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Computing Coarser Observation Functions Using Control-Compatible States of Supervisor

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Abstract

The paper discusses the problem of computing coarser observation functions in supervisory control of discrete event systems. It is shown that when a supervisor that realizes a given language L has certain properties, L-realizability of a coarser observation function is equivalent to control-compatibility of the states in some subsets of the state space of the supervisor. This characterization is then used to devise an iterative procedure of computing coarser L-realizable observation functions, where supervisor reduction and L-realizability verification of an observation function are performed at each iteration.

1 Introduction

In the supervisory control framework, first developed by Ramadge and Wonham [1] and then extended to the case of partial observations by Cieslak et al. [2], a discrete event dynamic system (DEDS) is modeled by an automaton and controlled by a supervisor that observes the occurrences of events in DEDS through an observation function. When a desired language (behavior of the system) L is found to be realizable, i.e., when there is a supervisor that realizes L with an observation function, we naturally ask if there is another supervisor that needs a coarser observation function (less information about the occurrences of the events in DEDS) and still be able to realize L. This is the observation function design problem first formulated in [6]. To solve this problem, we must have an effective method of verifying L-realizability of a coarser observation function (L-realizability of an observation function M guarantees the existence of a supervisor that realizes L with M; see [6]).

L-realizability of observation functions has been studied in [4] and shown to be equivalent to requiring a certain structural property to hold for a pair of automata representing the DEDS behavior and L. Also, in [7], it is shown that when a supervisor that realizes L has certain properties, L-realizability of a coarser observation function implies that each state of the image (see Section 2) of the supervisor consists of control-compatible states. The result of [7] is, however, restricted to the case where the supervisor realizes L with perfect observation and the coarser observation function under consideration is a projection. In this paper, we establish the same result as in [7] which is applicable to a wider class of cases. We also prove the reverse implication of the statement; thus a complete characterization of L-realizable coarser observation functions that utilizes only the supervisor structure is given.

Using these results, we also show that if a coarser observation function M turns out to be L-realizable, a reduced supervisor can be constructed in a simple manner. Thus we can use the reduced supervisor in computing yet another observation function coarser than M. Therefore we suggest in the paper an iterative procedure of computing coarser observation functions in which the above results are used repetitively.

2 Preliminaries

2.1 Supervisory Control of DEDS

In the supervisory control of a DEDS, we model the system by a finite automaton (FA) $G = (Q, \Sigma, \delta, q_0)$, where Q is a finite set of states, q_0 the initial state, δ a state transition function, and Σ a set of events. The language $L(G)$ defined by

$$L(G) := \{w \in \Sigma^* : \delta(w, q_0)!\}$$

represents the behavior of the "uncontrolled" DEDS (here, $\delta(w,q)!$ means that $\delta(w,q)$ is defined).

An observation function $M : \Sigma \to \Delta \cup \{\epsilon\}$, where ϵ is the empty string and Δ a set of output symbols, represents the partial observation of the DEDS. M is extented to Σ^* in an obvious manner [2].

A supervisor (controller) is modeled by a pair $\mathcal{S} = (S, \phi)$, where $S = (X, \Delta, \xi, x_0)$ is an FA and $\phi : X \to \Gamma$, Γ the set of all control patterns, is an output mapping. A control pattern $\gamma \subset \Sigma$ signifies the control action to the DEDS under which only the events in $\Sigma - \gamma$ are allowed to occur in the system. Usually, Σ is the union of the set of controllable events Σ_c and the set of uncontrollable events Σ_u. In this case, γ must contain Σ_u; thus the supervisor has no control over uncontrollable events.

The closed loop system consisting of G, M and $\mathcal{S}$ is represented by another FA and denoted by $\mathcal{S}/G$; specifically, $\mathcal{S}/G = (X \times Q, \Sigma, f, (x_0, q_0))$, where $f(\sigma,(x,q))!$ $(= (\xi(M(\sigma), x), \delta(\sigma, q)))$ iff $\delta(\sigma,q)!$, $\xi(M(\sigma),x)!$ and $\sigma \in \phi(x)$. Thus the language $L(\mathcal{S}/G)$ describes the closed loop system behavior. We note here that $f(w,(x_0,q_0)) = (\xi(M(w),x_0), \delta(w,q_0))$ whenever it is defined. Now, if $\mathcal{S}$ has a property called completeness ($\mathcal{S}$ is *complete* if $\xi(M(w\sigma),x_0)!$ whenever $f(w,(x_0,q_0))!$, $\delta(w\sigma,q_0)!$ and $\sigma \in \phi(\xi(M(w),x_0))$, the language $L(\mathcal{S}/G)$ can be conveniently defined recursively by i) $\epsilon \in L(\mathcal{S}/G)$ and ii) $w\sigma \in L(\mathcal{S}/G)$ iff $w \in L(\mathcal{S}/G)$, $\sigma \in \phi(\xi(M(w),x_0))$ and $w\sigma \in L(G)$. In this paper, we assume that every supervisor is complete unless otherwise stated.

