MRMS translation of 18m in 60 sec, whereas, Wie et al [25] impose a translation of 5m in 300 sec.

Nevertheless, the detuned regulators still produce divergent trajectories, although they are somewhat less dramatic. The trajectories corresponding to the last column, i.e., the least aggressive design are also divergent. Reduction of the initial attitude errors to .01 rad, however, provides convergent trajectories. We can conclude that the anticipated stable linear behavior is indeed observed in very small signal excursions. The significance of the nonlinear interactions which arise through the inertial couplings is quite striking. It is anticipated that further detuning would lead to a larger domain of attraction for the stable equilibrium point, although we have not confirmed this. Even so, it is clear that the achievable performance with linear regulators is severely limited.

Decoupling and Stabilization via PFL

We first consider attitude regulation with an MRMS maneuver combined with initial attitude errors and with perfect knowledge of all parameters. The PFL control results are illustrated in Figure 3.

Parameter uncertainty and adaptive PFL

We begin by illustrating the effect of a 5% stiffness uncertainty on the performance of the decoupling and stabilizing PFL controller. These results are shown in Figure 4. Note that regulation is seriously degraded even with this rather minimal uncertainty. However, this sensitivity is consistent with our prior observations about the linear regulator and in fact, it is likely that sensitivity would be substantially reduced by detuning of the stabilizer and reduction of the rate of MRMS motion.

Figure 5 illustrates the adaptive PFL with MRMS motion and 5% - 10% stiffness uncertainty. Somewhat less satisfactory results have been achieved with 15% uncertainty. However, 20% uncertainty results in serious degradation of performance.

5 Conclusions

This paper summarizes results of a study of the application of partial feedback linearization methods to the attitude control of an articulated spacecraft representative of the Space Station Freedom with a Mobile Remote Manipulator System. Computer studies contrast linear state feedback attitude stabilizers with PFL based attitude stabilizers. The results presented herein confirm previous observations that MRMS motion can significantly degrade and even destabilize attitude regulation when linear controllers are applied to this highly nonlinear dynamical system. Our results show that in the flexible case the linear regulator must be significantly detuned in order to achieve stable responses. As a matter of fact, even with detuning, the attitude errors must be very small in order to observe the behavior predicted by linear theory. Parameter uncertainty is not tolerable. Although the studies conducted to date are far from exhausting, it is clear that PFL design is promising. It is shown that the PFL controller performs quite well with perfect knowledge (no parameter uncertainty) both with respect to decoupling and stabilization. However, performance deteriorates rapidly with even small parametric uncertainties. Adaptive PFL is shown to restore the excellent PFL performance with uncertainties of 10%. Controller detuning will certainly improve robustness and studies which address the tradeoff between performance and sensitivity would be required in any given design situation.

References

[1] V. I. Arnold, V. V. Kozlov, and A. I. Neishtadt, "Mathematical Aspects of Classical and Celestial Mechanics," in *Dynamical Systems III*, V. I. Arnold, ed., Springer-Verlag Berlin, 1988.

[2] W. H. Bennett, H. G. Kwatny, C. LaVigna and G. L. Blankenship, "Nonlinear and Adaptive Control of Flexible Space Structure," in *Control of Systems With Inexact Dynamic Models*, DSC-Vol. 33, pp. 73-91, ASME 1991.

[3] W. H. Bennett, H. G. Kwatny, and M.-J. Baek, "Nonlinear Dynamics & Control of Flexible, Articulated Spacecraft: Application to SSF/MRMS," AIAA Conf. on GNC, August 1992.

[4] N. G. Chetaev,*Theoretical Mechanics*, Springer-Verlag, New York, 1989.

[5] T. A. W. Dwyer, "Exact Nonlinear Control of Large Angle Rotational Maneuvers," *IEEE Trans. on Auto. Contr.*, Vol. AC-29, No. 9, pp.769-774, 1984.

[6] A. Isidori, *Nonlinear Control Systems: An Introduction*, 2nd ed., Springer-Verlag: New York, 1989.

[7] A. Jain, "Unified Formulation of Dynamics for Serial Rigid Multibody Systems," *AIAA Jrl. Guidance and Control*, Vol. 14, No.3, pp. 531-542, 1991.

[8] H. G. Kwatny, F. M. Massimo and L. Y. Bahar, "The Generalized Lagrange Formulation for Nonlinear RLC Networks", *IEEE Trans. on Circuits and Systems*, Vol. CAS-29, No. 4, pp. 220-233, April, 1982.

