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Compliance Paradigms for Kinematically Redundant Manipulators

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ABSTRACT

The kinematic resolutions of redundancy is addressed in this paper. The govering equation for quasistatic behavior of compliance governed redundant manipulators is formulated and the repeatable property of the manipulator is proposed. Then the compliance paradigm is used to resolve the redundancy in a repeatable way. The compliance paradigm is one under which the controller simulates the imaginary manipulator which is governed to move by real joint stiffness. The equation is expressed as the weighted pseudoinverse with the configuration dependent weighting matrix. Algorithmic singularities arisen from this scheme are also discussed.

1 Introduction

A robotic system is considered to have kinematic redundancy if the dimension of its configuration space is greater than that of its task space. From application viewpoint, prerequisite is the inverse kinematic problem of specifying the displacements of the joint variables algorithmically corresponding to the position[1] of the hand. That kind of algorithm is termed as the inverse kinematic resolution scheme for redundant manipulators.

The basic idea to resolve the redundancy is to add enough internal or external constraints which are either explicit or implicit. The constraints can be local or instantaneous in nature, or more global and can consider the dynamics of robot or not. The constraints help to make the system well-posed which is originally ill-posed. Once a specific algorithm and an initial setting are chosen, there should be a unique joint-space path for each hand-space trajectory. In other words the arm should always reproduce the same joint path in case of the same situation.

Suppose the manipulator has n degrees of freedom at the joints and operates in an m-dimensional space (where $m \leq 6$). Let Q be the n-dimensional configuration space (or joint space), W the m-dimensional operational space (or task space), and $f : Q \rightarrow W$ the forward kinematic function. We assume the manipulator be redundant, *i.e.* $n > m$.

If $q = (q_1, q_2, \cdots, q_n)^T$ is a configuration and $x = (x_1, x_2, \cdots, x_m)^T$ a position, the relation can be described as follows:

$$x = f(q) \quad (1)$$

$$\dot{x} = J(q)\dot{q}, \quad (2)$$

where $J(q)$ is the Jacobian matrix of partial derivatives of f, evaluated at the current configuration q. Since $n > m$, J is a rectangular matrix having m rows and n columns. Eq. (1) describes the relation in position level and Eq. (2) shows the linearized or instantaneous relation in rate level between two spaces.

The most important factors considered in the kinematic resolution are: (1) singularity avoidance and (2) repeatability. A local path tracking strategy is said to be *repeatable* if the configuration of the arm is driven to repeat itself when the hand traces a closed path. The null motion is not allowed to retain repeatability in this definition. Nonrepeatability of the pseudoinverse method was observed by Klein & Huang [1]. The repeatability problem was mentioned in several other papers dealing with redundant manipulators, but was not fully treated until Baker & Wampler [2] used topological arguments to show that, over a simply connected region of the workspace, a path tracking algorithm is repeatable iff it can be described in terms of an inverse function. Independently, Brockett [3] stated with differential geometrical context that repeatability is a question about the *integrability* of the distribution in $\Re^n$ defined by range of generalized inverse of J. If this distribution is integrable [4], then the pseudoinverse approach will map closed curve in operational space into closed curves in configuration space, otherwise it will not. Analytically, the integrability is equivalent to the solvability of the partial differential equation system defined by the distribution and thus the integra-

[1]The position just denotes the position and orientation of the hand from now on.

bility guarantees the existence of local inverse function.