When M is an observation function for $\mathcal{S}$ and $L(\mathcal{S}/G) = L$, we say that $\mathcal{S}$ *realizes* L *with* M. The necessary and sufficient conditions for the existence of a supervisor that realizes a prefix-closed language L are found to be [2]: (a) L is $(\Sigma_u, L(G))$*-invariant*; i.e., $L\Sigma_u \cap L(G) \subset L$, and (b) L is $(M, \Sigma_c, L(G))$*-controllable*; i.e., $s,t \in L$, $\sigma \in \Sigma_c$, $s\sigma \in L$, $t\sigma \in L(G)$ and $M(s) = M(t) \Rightarrow t\sigma \in L$. We say for convenience that M is L*-realizable* when L satisfies the above conditions (a) and (b).

A supervisor $\mathcal{S} = (S, \phi)$ is called (M,L)*-normal* if ϕ is given in the form $\phi = (\phi_0, \phi_1)$ where

$$\begin{aligned}\phi_0(x) = &\ \{\sigma \in \Sigma_c : \exists\, s \in L \text{ such that } \xi(M(s),x_0) = x \\ &\ \text{and } s\sigma \in L(G) - L\,\},\\ \phi_1(x) = &\ \{\sigma \in \Sigma_c : \exists\, s \in L \text{ such that } \xi(M(s),x_0) = x \\ &\ \text{and } s\sigma \in L\,\},\end{aligned}$$

and $\phi_0(x) \cap \phi_1(x) = \emptyset$ for all x. (Note that if $L \subset L(S)$, then the condition that $\phi_0(x) \cap \phi_1(x) = \emptyset$ for all x implies that L is $(M, \Sigma_c, L(G))$-controllable.) When $\mathcal{S}$ is a normal supervisor, the control pattern $\phi(x)$ can be defined to be any set satisfying $\phi_1(x) \cup \Sigma_u \subset \phi(x) \subset \Sigma - \phi_0(x)$; in other words, the events in the set $\Sigma_c - (\phi_0(x) \cup \phi_1(x))$ are redundant in determining the language $L(\mathcal{S}/G)$([7,3]).

Frequently, S is the *image under* M of a recognizer for L. That is, if $A = (Z, \Sigma, \eta, z_0)$ is the *recognizer* for L (i.e., $L(A) = L$), then $S = Ac(2^Z - \{\emptyset\}, \Delta, \xi, x_0)$ where $\Delta = M(\Sigma) - \{\epsilon\}$, $x_0 = \{\eta(s,z_0) : M(s) = \epsilon\}$, and

$$\xi(\delta, x) = \begin{cases} \{\eta(s,z) : z \in x,\ M(s) = \delta\}, & \text{if it is nonempty} \\ \text{undefined, otherwise} \end{cases}$$

(Here Ac means that S is the accessible component [1] of the automaton defined above.) We note that for $d \in \Delta^*$, $\xi(d, x_0) = \{\eta(w,z_0) : M(w) = d\}$ whenever the right hand side is not empty. Thus if S is the image under M of a recognizer for L, then $L(S) = M(L)$ where $M(L) := \{M(w) : w \in L\}$. We can also show that if S is the image of a recognizer for L and if L is realizable, the sets ϕ_0 and ϕ_1 defined above have the property that $\phi_0(x) \cap \phi_1(x) = \emptyset$ $\forall\ x$ (refer to [1] for perfect observation case and to [7] for the case of M being a projection).

For a normal supervisor $\mathcal{S} = (S, \phi)$, we can introduce the notion of control-compatible states; x_1 is *control-compatible* to x_2, written $x_1 \sim x_2$, if $\phi_0(x_1) \cap \phi_1(x_2) = \emptyset = \phi_1(x_1) \cap \phi_0(x_2)$. The notions of normal supervisors and control-compatibility proved to be useful in treating the supervisor reduction problem [3] and in computing a maximal projection [7].