[9] H. W. Mah, V. J. Modi, Y. Morita, and H. Yokoto, "Dynamics and Control During Slewing Maneuvers," *Acta Automatica*, Vol. 19, No. 2, pp.125-143, 1989.

[10] L. Meirovich, *Analytical Methods in Vibrations*, McMillan: New York, 1967.

[11] Ju. I. Neimark, and N. A. Fufaev, *Dynamics of Nonholonomic Systems*, American Mathematical Society, 1972.
[12] G. Rodriguez, "Kalman Filtering, Smoothing and Recursive Robot Arm Forward and Inverse Dynamics," *IEEE Jrl. of Robotics and Automation*, Vol. RA-3, No. 6, pp. 624-639, 1987.
[13] S. S. Sastry and A. Isidori, "Adaptive Control of Linearizable Systems," *IEEE Trans. Auto. Contr.*, Vol. 34, No. 11, pp. 1123-1131, 1989.
[14] B. Wie, A. Hunt and R. Singh, "Multibody Interaction Effects on Space Station Attitude Control and Momentum Management," *AIAA J. Guidance, Navigationa nd Control*, Vol. 13, No. 6, Nov.-Dec. 1990.

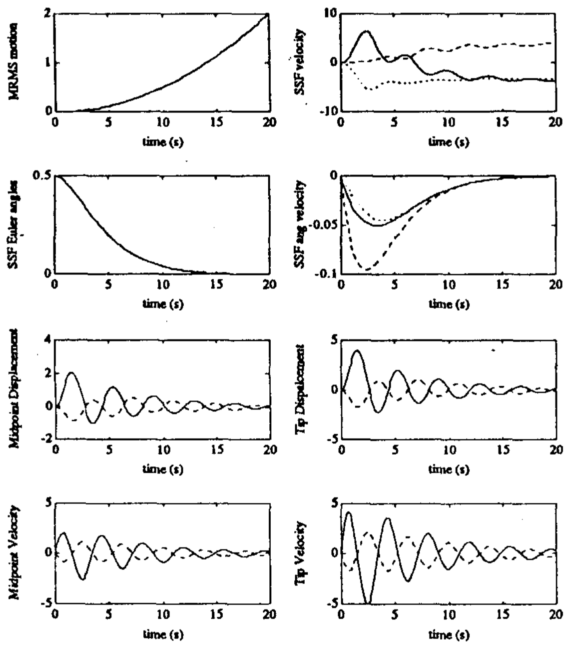

Figure 3. The effectiveness of combined PFL decoupling and attitude stabilization is clearly illustrated in this figure.

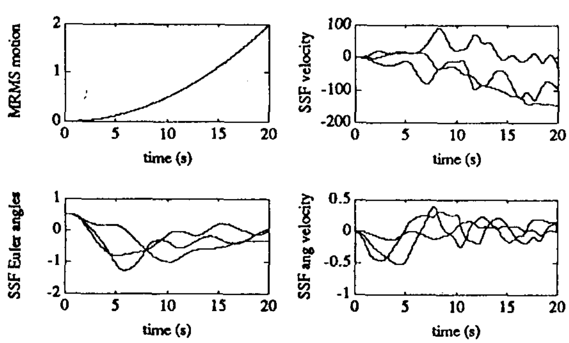

Figure 4. Notice the severe degradation of performance which is the result of a 5% uncertainty in stiffness parameters.

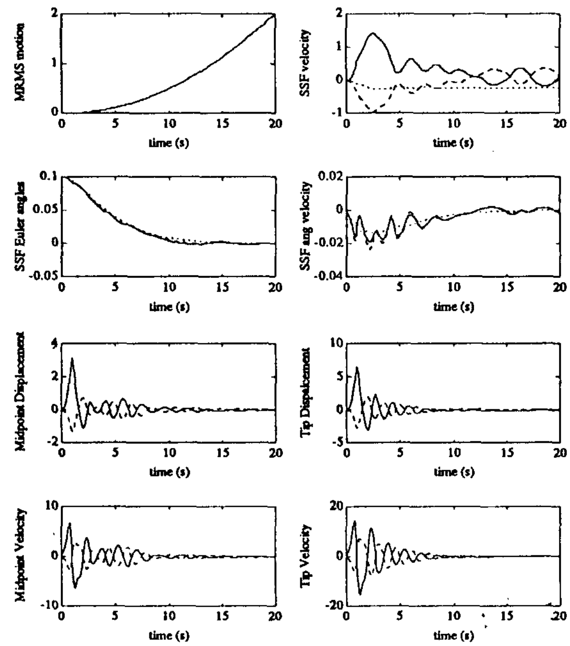

Figure 5. Adaptive PFL with 5% uncertainty.