The application of Frobenius' theorem over simply connected regions, by Shamir & Yomdin [5], yields a computable condition which is termed as the *Lie bracket condition*(LBC). If the Lie bracket of any vector fileds in the distribution is closed, the distribution is integrable and induces the integral manifold whose tangent space is generated by the distribution itself. The extended Jacobian method was shown repeatable by their arguments, and they also showed that some initial configurations satisfying LBC drives the manipulator by pseudoinverse control in a repeatable manner. As a matter of fact, by Baker and Wampler's characterization, the algorithms based on the resolved motion rate control method would not be repeatable in general because the second term makes it impossible to consider q as a single valued function of x.

the resolved rate approach is not repeatable and nor is the pseudoinverse approach [1]. The extended Jacobian approach can produce the repeatable trajectory but induce another singularity, so called *the algorithmic singularity* [6]. Moreover, the extended Jacobian approach suffers from the high symbolic computational burden when applied to multiple redundant spatial manipulator, as it requires the closed forms of all equations. As a numerical solution to inverse kinematic problem, Asada & Slottine [7] suggested that the impedance control scheme can be applied to the stable inverse kinematic solutuion. Mussa-Ivaldi & Hogan [8] extended his idea to redundant manipulator to attain repeatability.

In this paper, the idea is utilized that the repeatability will be guaranteed if the resolution method simulate certain ideal system which is originally repeatable. In other words, the imaginary system is the model of the kinematic resolution. The chosen imaginary system is the compliance-governed manipulator system thus this scheme is refered to as *compliance paradigm*. Under this paradigm the pseudoinverse approach will be complemented to attain repeatability. There are also some singularities arisen from the algorithm, so called algorithmic singularity. Section 2 formulates the model of the imaginary system and present the characteristics of the system. The repeatable property is proposed in Section 3 with some comments about algorithmic singularities. The method is verified effective with numerical experiments with 4-DOF planar redundant manipulator in Section 4 and Section 5 concludes this paper.

2 Quasistatic behaviors of compliance governed redundant manipulators

Compliance paradigm can be used to the trajectory planning of redundant manipulators by the following scheme. Given a manipulator to control, imagine another manipulator with the joint-distributed stiffness which has the same kinematic structure as one to be controlled (See Fig. 1). The inverse kinematic resolution of the original manipulator is done by immitating the motion of the imaginary manipulator. Suppose the manipulator is now in static equilibrium and we want to move its tip by dx. The tip of the imaginary manipulator is then moved by the force exterted at the tip. The force should be exactly compatible with the tip displacement. This means that the manipulator will be in another equilibrium when disturbed by the force from the previous equilibrium. The manipulator to be resolved follows after the imaginary manipulator.

To verify the validity of this paradigm, two propositions must be claimed. One is that the exact formulation of the motion of the imaginary manipulator is possible so that the exact simulation can be made. Since it is almost impossible to make an imaginary system at each case, we had better have the mathematical model to simulate the system and apply it to resolution. The other is, moreover, that the imaginary manipulator which is govened to move by the real stiffness mechanism exhibits repeatability.

The imaginary manipulator has the characteristic that the equilibrium configuration without any tip force is unique. With this manipulator the equilibrium can be changed with the force applied and it gets into another equilibrium with the tip force statically equilibriated with the restoring force from stiffness. One should not be misconceived that given dx, applying the force calculated by just multiplying the current tip stiffness to dx can cause the manipulator to move by dx. Since the next equilibrium configuration is determined by the compliance at the next state and the mapping between the configuration and the operational space is nonlinear, much care is needed. In this section, we formulate the governing equation for static behavior of the stiffness-governed manipulator system and show that this system is repeatable.

There exist four domains related to robot kinematics and statics, or joint space Q, task space W, joint torque T, and tip force F. Fig. 2 describes the relations between them and their respective tangent spaces, *i.e.* incremental displacement spaces. It looks similar with the premultiplier diagram proposed by Kim *et al.* [9], but they assumed there be no change in joint variables q and they neglected the change of Jacobian due to change of q. In this figure, some relations are well known like J between dx and dq and J^T between F and T, but others should be taken more cares.

The manipulator is assumed to act as its motion at equlibrium states is governed by the compliance relation as follows:

$$q - q_0 = C(q)\tau, \tag{3}$$

where q_0 is the initial equilibrium configuration without any tip force, and $C(q)$ is the compliance distributed at the joint at q which can be either constant or varying.