2.2 Equivalence and Coarseness of Observation Functions

Let M_1 and M_2 be observation functions. M_1 is said to be *equivalent* to M_2, written $M_1 \equiv M_2$, if (i) $M_1^{-1}(\epsilon) = M_2^{-1}(\epsilon)$ and (ii) $\forall \sigma_1, \sigma_2 \in \Sigma$, $M_1(\sigma_1) = M_1(\sigma_2)$ iff $M_2(\sigma_1) = M_2(\sigma_2)$. Also, M_2 is said to be *coarser* than M_1, written $M_1 \leq M_2$, if (i) $M_1^{-1}(\epsilon) \subset M_2^{-1}(\epsilon)$ and (ii) $\forall \sigma_1, \sigma_2 \in \Sigma - M_2^{-1}(\epsilon)$, $M_1(\sigma_1) = M_1(\sigma_2) \Rightarrow M_2(\sigma_1) = M_2(\sigma_2)$. Equivalent observation functions convey exactly the same amount of information to the supervisor; thus we consider them identical. Note that if $M_1 \equiv M_2$, then $M_1 \leq M_2$ and $M_2 \leq M_1$, and vice versa. The following fact is also an immediate consequence of the definition: if $M_1 \leq M_2$, then $M_1(s) = M_1(t)$ implies $M_2(s) = M_2(t)$.

3 Characterization of L-Realizable Coarser Observation Functions

In this section, we characterize the L-realizable coarser observation functions in terms of some structural properties of the supervisor. Let $L(G)$ and L be given, and assume that L is $(\Sigma_u, L(G))$-invariant. Let $M_0 : \Sigma \to \Delta$ be an L-realizable observation function and let $\mathcal{S} = (S, \phi)$ with $S = (X, \Delta, \xi, x_0)$ be a (M_0, L)-normal supervisor that realizes L with M_0. Finally, we let $M : \Sigma \to \Delta$

be an observation function such that $M_0 \leq M$. In what follows, we restrict ourselves to the following cases:

$$\text{(M1)} \quad M_0 \text{ and } M \text{ are projections}$$
$$\text{(M2)} \quad M_0^{-1}(\epsilon) = M^{-1}(\epsilon).$$

Now define a mapping $\hat{M} : M_0(\Sigma) \to \Delta$ by

$$\hat{M}(\delta) = M(\sigma) \text{ where } \sigma \in M_0^{-1}(\delta).$$

Lemma 1 *$\hat{M}$ has the following properties:*

(a) $\hat{M}(\epsilon) = \epsilon$

(b) $\hat{M}$ is well defined

(c) for the case (M1), $\hat{M}\,|_{M_0(\Sigma)-\{\epsilon\}} = M\,|_{M_0(\Sigma)-\{\epsilon\}}$

(d) $M(\sigma) = \hat{M}(M_0(\sigma))$

Proof: (a) Trivial.
(b) For the case (M1), $M_0^{-1}(\delta)$, $\delta \neq \epsilon$, is a singleton set. Thus $\hat{M}$ is well defined. Consider the case (M2). If $\sigma_1,\ \sigma_2 \in M_0^{-1}(\delta)$, $\delta \neq \epsilon$ (i.e., $M_0(\sigma_1) = M_0(\sigma_2) \neq \epsilon$), then $\sigma_1,\ \sigma_2 \in \Sigma - M_0^{-1}(\epsilon) = \Sigma - M^{-1}(\epsilon)$. Since $M_0 \leq M$, $M(\sigma_1) = M(\sigma_2)$. Thus $\hat{M}$ is well defined.
(c) Trivial.
(d) If $\sigma \in M_0^{-1}(\epsilon)$, then $\sigma \in M^{-1}(\epsilon)$ since $M_0 \leq M$. Thus $M(\sigma) = \epsilon$ and $\hat{M}(M_0(\sigma)) = \hat{M}(\epsilon) = \epsilon$. Suppose that $\sigma \notin M_0^{-1}(\epsilon)$. For the case (M1), $\hat{M}(M_0(\sigma)) = \hat{M}(\sigma) = M(\sigma)$ where the last equality follows from (c). Consider the case (M2). By the definition, $\hat{M}(M_0(\sigma)) = M(\sigma')$ where $\sigma' \in M_0^{-1}(M_0(\sigma))$, i.e., $M_0(\sigma') = M_0(\sigma)$. Since $M_0 \leq M$ and $\sigma, \sigma' \notin M_0^{-1}(\epsilon) = M^{-1}(\epsilon)$, $M(\sigma') = M(\sigma)$. Hence $\hat{M}(M_0(\sigma)) = M(\sigma)$. ∎

We extend $\hat{M}$ to $(M_0(\Sigma))^*$ by defining $\hat{M}(d\delta) = \hat{M}(d)\hat{M}(\delta)$ for all $d \in (M_0(\Sigma))^*$, $\delta \in M_0(\Sigma)$. It then follows that if $M_0 \leq M$, $M(s) = \hat{M}(M_0(s))$ for all $s \in \Sigma^*$.

