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Optimal Actuator Selection for Output Variance Constrained Control

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Abstract

In this paper, a specified number of actuators are selected from a given set of admissible actuators. The selected set of actuators is likely to use minimum control energy while required output variance constraints are guaranteed to be satisfied. The actuator selection procedure is an iterative algorithm composed of two parts; an output variance constrained control and an input variance constrained control algorithm. The idea behind this algorithm is that the solution to the first control problem provides the necessary weighting matrix in the objective function of the second optimization problem, and the sensitivity information from the second problem is utilized to delete one actuator. For variance constrained control problems, by considering a dual version of each control problem an efficient algorithm is provided, whose convergence properties turn out to be better than an existing algorithm. Numerical examples with a simple beam are given for both the input/output variance constrained control problem and the actuator selection problem.

1. Introduction

A most fundamental and important problem in the synthesis of a control system is the selection of the actuators and/or sensors (their number, type, size and location). The cost, reliability and weight of the input/output devices limit their characteristics. Superfluous sensors/actuators can increase plant disturbances or model uncertainty without contributing much to the system performance. Furthermore, an unfavorable selection resulting in a plant poorly conditioned in a sense of input-to-output characteristics, may lead to controller design difficulties and hence, simplistic design methods have to be replaced by more sophisticated ones so that unnecessarily excessive effort must be made. The size (i.e., dynamic range) of the devices must also be taken into account during the selection process because any physical device has its own limitation.

It is desirable to have a computationally less demanding selection algorithm. In general, a good choice of actuators and sensors can be made when a closed-loop (rather than open-loop) system performance is considered as a selection criterion. So the selection algorithm must be integrated with the controller design process. In this paper, the actuator selection problem will be considered for output variance constrained control problem.

Consider a plant described by

$$\left.\begin{aligned} \dot{x}_p &= A_p x_p + B_p \beta (u + w_a) + D_p w_p , \\ y &= C_p x_p \end{aligned}\right\} \quad (1.1)$$

where $x_p \in R^{n_x}$, $u \in R^{n_u}$ and $y \in R^{n_y}$ are the vectors of plant state, input and output. The diagonal matrix β is to be selected such that

$$\beta = \operatorname{diag}[\beta_1, \beta_2, \cdots, \beta_{n_u}] , \quad (1.2a)$$

$$\beta_i = 0 \text{ or } 1 \text{ for } i = 1, 2, \cdots, n_u \quad (1.2b)$$

where n_u is the number of all available actuators. $\beta_i = 1$ means that the i-th actuator is selected for a controller design. The columns of B_p matrix are dictated by the locations and types of available actuators. The actuator noise w_a and the plant disturbance w_p are assumed to be zero mean Gaussian white noise processes with intensity of W_a and W_p , respectively. The problem under consideration may be stated as follows:

<u>Optimal Actuator Selection (OAS) Problem</u>

Given a positive definite input weighting matrix R_o and upper bounds required of output variances σ^2, find the N actuator locations (or location index matrix β) out of n_u (>N) available locations, and the corresponding control law u such that

$$\left.\begin{aligned} &\min_{(\beta,u)} \; E_\infty (\beta u)^T R_o (\beta u) , \\ &\text{subject to } E_\infty y_i^2 , \; i = 1, 2, \cdots, n_y \end{aligned}\right\} \quad (1.3)$$

where $E_\infty \triangleq \lim_{t\to\infty} E$ is the expectation operator and β is defined by (1.2).

Notice that when β_i are fixed for all i, the above OAS Problem is reduced to the well-known Output Variance Constrained (OVC) control problem [1~3].

This paper is organized as follows: In the next section, the input/output variance constrained control problem will be solved by a dual approach. An actuator selection algorithm is proposed in section 3 by successively iterating the output variance constrained control and input variance constrained control. Numerical examples and concluding remarks are given in section 4.