The above relation is implicit expression with respect to q, so the differential relation is not $dq = C(q)d\tau$ unless C is constant. Thus by the help of implicit function theorem [4], if

$$\det[I - \frac{\partial C}{\partial q}\tau] \neq 0 \quad (4)$$

then we get the differential relation as:

$$dq = (I - \frac{\partial C}{\partial q}\tau)^{-1}C(q)d\tau. \quad (5)$$

Let the transformation matrix in rhs of the equation,

$$\begin{aligned} C^*(q) &= (I - \frac{\partial C}{\partial q}\tau)^{-1}C(q) \\ &= (K(q) - K(q)\frac{\partial C}{\partial q}\tau)^{-1} \end{aligned} \quad (6)$$

where

$$K = C^{-1}. \quad (7)$$

Eq. (5) can be considered as the Taylor approximation up to the first order and, as such, all the values are evaluated at the present state, q, which approximate the change to the next state, dq.

Tip force and joint torque[2] are related using the transpose of the Jacobian:

$$\tau = J(q(\tau))^T F. \quad (8)$$

Since it is also implicitely expressed, with the same arguments as above the following equation is derived.

$$d\tau = [I - (\frac{\partial J^T}{\partial q}F)C^*]^{-1}J^T dF \quad (9)$$

which is satisfied whenever $\det(I - (\frac{\partial J^T}{\partial q}F)C^*) \neq 0$. Denoting the term containing the derivative of the Jacobian and the force as Γ, *i.e.*

$$\Gamma = \frac{\partial J^T}{\partial q}F. \quad (10)$$

The i^{th} column of Γ, Γ^i, is given by:

$$\Gamma^i = \frac{\partial J^T}{\partial q_i}F. \quad (11)$$

Γ is geometrically interpreted as the intrinsic impedance component due to the nonlinear mapping [8]. This comes from the change of the Jacobian when the manipulator is disturbed from the static equlibrium. The tip stiffness induced from the joint stiffness in the imaginary manipulator and the geometric stiffness term should be considered in simulating the motion.

Adjoining Eq. (5) and Eq. (9),

$$\begin{aligned} dq &= C^*(I - \Gamma C^*)^{-1}J^T dF \\ &= (K^* - \Gamma)^{-1}J^T dF, \end{aligned} \quad (12)$$

where

$$K^* = (C^*)^{-1}. \quad (13)$$

[2]Tip force and joint torque represent the generalized force and torque in static state.

The incremental displacements in both spaces is approximated using the Jacobian of its forward kinematic map as:

$$\begin{aligned} dx &= J(q)dq \\ &= J(K^* - \Gamma)^{-1}J^T dF. \end{aligned} \quad (14)$$

Assumtion that

$$\det[J(K^* - \Gamma)^{-1}J^T] \neq 0 \quad (15)$$

yields the following equation:

$$dF = K_e dx \quad (16)$$

by letting

$$K_e = [J(K^* - \Gamma)^{-1}J^T]^{-1}. \quad (17)$$

K_e is the tip stiffness induced by the joint stiffness and its rate of change and the nonlinear compensation impedance. Then the manipulator tip is ideally governed to move by this stiffness.

The desired kinematic resolution is obtained when dF in Eq. (16) is introduced to Eq. (12), which in form looks alike the weighted pseudoinverse:

$$dq = (K^* - \Gamma)^{-1}J^T[J(K^* - \Gamma)^{-1}J^T]^{-1}dx \quad (18)$$

The above equation describes the joint displacement to occur when disturbed from the present state only with the present state. This equation does satisfy the two claims delineated above, as is shown in the next section.

This remark will be suitable at this point. The resolved trajectory does not depend on the actual value of the compliance itself, but on the ratio of the values. When the diagonal compliance matrix is used, the ratio, *i.e.* only $(n-1)$ independent parameters have to be determined. The negative compliance element is also allowed. For example, two compliance matrices opposite in sign will generate the same trajectory, but the imaginary force will reverse the sign to each other. The resolved trajectories can be parametrized with the initial configuration and the ratio of the compliance elements.