Now we define a set $X_M(d)$ of states of the supervisor S for each $d \in (M(\Sigma))^*$:

$$X_M(d) := \{\xi(e, x_0) : \hat{M}(e) = d\}.$$

Note that $X_M(d)$ is a state of the image of S under $\hat{M}$, and thus the set of different $X_M(d)$'s is finite.

Proposition 1 *If $X_M(d)$ consists of control-compatible states for each $d \in (M(\Sigma))^*$, then M is L-realizable.*

Proof: Suppose to the contrary that M is not realizable. Then L is not $(M, \Sigma_c, L(G))$-controllable since L is $(\Sigma_u, L(G))$-invariant. Thus there exist $s, t \in L$ with $M(s) = M(t) := d$ such that for some $\sigma \in \Sigma_c$, $s\sigma \in L$ and $t\sigma \in L(G) - L$. Hence, if we let $x = \xi(M_0(s), x_0)$ and $y = \xi(M_0(t), x_0)$, then $\sigma \in \phi_1(x)$ and $\sigma \in \phi_0(y)$. Thus x and y are not control-compatible. Now note that $\hat{M}(M_0(s)) = M(s) = d = M(t) = \hat{M}(M_0(t))$. Thus $x, y \in X_M(d)$. Therefore we have the set $X_M(d)$ where some states are not control-compatible, which contradicts the assumption. ∎

The converse to Proposition 1 is not true in general. However, if the supervisor satisfies certain conditions, the converse statement can be shown to hold. Before stating what conditions are required for the supervisor, we introduce the notion of subautomaton.

Let $A = (Q_A, \Sigma, \delta_A, q_{A0})$ and $B = (Q_B, \Sigma, \delta_B, q_{B0})$ be two accessible FA's with $L(B) \subset L(A)$. The FA B is said to be a *subautomaton* of A if $\delta_B(s, q_{B0}) = \delta_A(s, q_{A0})$ for all $s \in L(B)$. In this case, we necessarily have that $Q_B \subset Q_A$ and $q_{A0} = q_{B0}$. We also note that given two FA's G_1 and G_2 with $L(G_2) \subset L(G_1)$, we can always construct two FA's A and B such that B is a subautomaton A, $L(A) = L(G_1)$ and $L(B) = L(G_2)$ (see [5] for details).

Now consider the following conditions for the FA S:

(S1) $L(S) = M_0(L)$

(S2) if $\xi(M_0(w_1), x_0) = \xi(M_0(w_2), x_0)$ for $w_1, w_2 \in L$, then the following implications hold:
  i) $w_1\sigma \in L \Rightarrow \exists\, w_3 \in L$ such that $M_0(w_3) = M_0(w_2)$ and $w_3\sigma \in L$
  ii) $w_1\sigma \in L(G) - L \Rightarrow \exists\, w_3 \in L$ such that $M_0(w_3) = M_0(w_2)$ and $w_3\sigma \in L(G) - L$

The condition (S2) above looks hard to satisfy. However, an FA S satisfying the above conditions can be easily constructed as demonstrated in the following lemma.

Lemma 2 *Let G_s be a subautomaton of G with $L(G_s) = L$, and let S be the image of G_s under M_0. Then S satisfies the conditions (S1) and (S2).*

Proof: Clearly, (S1) holds since S is the image of G_s under M_0. Suppose that $w_1, w_2 \in L$ and $\xi(M_0(w_1), x_0) = \xi(M_0(w_2), x_0)$. Let $q = \delta_s(w_1, q_0)$. Then $q = \delta(w_1, q_0)$ (G_s is a subautomaton of G). Moreover, $q \in \xi(M_0(w_1), x_0)$ since S is the image of G_s under M_0. Thus we have that $q \in \xi(M_0(w_2), x_0)$. Hence there is w_3 such that $M_0(w_3) = M_0(w_2)$ and $\delta_s(w_3, q_0) = q$. Note that $w_3 \in L(G_s) = L$. Also, $\delta(w_3, q_0) = q$ since G_s is a subautomaton of G. Now if $w_1\sigma \in L$, then $\delta_s(\sigma, q)!$ so that $\delta_s(w_3\sigma, q_0)!$, i.e., $w_3\sigma \in L$. If $w_1\sigma \in L(G) - L$, then $\delta(\sigma, q)!$ and $\delta_s(\sigma, q)$ is undefined. Therefore $\delta(w_3\sigma, q_0)!$, i.e., $w_3\sigma \in L(G)$. Also, $\delta_s(w_3\sigma, q_0)$ is undefined, i.e., $w_3\sigma \notin L$. Hence $w_3\sigma \in$

$L(G) - L$. ∎

We now present a technical lemma that is useful when we prove the converse statement to Proposition 1.