2. A Dual Approach to Input/Output Variance Constrained Control Problem

Consider the following stabilizable and detectable plant

$$\left.\begin{aligned} \dot{x}_p &= A_p x_p + B_p u + D_p w_p \ , \\ y &= C_p x_p \ . \end{aligned}\right\} \tag{2.1}$$

Note that (2.1) is obtained by setting $\beta = I_{n_u}$ and $w_u = 0$ in (1.1). The control problem under consideration is following:

Input/Output Variance Constrained (IOVC) Control Problem

Find a stabilizing linear control law u attaining a minimum value of a quadratic objective function subject to the inequality constraint on each input and output variance for the given system (2.1), i.e.,

$$\left.\begin{aligned} &\min_{u\in\Omega} J(u) \triangleq E_\infty(y^T Q_o y + u^T R_o u) \\ &\text{subject to } \begin{Bmatrix} E_\infty y_i^2 \le \sigma_i^2, & i = 1,\cdots, n_y \\ E_\infty u_j^2 \le \mu_j^2, & j = 1,\cdots, n_u \end{Bmatrix} \end{aligned}\right\} \tag{2.2a}$$

where

$$\Omega = \{\ u \mid u = \text{stabilizing control law}\ \} \ , \tag{2.2b}$$

Q_o is a given positive semi-definite output weighting matrix, R_o a given positive definite input weighting matrix, σ_i^2 the given upper bound of the i-th output variance, μ_j^2 the given upper bound of the j-th input variance.

We shall confine ourself only to state feedback and full-order dynamic feedback control laws. A similar control problem was considered in [4]. When only the output variance constraints are imposed with $Q_o = 0$, the problem is called Output Variance Constrained (OVC) control problem and has been extensively investigated by several authors [1-3, 5]. A companion problem called the Input Variance Constrained (IVC) control problem is also considered with $R_o = 0$ and only the input variance constraints [1,6].

We shall provide a dual version of IOVC control problem (2.2), whose solution is also a solution to the above problem. Hence a sufficiency is guaranteed. We first consider the following problem intimately related to the problem (2.2):

Lagrangian Problem

$$\min_{u\in\Omega} L(u,q,r) \triangleq J(u) + \sum_{i=1}^{n_y} q_i(E_\infty y_i^2 - \sigma_i^2) + \sum_{j=1}^{n_u} r_j(E_\infty u_j^2 - \mu_j^2) \tag{2.3a}$$

where the parameters q and r are fixed vectors whose elements are nonnegative, i.e.,

$$q = [\ q_1, q_2, \cdots, q_{n_y}]^T, \quad q_i \ge 0\ , \quad i = 1, \cdots, n_y\ , \tag{2.3b}$$

$$r = [\ r_1, r_2, \cdots, r_{n_u}]^T, \quad r_j \ge 0\ , \quad j = 1, \cdots, n_u\ . \tag{2.3c}$$

Then a dual version of the IOVC control problem is given as follows:

A Dual Version of IOVC Control Problem

$$\max_{(q,r)\in\Psi} h(q,r) \triangleq \min_{u\in\Omega} L(u,q,r) \tag{2.4a}$$

where

$$\Psi \triangleq \{\ (q,r) \mid q \ge 0\ , r \ge 0\ , \text{ and } \min_{u\in\Omega} L(u,q,r) \text{ exists }\ \}\ . \tag{2.4b}$$

It can be shown that $h(q,r)$ is differentiable at some $(\bar{q},\bar{r}) \in \Psi$, and the partial derivative is given by

$$\left.\frac{\partial h}{\partial q_i}\right|_{(q,r)=(\bar{q},\bar{r})} = E_\infty \bar{y}_i^2 - \sigma_i^2\ , \tag{2.5a}$$

$$\left.\frac{\partial h}{\partial r_j}\right|_{(q,r)=(\bar{q},\bar{r})} = E_\infty \bar{u}_j^2 - \mu_j^2 \tag{2.5b}$$

where $\bar{y}$ is the output vector when the control law is $\bar{u}$, which is the solution to the Lagrangian problem (2.3) with $(q,r) = (\bar{q},\bar{r})$. More details about underlining derivation may be found in [7].

We now provide a new algorithm to solve the IOVC control problem.