3 Repeatability of the compliance paradigms

Following the paradigm proposed above, the inverse kinematics is solved if the manipulator can simulate the imaginary manipulator exactly. The tip force can be predicted as follows. F is set to zero at a initial configuration that reaches the desired initial tip position, and iterate at each step $dF_k = K_e(q_k)dx_k$ and add to F_k to get F_{k+1}. The manipulator is assumed to be governed by the compliance mechanism as in Eq. (3).

PROPOSITION 3.1 *Let f be the forward kinematic map. At t_0, the manipulator is assumed to be in equilibrium at q_0, when the tip reaches $x_0 = f(q_0)$. Assume that no force exerted at the tip at t_0. Then the manipulator is*

in configuration q_0 *at* $t > t_0$ *if and only if the tip force* $F = 0$.
Proof
Assume that q_0 is the nonsingular configuration. Suppose that at that time, the manipulator's configuration is q_0. Then,

$$q - q_0 = q_0 - q_0 = 0 = C(q)\tau$$

which implies that whenever $C(q)$ is nonsingular, $\tau = 0$, and since q_0 is not singular configuration, $F = 0$.

Suppose that $F = 0$. Then $\tau = 0$, which implies that $q - q_0 = 0$. So, we get $q = q_0$.

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The above proposition is no more than the guarantee of repeatability of the imaginary manipulator. To assert the repeatability of the inverse kinematic resolution requires that the tip force is zero when $x = x_0$. This comes from the proposition below.

PROPOSITION 3.2 *Let* $dF = K_e(x)dx$ *for* $dF \in \Re^m$, $dx \in \Re^m$. *Assuming that at* t_0, $x = x_0$ *and* $F = F_0$. *Then* $F(t_f) = F_0$ *iff* $x(t_f) = x_0$ *whenever* $K_e(x)$ *is nonsingular during* $[t_0, t_f]$, $t_f > t_0$.

In order to show the exactness of the formulation, the claim should be satisfied that the manipulator will get into another equilibrium when the tip is disturbed by the desired displacement. The following proposition makes certain this statement.

PROPOSITION 3.3 *Let the manipulator at the joint configuration* q_p *and with the joint stiffness* $K(q_p)$ *is in equilibrium with the tip force* F_p. *Then the manipulator will be in another equilibrium at* $q_p + dq$, *that is,*

$$J^T(q_p + dq)(F + dF) = K(q_p + dq)(q_p + dq), \quad (19)$$

where dq *is such as in* Eq. (18),

$$dq = (K - K\frac{\partial C}{\partial q}\tau - \Gamma)^{-1}J^T dF,$$

where all quantities are evaluated at q_p.
Proof
Use Taylor's approximation to Eq. 19 to get Eq. 12.

∎

Algorithmic singularities are refered to singularities arisen from the specific resolution scheme other than kinematic singularities, due to the noninvertibility of the whole system which consists of the mechanical system and the control system. During the mathematical derivation of the compliance paradigm, many assumptions of nonsingularity of certain matrices were made, especially on the use of implicit function theorem to get Taylor's approximation.

Checking the derivation, the system in all is not invertible whenever

$$A = \det\left[J(K^* - \Gamma)^{-1}J^T\right] = 0. \quad (20)$$

the above equation is satisfied in three cases:

- $\det J = 0$,
- $\det\left(I - \frac{\partial C}{\partial q}\tau\right) = 0$, and
- $\det(K^* - \Gamma) = 0$.

The first of above equations is the usual kinematic singularity condition. Possibility of the second can be removed by using constant compliance values. Consider the third. If $(K^* - \Gamma)$ is singular, zero dF does not make $dq = 0$. In other words, the manipulator is able to reconfigure itself with tip force unchanged as there is the null space generated by its singular direction.

4 Numerical Examples

Here we present the results of numerical simulations of planar 4-DOF manipulators. Link lengths are 3, 3, 1, and 1 units from base to proximal link. The manipulator is commanded to trace the circle located on the x axis. The 4-th order Runge-Kutta method was adopted for numerical integration of ODE system defined by Eq. (18) with 2000 integration steps.