Lemma 3 *Assume (S1) and let $d \in (M(\Sigma))^*$. If $x \in X_M(d)$, then there exists $s \in L$ such that $M(s) = d$ and $\xi(M_0(s), x_0) = x$.*

Proof: Let $x \in X_M(d)$. By the definition of $X_M(d)$, there exists $d_1 \in (M_0(\Sigma))^*$ such that $\hat{M}(d_1) = d$ and $\xi(d_1, x_0) = x$. Note that $d_1 \in L(S)$. Since $L(S) = M_0(L)$ by (S1), there exists $s \in L$ such that $M_0(s) = d_1$. Note that $\xi(M_0(s), x_0) = x$. Also, $M(s) = \hat{M}(M_0(s)) = \hat{M}(d_1) = d$. ∎

Proposition 2 *Assume (S1) and (S2). If M is L-realizable, then $X_M(d)$ consists of control-compatible states for each $d \in (M(\Sigma))^*$.*

Proof: Suppose that the conclusion does not hold for a set $X_M(d)$, $d \in (M(\Sigma))^*$. Then there exist $x_1, x_2 \in X_M(d)$ and $\sigma \in \Sigma_c$ such that $\sigma \in \phi_0(x_1) \cap \phi_1(x_2)$. It follows from the definition of ϕ_0 and ϕ_1 that

(*) $\exists\ u, v \in L$ such that $\xi(M_0(u), x_0) = x_1$, $\xi(M_0(v), x_0) = x_2, u\sigma \in L(G) - L$ and $v\sigma \in L$.

Also, $x_1, x_2 \in X_M(d)$ implies (Lemma 3) that

(**) $\exists\ s, t \in L$ such that $M(s) = M(t) = d$, $\xi(M_0(s), x_0) = x_1$ and $\xi(M_0(t), x_0) = x_2$.

From (*) and (**), we have two strings $u, s \in L$ such that $\xi(M_0(u), x_0) = \xi(M_0(s), x_0)$ and $u\sigma \in L(G) - L$. By (S2), there exists $s' \in L$ such that $M_0(s') = M_0(s)$ and $s'\sigma \in L(G) - L$. Similarly, there is $t' \in L$ such that $M_0(t') = M_0(t)$ and $t' \in L$. Note that $M(s') = \hat{M}(M_0(s')) = \hat{M}(M_0(s)) = M(s)$. Similarly, $M(t') = M(t)$. Recall from (**) that $M(s) = M(t)$. So $M(s') = M(t')$. Hence we have two strings of events $s', t' \in L$ and $\sigma \in \Sigma_c$ such that $M(s') = M(t')$, $s'\sigma \in L(G) - L$ and $t'\sigma \in L$. Therefore M is not L-realizable, which is a contradiction. Thus we proved the proposition. ∎

Proposition 2 is a generalization of the result in [7] where only the projections are considered as observation functions and M_0 is restricted to be the identity mapping. Now Proposition 1 and Proposition 2 give a complete characterization of L-realizable coarser observation functions; that is, if the supervisor has the properties (S1) and (S2), then L-realizability of a coarser observation function M is equivalent to control-compatibility of the states in $X_M(d)$ for all $d \in (M(\Sigma))^*$.

4 Iterative Procedure of Computing Coarser Observation Functions

Let $\mathcal{S} = (S, \phi)$ be a (M_0, L)-normal supervisor where S satisfies (S1) and (S2). Suppose that an observation function M with $M_0 \leq M$ has been verified, by the use of the result in Section 3, to be L-realizable. Thus each $X_M(d)$ consists of control-compatible states. As noted earlier, each $X_M(d)$ is a state of the image S_M of S under $\hat{M}$. In other words, if we let $S_M = (Y, \Delta, \varsigma, y_0)$, then $\varsigma(d, y_0) = X_M(d) = \{\xi(e, x_0) : \hat{M}(e) = d\}$. Note that the set of events in S_M, i.e., $M(\Sigma) - \{\epsilon\}$, has a smaller cardinality than that in S. Also, the number of states of S_M is frequently smaller than that of S. Now we ask: can we construct a (reduced) supervisor $\mathcal{S}_M = (S_M, \phi_M)$ that realizes L with M? If so, does (or can we make) the supervisor $\mathcal{S}_M$ have the property that $\mathcal{S}_M$ is (M, L)-normal and S_M satisfies the conditions (S1) and (S2) with M_0 replaced by M? If the answers to these questions are yes, then we can attempt to get, by investigating the control-compatibility between the states in S_M, yet another observation function that is coarser than M; thus we could devise an iterative procedure where the original observation function gets coarser and coarser as the number of iterations increases. Before we answer the quetions raised above, we present an important result on the closed loop system behavior for the case of normal supervisors.