IOVC Algorithm

Enter A_p, B_p, C_p, D_p, W_p, $Q_o \ge 0$, $R_o > 0$, $q^{(0)} \ge 0$, $r^{(0)} \ge 0$.

Step 1 Set $k = 0$.

Step 2 Solve the Lagrangian problem (2.3) with $Q(q) \triangleq Q_o + \text{diag}[q^{(k)}]$ and $R(r) \triangleq R_o + \text{diag}[r^{(k)}]$.

Step 3 Calculate the value of the dual objective function $h(q,r)$ in (2.4) and, if necessary, the derivatives $\frac{\partial h}{\partial q}$ and $\frac{\partial h}{\partial r}$ by (2.5).

Step 4 Set $k = k + 1$ and utilize the information obtained in Step 3 to update $q^{(k)}$ and $r^{(k)}$ as follows ;

$$\begin{pmatrix} q^{(k+1)} \\ r^{(k+1)} \end{pmatrix} = \begin{pmatrix} q^{(k)} \\ r^{(k)} \end{pmatrix} + \alpha s^{(k)}$$

where α is the step size and $s^{(k)}$ is the search direction.

Step 5 Repeat Steps 2 to 4 until convergence.

Notice that in Step 4, α and $s^{(k)}$ should be calculated so that $q^{(k+1)} \geq 0$ and $r^{(k+1)} \geq 0$ must be satisfied. They may be obtained by any existing algorithm [8] under the constraints $q^{(k+1)} \geq 0$ and $r^{(k+1)} \geq 0$.

3. An Actuator Selection Algorithm by OVC/IVC Algorithm

In this section the close relationship between output variance constrained control problem and input variance control problem will be utilized to solve the optimal actuator selection problem posed in section 1, which is a nonlinear (0,1) integer programming problem. Our method is an iterative algorithm instead of directly solving the integer program which requires extremely intensive computation even for moderately-sized problems.

The following theorem provides the relationship between OVC and IVC control problems:

Theorem [7]

I. Let $\bar{q}$ solve the dual version of the OVC Control Problem and let $\bar{u}$ be the solution to its Lagrangian Problem with $q = \bar{q}$. Suppose that for the IVC Control Problem we choose

$$Q_o = \text{diag}(\bar{q}) \quad \text{and} \quad \mu_j^2 = E_\infty \bar{u}_j^2, \quad j = 1, 2, \cdots, n_u, \tag{3.1}$$

then $\bar{u}$ is also a solution to the IVC Control Problem and the corresponding multipliers are the diagonal elements of R_o given in the OVC control problem.

II. Let $\hat{r}$ solve the dual version of the IVC Control Probelm and let $\hat{u}$ be the solution to its Lagrangian Problem with $r = \hat{r}$. Suppose that for the OVC Control Problem we choose

$$R_o = \text{diag}(\hat{r}) \quad \text{and} \quad \sigma_i^2 = E_\infty \hat{y}_i^2, \quad i = 1, 2, \cdots, n_y, \tag{3.2}$$

where $E_\infty \hat{y}_i^2$ is the i-th output variance in the IVC control problem when $\hat{u}$ is applied, then $\hat{u}$ is also a solution to the OVC Control Problem and the corresponding multipliers are the diagonal elements of Q_o given in the IVC control problem.

The above Theorem is the basis of the following algorithm for solving the OAS problem posed in section 1:

An OAS Algorithm

Enter $A_p, B_p, C_p, D_p, W_p, R_o > 0, \sigma^2, \beta_j^{(0)} = 1$ for all j.

Step1 Set $k = 0$.

Step2 Use a dual version to solve the OVC control problem with $R_o > 0$ and calculate the optimal input variances;

$$E_\infty \bar{u}_j^2 \tag{3.3}$$

for all j such that $\beta_j^{(k)} = 1$. The multiplier vector $\bar{q}$ is a solution.

Step3 Use a dual version to solve the IVC control problem with the output weighting matrix

$$Q_o = \text{diag}[\bar{q}_1, \bar{q}_2, \cdots, \bar{q}_{n_y}] \tag{3.4}$$

and an appropriate upper bound (from (3.7a) below) of the input variance, μ^2. The multiplier vector $\bar{r}$ is a solution.