The commanded circle is located at $(4,0)$*units* with radius of 3*units*. The initial arm configuration is (20.0000, −52.0227, 21.0509, 60.0000)(°) corresponding to the tip position of $(7.0, 0.0)$*units*. The pseudoinverse method (PI) with the weight I and the proposed method (CP) with the compliance I were simulated with the governing differential equations:

(PI) $dq = J^T(JJ^T)^{-1}dx$

(CP) $dq = (I - \Gamma)^{-1}J^T[J(I - \Gamma)^{-1}J^T]^{-1}dx$.

The simulation results are depicted in Figs. 3 and 4, where (a) shows the joint space trajectories and (b) is the graph of the tip force to simulate the imaginary manipulator along the circular trajectory. The pseudoinverse method leads to the configuration drift[3] of 184.69° while the proposed method results in the drift of 0.00°. Thus, the repeatability was guaranteed by the proposed scheme not by the pseudoinverse method. Fig. 4(b) shows the tip force of the imaginary manipulator during resolution, which exhibits that the tip force becomes zero at final configuration. Pseudoinverse method cannot simulate the imaginary compliance-gorverned manipulator because it does not predict the tip force which is compatible to the desired tip displacement. The tip force also does not return to zero, which implies that the repeatability is broken.

The repeatability ceases to hold along any tip path enclosing one or more singular points, which is well-known

[3]The configuration drift(CD) is defined as: $CD = (\sum_{i=1}^{n}(q_{i,final} - q_{i,initial})^2)^{1/2}$.

results of differential calculus [2, 4, 8]. Thus the nonrepeatability with the proposed scheme can be induced from two causes: one is the algorithmic singularity and the other is the *non*-simply connectedness of the workspce closed path.

5 Conclusion

A method of kinematic resolutions of redundancy called compliance paradigm was proposed which has the repeatable property. When the compliance matrix is constant, this scheme can complement the nonconservativity of weighted pseudoinverse approach as shown with numerical experiments of 4-DOF planar redundant manipulator. The governing equation of static behavior of imaginary compliance-governed redundant manipulator was formulated and the repeatable property was proved also with the help of physical intuitions. Algorithmic singularity problem seems to accompany the repeatable behavior. With this scheme, algorithmic singularities can be identified with the simple determinant condition and they result in the invalidity of Taylor's approximation up to first order. This kind of algorithm is very effective compared to the extended Jacobian method when applied to multiple redundancy case.

The exact representation of four domains related to robot kinematics and statics, or joint space Q, work space W, joint torque T, and tip force F and the relations between them and their respective tangent space can be easily captured with this paradigm. The nonrepeatability of the pseudoinverse control can be considered that it neglects the kinematic nonlinearity between Q and W. The kinematic nonlinearity can be complemented by a single correction matrix that has the physical meaning of an apprent joint stiffness due to change of the Jacobian and to the nonzero tip force in statical equilibrium.

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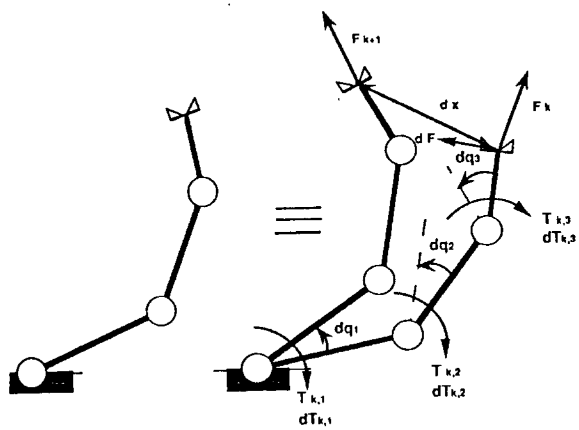