Proposition 3 *Let $L \subset L(G)$ be closed and $(\Sigma_u, L(G))$-invariant. Let $\mathcal{T} = (T, \psi)$ be a (M, L)-normal supervisor with $M(L) \subset L(T)$. Then $\mathcal{T}$ is complete and realizes L with M.*

Proof: Let $T := (Z, \Delta, \eta, z_0)$ and let $\mathcal{T}/G := (Z \times Q, \Sigma, g, (z_0, q_0))$. We prove the statement in three steps.

(a) $(L(\mathcal{T}/G) \subset L)$ We prove by induction. First, note that $\epsilon \in L(\mathcal{T}/G)$ and $\epsilon \in L$. Suppose now that $w\sigma \in L(\mathcal{T}/G)$, i.e., $g(w\sigma, (z_0, q_0))!$ By the definition of g, $g(w, (z_0, q_0))!$ (i.e., $w \in L(\mathcal{T}/G)$), $\delta(w\sigma, q_0)!$ (i.e., $w\sigma \in L(G)$), $\eta(M(w\sigma), z_0)!$ and $\sigma \in \psi(z)$ where $z = \eta(M(w), z_0)$. By the induction hypothesis, $w \in L$. Now if $\sigma \in \Sigma_u$, $w\sigma \in L$ since L is $(\Sigma_u, L(G))$-invariant. Suppose that $\sigma \in \Sigma_c$. If $w\sigma \notin L$, then $\sigma \in \psi_0(z)$, which contradicts the fact that $\psi(z) \cap \psi_0(z) = \emptyset$. Thus $w\sigma \in L$.

(b) ($\mathcal{T}$ is complete) Suppose that $g(w, (z_0, q_0))!$, $\delta(w\sigma, q_0)!$ and $\sigma \in \psi(z)$ where $z = \eta(M(w), z_0)$. By the result (a), $w \in L$. Since $w\sigma \in L(G)$, we must have that $\sigma \in \psi_1(z) \cup \Sigma_u$. Now if $\sigma \in \Sigma_u$, then $w\sigma \in L$ since L is $(\Sigma_u, L(G))$-invariant. Since $M(L) \subset L(T)$, $M(w\sigma) \in L(T)$ and therefore $\eta(M(w\sigma), z_0)!$ Suppose that $\sigma \in \psi_1(z)$. Then there exists $s \in L$ such that $\eta(M(s), z_0) = z$ and $s\sigma \in L$. Again, by the assumption that $M(L) \subset L(T)$, $\eta(M(s\sigma), z_0)!$, which in turn implies that $\eta(M(\sigma), z)!$ Recall that $\eta(M(w), z_0) = z$. Thus $\eta(M(\sigma), z) = \eta(M(w\sigma), z_0)$. Therefore $\eta(M(w\sigma), z_0)!$ Hence we have established that $\mathcal{T}$ is complete.

(c) ($L \subset L(\mathcal{T}/G)$) We prove by induction. If $w\sigma \in L$, then $w \in L$. By the induction hypothesis, $w \in L(\mathcal{T}/G)$. Let $\eta(M(w), z_0) = z$. It follows from the definition of ψ_1, $\sigma \in \psi_1(z) \subset \psi(z)$. Hence, by the definition of $L(\mathcal{T}/G)$ for complete supervisors, $w\sigma \in L(\mathcal{T}/G)$. ∎

In what follows, we will answer the questions of the beginning of this section.

Proposition 4 *Let S satisfy (S1) and (S2), and let $M_0 \leq M$. Then the image S_M of S under $\hat{M}$ also has the properties (S1) and (S2) with M_0 replaced by M.*

Proof: Since S_M is the image of S under $\hat{M}$ and since S satisfies (S1), $L(S_M) = \hat{M}(L(S)) = \hat{M}(M_0(L)) = M(L)$. Thus S_M satisfies (S1) with M_0 replaced by M.