Step4 Calculate the actuator effectiveness values

$$\alpha_j = \bar{r}_j(\text{from Step 3}) \times \frac{E_\infty \bar{u}_j^2(\text{from Step 2})}{\mu_j^2} \tag{3.5}$$

for j such that $\beta_j^{(k)} = 1$.

Step5 Set $k = k + 1$ and delete one actuator which has the smallest value of α_j (i.e., set $\beta_j^{(k)} = 0$ for the corresponding actuator).

Step6 Repeat Steps 2 thru 5 until

$$N = \sum_{j=1}^{n_u} \beta_j^{(k)}. \tag{3.6}$$

The final controller would be the one from Step 2. Hence the output variance constraints are guaranteed to be satisfied. In case that there are variance constraints on each actuator, they should be used for the upper bound μ_j^2 in Step 3 to solve the IVC control problem. Otherwise, the following value is recommended:

$$\mu_j^2 = \gamma_j E_\infty \bar{u}_j^2 \text{ for all j such that } \beta_j^{(k)} = 1 \tag{3.7a}$$

where $E_\infty \bar{u}_j^2$ is available from the solution of OVC control problem in Step 2 and

$$\gamma_j = 1 - \frac{E_\infty \bar{u}_j^2}{\sum_k E_\infty \bar{u}_k^2}. \tag{3.7b}$$

Notice that the above algorithm does not use any integer programming strategy. This algorithm does not guarantee that the solution will be globally optimal. However, for most of our numerical examples, the algorithm yields the optimal solution.

4. Numerical Examples

Consider the simply-supported Euler-Bernoulli beam shown in Figure 1 for which three force actuators and three torque actuators are available. The problem is to find two actuators which consume minimum control energy while satisfying output variance constraints. The system dynamics of the beam is described by modal data as follows:

$$\left.\begin{aligned} \ddot{\eta} + 2\zeta\omega\,\dot{\eta} + \omega^2\,\eta &= B(u + w_a) + w_p, \\ y &= C\eta \end{aligned}\right\} \tag{4.1a}$$

where

$$\omega = \text{diag}[1,\ 4,\ 9,\ 16,\ 25]\ , \qquad (4.1b)$$

$$\xi = \text{diag}[5.0e-3,\ 8.732e-3,\ 1.829e-2,\ 3.211e-2,\ 5.0e-2] \qquad (4.1c)$$

and η is the vector of the modal coordinates. The input distribution matrix B and the output matrix C can be obtained analytically from the mode shapes and the locations of actuators and sensors, respectively [6]. The actuator process noise w_a and the plant disturbance w_p are assumed to be zero-mean Gaussian white noise processes. The intensity of w_p is taken as $W_p = I_5$.
We shall use two different intensities of w_a, $W_a = 0.1\ I_6$ and $W_a = 0.5\ I_6$, to see that there is an optimal number of noisy actuators when the actuator noises are big enough relatively to the actuator signals u. The upper bounds of the output variances are

$$\sigma^2 = [7.9965e-2,\ 8.9080e-2,\ 9.7204e-2,\ 1.1765e-1]^T\ . \qquad (4.2)$$

For the IVC control problem, we used the formula given by (3.7) to get the input variance constraints. The program CONSTR.M in MATLAB Optimization Toolbox [9] was utilized to solve the dual version (2.4) for both OVC and IVC control problems.