Figure 1: Compliance paradigm for kinematic resolutions of redundancy

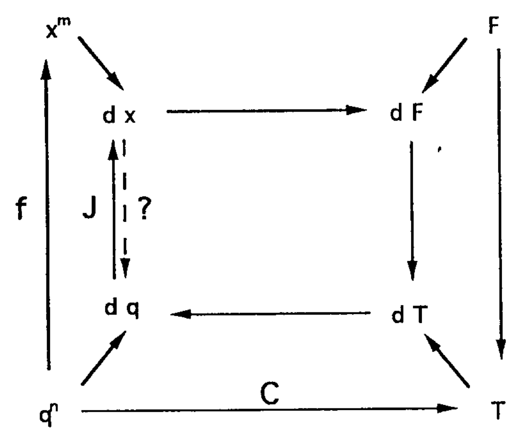
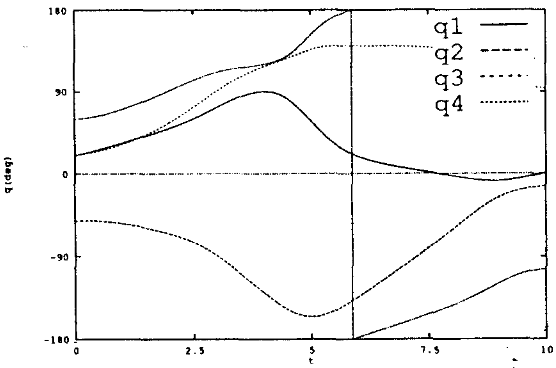
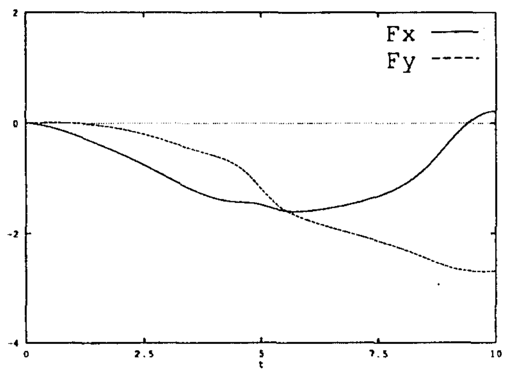


Figure 2: Relations between four spaces and their respective tangent spaces

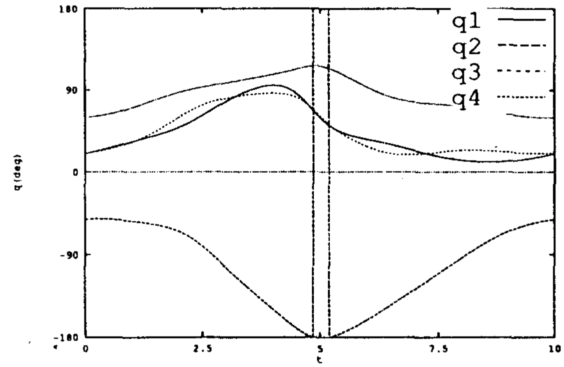


(a) The resolved trajectory

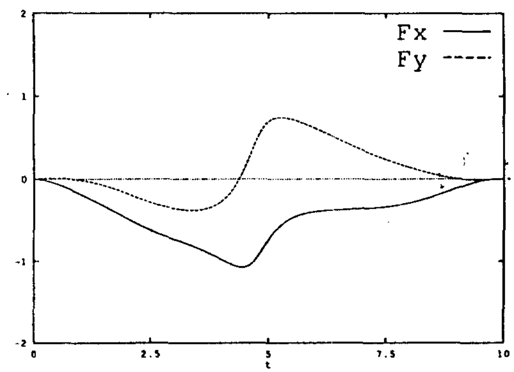


(b) The imaginary force on the imaginary manipulator

Figure 3: Simulation results with the pseudoinverse method



(a) The resolved trajectory



(b) The imaginary force on the imaginary manipulator

Figure 4: Simulation results with the compliance paradigm