Recall that $S_M = (Y, \Delta, \varsigma, y_0)$ and

$$\varsigma(d, y_0) = X_M(d) = \{\xi(e, x_0) : \hat{M}(e) = d\}.$$

Now let $w_1, w_2 \in L$ and suppose that $\varsigma(M(w_1), y_0) = \varsigma(M(w_2), y_0)$. Let $x = \xi(M_0(w_1), x_0)$. Since $\hat{M}(M_0(w_1)) = M(w_1)$, $x \in \varsigma(M(w_1), y_0)$, and therefore $x \in \varsigma(M(w_2), y_0)$. Thus there exists $e \in \Delta^*$ such that $\hat{M}(e) = M(w_2)$ and $\xi(e, x_0) = x$. Since $e \in L(S)$ and $L(S) = M_0(L)$ by (S1) for S, we can choose $u \in L$ such that $M_0(u) = e$. Thus we have that $w_1, u \in L$ and $\xi(M_0(w_1), x_0) = \xi(M_0(u), x_0)$. Suppose now that $w_1\sigma \in L$. Since S satisfies (S2), there exists $v \in L$ such that $M_0(v) = M_0(u)$ and $v\sigma \in L$. Note that $M(v) = \hat{M}(M_0(v)) = \hat{M}(M_0(u)) = \hat{M}(e) = M(w_2)$. Similarly, if $w_1\sigma \in L(G) - L$, then there exists $v \in L$ such that $M(v) = M(w_2)$ and $v\sigma \in L(G) - L$. Hence S_M satisfies (S2) with M_0 replaced by M. ∎

We now construct a supervisor $\mathcal{S}_M = (S_M, \phi_M)$ for G by defining ϕ_M as follows: $\phi_M = (\phi_{M0}, \phi_{M1})$ where

$$\phi_{M0}(y) = \bigcup_{x \in y} \phi_0(x) \quad \text{and} \quad \phi_{M1}(y) = \bigcup_{x \in y} \phi_1(x)$$

Proposition 5 *Let S satisfy (S1) and (S2), and let M be L-realizable. Then $\mathcal{S}_M$ is (M, L)-normal and complete, and realizes L with M.*

Proof: We prove in three steps.

(i) Let

$$\Sigma_y^0 = \{\sigma \in \Sigma_c : \exists\, s \in L \text{ such that } \varsigma(M(s), y_0) = y \text{ and } s\sigma \in L(G) - L\},$$

$$\Sigma_y^1 = \{\sigma \in \Sigma_c : \exists\, s \in L \text{ such that } \varsigma(M(s), y_0) = y \text{ and } s\sigma \in L\}.$$

We show that $\phi_{M0}(y) = \Sigma_y^0$ and $\phi_{M1}(y) = \Sigma_y^1$. If $\sigma \in \Sigma_y^0$, there is $s \in L$ such that $\varsigma(M(s), y_0) = y$ and $s\sigma \in L(G) - L$. Let $x = \xi(M_0(s), x_0)$. Then $\sigma \in \phi_0(x)$. Moreover, $x \in \varsigma(M(s), y_0) = y$ since $\hat{M}(M_0(s)) = M(s)$. Hence $\sigma \in \phi_{M0}(y)$. Now suppose that $\sigma \in \phi_{M0}(y)$. Then there is $x \in y$ such that $\sigma \in \phi_0(x)$. By the definition of ϕ_0, there is $s \in L$ such that $\xi(M_0(s), x_0) = x$ and $s\sigma \in L(G) - L$. Now let $d \in \Delta^*$ be such that $\varsigma(d, y_0) = y$ (such d exists since S_M is accessible). Since $x \in y$, $x \in \varsigma(d, y_0)$. Thus $x = \xi(e, x_0)$ for some e with $\hat{M}(e) = d$. Since $e \in L(S)$ and $L(S) = M_0(L)$ by the property (S1), there exists $w \in L$ such that $M_0(w) = e$. Hence we have $s, w \in L$ such that $\xi(M_0(s), x_0) = x = \xi(M_0(w), x_0)$. By the property (S2), $s\sigma \in L(G) - L$ implies that there is $t \in L$ such that $M_0(t) = M_0(w)$ and $t\sigma \in L(G) - L$. Note that $M(t) = \hat{M}(M_0(t)) = \hat{M}(M_0(w)) = \hat{M}(e) = d$. Thus $\varsigma(M(t), y_0) = \varsigma(d, y_0) = y$. Hence $\sigma \in \Sigma_y^0$. We thus established that $\Sigma_y^0 = \phi_{M0}(y)$. Similarly, $\Sigma_y^1 = \phi_{M1}(y)$.