The actuator selection process proposed in OAS algorithm is shown in Table 1, where the actuator effectiveness values defined by (3.5) and the total control energy are given at each iteration. N represents the number of actuators retained for controller design at each iteration. According to the minimum value of α_j, one actuator is deleted at each iteration of OAS algorithm. Notice that for this example deleting 4 actuators (F_1, F_2, T_2 and T_3) at the first iteration (N = 6) results in the same set of actuators as one-at-a-time deletion. For both cases of $W_a = 0.1\ I_6$ and $W_a = 0.5\ I_6$, the actuator to be deleted according to the α_j is the same at each iteration. When $W_a = 0.1\ I_6$, the total control energy required to meet the output variance constraints are monotonically increasing as the number of actuators decreases. By exhausting all possible combinations of the specified number of actuators, we confirmed that at each iteration the remaining set of actuators shown in Table 1 consumes minimum energy with respect to other set of actuators. For example, at the iteration of N = 4, there are 15 possible combinations of 4 actuators ($_6C_4$), out of which the set in Table 1 {F_2, F_3, T_1 and T_2} turn out to consume minimum control energy. Table 2 shows the input variances calculated after the OVC control problem is solved at the first iteration. Notice that the second case ($W_a = 0.5\ I_6$) use very noisy actuators (the squared RMS values of the actuator signals are smaller than the intensity of the noises). As shown in the last row of Table 1, for the case of $W_a = 0.5\ I_6$, the minimum value of control energy is attained at N = 4. Hence the optimal number of actuators is 4.

Once the actuator selection problem is solved, the OAS algorithm provides valuable information: actuator sizes necessary to achieve the required performance. For our examples, the required RMS dynamic range of actuators, F_3 and T_1, are

$$\left.\begin{aligned} F_3\,(S) &= \sqrt{0.45713} = 0.6761 \\ T_1\,(S) &= \sqrt{0.57019} = 0.7551 \end{aligned}\right\} \text{ for } W_a = 0.1\ I_6,\ (4.3a)$$

$$\left.\begin{aligned} F_3\,(S) &= \sqrt{0.64982} = 0.8061 \\ T_1\,(S) &= \sqrt{0.57019} = 0.9390 \end{aligned}\right\} \text{ for } W_a = 0.5\ I_6.\ (4.3b)$$

It is concluded that the proposed actuator selection algorithm produces the best set of actuators while the required performance is achieved. In addition, the information about actuator size necessary to meet the required performance is obtained as a by-product of the algorithm. When noisy actuators are to be used, the algorithm can yield the optimal number of actuators. However, it is found that if the actuator noise is much greater than the external plant disturbance our algorithm fails to find the optimal solution in one example. It is worthwhile to investigate under what conditions the algorithm produces the optimal solution.

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Figure 1. Simply-Supported Euler-Bernoulli Beam

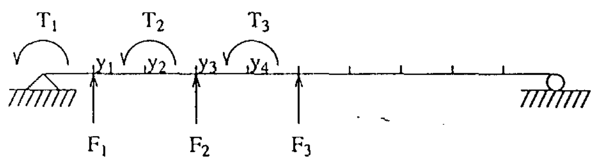

Table 1. Actuator Selection Process for the Euler-Bernoulli Beam

Actuator	Actuator Effectiveness Value a_j				
	N = 6	N = 5	N = 4	N = 3	N = 2
F_1	1.6754 (1.6323)				
F_2	2.1184 (2.0610)	2.2031 (2.1439)	2.4882 (2.4510)	3.3083 (3.2139)	
F_3	2.4057 (2.2693)	2.5054 (2.3672)	2.8612 (2.7246)	4.1211 (3.7757)	8.1223 (7.3020)
T_1	2.7777 (2.8560)	2.9289 (3.0116)	3.6281 (3.6558)	5.0921 (5.1989)	11.396 (11.261)
T_2	2.0936 (2.0263)	2.1750 (2.1057)	2.4313 (2.3948)		
T_3	1.6791 (1.7059)	1.7236 (1.7522)			
Total Control Energy	7.2291e-1 (1.6060e+0)	7.3143e-1 (1.5926e+0)	7.5902e-1 (1.4688e+0)	8.5565e-1 (1.4697e+0)	1.0273e+0 (1.5316e+0)

*. The quantities in ( ) are for W_a = 0.5 I_6.

Table 2. Input Variances at the First Iteration

Actuator Noise Intensity W_a	F_1	F_2	F_3	T_1	T_2	T_3
0.1 I_6	0.02052	0.11848	0.17218	0.22248	0.11623	0.07307
0.5 I_6	0.04880	0.25799	0.34308	0.53635	0.24645	0.17336