(ii) We show that $\phi_{M0}(y) \cap \phi_{M1}(y) = \emptyset$ for all y. Let $y \in Y$ be arbitrary. Then $y = X_M(d)$ for some $d \in (M(\Sigma))^*$. Since M is L-realizable, $X_M(d)$ consists of control-compatible states (Proposition 2). In other words, $\phi_0(x_1) \cap \phi_1(x_2) = \emptyset$ for all $x_1, x_2 \in y$. Hence we must have that $\phi_{M0}(y) \cap \phi_{M1}(y) = \emptyset$.

(iii) By (i) and (ii), we showed that $\mathcal{S}_M$ is (M, L)-normal. Also, $L(S_M) = M(L)$ by Proposition 4. It therefore follows from Proposition 3 that $\mathcal{S}_M$ is complete and realizes L with M. ∎

Now we have an (M, L)-normal supervisor $\mathcal{S}_M = (S_M, \phi_M)$ that observes the DEDS G through a coarser mapping M and realizes L. Moreover, S_M satisfies (S1) and (S2), and therefore we could continue to seek for another observation function, which is coarser than M, by using the structural properties of S_M. We note that the sets $X_M(d)$'s form a "cover", for the case of partial observation, which is a key notion used in supervisor reduction problems ([3,7]). We summarize the above discussion in the form of an iterative procedure of computing coarser observation functions.

Procedure A: Given an (M_0, L)-normal supervisor $\mathcal{S}_0 =$

(S_0, ϕ^0) with S_0 satisfying (S1) and (S2),

Step 0. $i = 0$.

Step 1. Select a coarser observation function M_{i+1} such that $M_i \leq M_{i+1}$.

Step 2. Construct the image S_{i+1} of S_i under M_{i+1}.

Step 3. Check if each state $X_{M_{i+1}}(d)$ of S_{i+1} consists of control-compatible states of S_i. If not, go to Step 1.

Step 4. Construct the reduced supervisor $S_{i+1} = (S_{i+1}, \phi^{i+1})$ by defining ϕ^{i+1} in the same manner as shown before Proposition 5.

Step 5. $i = i + 1$ and go to Step 1.

We should note here that there is already an effective method [4] of verifying $(M, \Sigma_c, L(G))$-controllability of L, or equivalently L-realizability of an observation function M. In the method of [4], the image of G_s under M must be constructed and a condition equivalent to the control-compatibility should be verified for every state of the image. Thus both the result in Section 3 and the method in [4] require the construction of the image of an automaton under M. The differences between the two are: first, the result in Section 3 utilizes only the supervisor structure while the method in [4] need to look into the structures of both G and G_s. Verification of the condition required for each state of the image is simpler in the result of this paper. Second, there is a restriction in using the result in Section 3 that either (M1) or (M2) must be the case for M_0 and M. On the other hand, any observation function M can be tested by the method in [4]. When we attempt to coarsen the original observation function successively, however, the use of the result of this paper is much more advantegeous because we can work with the supervisor whose structure continues to get simpler as we proceed. Moreover, a reduced supervisor is obtained as a by-product at almost no cost in each iteration. Therefore whenever we stop the iteration, a reduced supervisor for the final observation function is right there for use.

Finally, we consider again the restriction that in Procedure A, either M_i's are projections or $M_i^{-1}(\epsilon) = M_0^{-1}(\epsilon)$ for all i. In view of this restriction, we suggest to use Procedure A in the following manner: when M_0 is the identity mapping or a projection, use Procedure A to get a maximal projection P_{max}. Let $M_0 = P_{max}$. Use Procedure A to get a maximal observation function M_{max} with $M_{max}^{-1}(\epsilon) = P_{max}^{-1}(\epsilon)$. Then M_{max} is a coarsest L-realizable observation function.

5 Conclusion

In this paper we showed that if a supervisor that realizes L with M_0 has the properties (S1) and (S2), L-realizability of a coarser observation function M is equivalent to control-compatibility of the states in $X_M(d)$ for each $d \in M(\Sigma))^*$. We then showed that we can construct a reduced supervisor in a simple manner which has $X_M(d)$'s as states and possesses the properties (S1) and (S2). These results thus led to suggesting an iterative method by which the original observation function continues to get coarsened.

The results of the paper are applicable only to the case where the observation functions are projections or their inverse images of ϵ are the same. The paper did not deal with the problem of how to get a coarser L-realizable observation function; only the method of verifying L-realizability of a "candidate" observation function has been presented. The problem of obtaining such observation functions should be one of the future research topics in this area. Some results on this issue were reported in [6,7] for the case of projections.

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