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자이로 컴파스 얼라인먼트 오차특성을 고려한 스트랩다운 관성항법장치의 상호분산해석

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Covariance Analysis of Strapdown INS Considering Characteristics of Gyrocompass Alignment Errors

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ABSTRACT

Presented in this paper is a complete error covariance analysis for strapdown inertial navigation system(SDINS). We have found that in SDINS the cross-coupling terms in gyrocompass alignment errors can significantly influence the SDINS error propagation. Initial heading error has a close correlation with the east component of gyro bias error, while initial level tilt errors are closely related to accelerometer bias errors. In addition, pseudo-state variables are introduced in covariance analysis for SDINS utilizing the characteristics of gyrocompass alignment errors. This approach simplifies the covariance analysis because it makes the initial error covariance matrix to a diagonal form. Thus a real implementation becomes easier. The approach is conformed by comparing the results for a simplified case with the covariance analysis obtained from the conventional SDINS error model.

I. INTRODUCTION

The initial alignment of inertial navigation system(INS) is an important process performed prior to normal navigation. For most ground based applications, gyrocompassing is known to be a common self-alignment method[1-5]. Although the purpose of the gyrocompassing is to drive the alignment errors to zero, unfortunately the alignment errors in practical system do not reach zero due to inertial sensor errors[6-9]. The gyrocompass alignment errors of strapdown INS(SDINS) are known to have an interesting relationship with the bias errors of inertial sensors[10-13]. For example, when an SDINS in stationary navigation maintains the alignment attitude, there is a cancellation between the alignment errors and inertial sensor biases. On the other hand, if the SDINS changes the alignment heading, the cancellation between them is perturbed and relatively large navigation error may be generated.

To analyze the navigation error of SDINS considering these characteristics of gyrocompass alignment errors, the Monte Carlo method and individual parameter variation method have been employed[10]. However, covariance analysis method which is very efficient for terrestrial navigation error analysis[14] has not been appeared in the literature. In covariance analysis for SDINS, all states are generally dealt to be jointly Gaussian and to be initially uncorrelated with one another, while in case of considering the characteristics of gyrocompass alignment errors, we must take it into consideration that the initial attitude errors are correlated with the inertial sensor biases.

In this paper, considering such a point, the characteristics of gyrocompass alignment errors are investigated from a stochastic theoretical point of view and the two kinds of covariance analysis approaches are presented. One is to use an existing conventional SDINS error model considering the correlation between the initial attitude errors and the sensor biases. The other is to utilize a modified SDINS error model where the attitude error states are transformed into new state variables, so-called pseudo-states which are initially uncorrelated with the sensor biases. Here, we investigate how the characteristics of gyrocompass alignment errors can be shown to be conceptually equivalent in the two SDINS error models. Moreover, we explain the difference between the two approaches and show that the two approaches give the same result.

In the next section, the characteristics of gyrocompass alignment errors in the conventional SDINS error model are investigated and in section III, a modified SDINS error model is derived and compared with the conventional error model. In section IV, covariance analysis is performed, and the conclusion is given in the final section.

II. CHARACTERISTICS OF GYROCOMPASS ALIGNMENT ERRORS IN CONVENTIONAL SDINS ERROR MODEL

In this section, we look into the characteristics of gyrocompass alignment errors in the conventional error model. In [12], the heading-sensitive characteristic of initial north level tilt was analyzed through the mathematical and geometric interpretation. Here, we extend the analysis to investigate the characteristics of initial east level tilt and initial heading error in detail.

A conventional SDINS error model is obtained by adding the coordinate transformation matrix to the gimballed INS error model. For the purpose of analyzing the characteristics of gyrocompass alignment errors, we have modified Goshen-Meskin and Bar-Itzhack's INS error model[15]. A local level NED-frame is used as the navigation frame. When we consider the characteristics of gyrocompass alignment errors, since the position error model is not changed, it is omitted. The accelerometer and gyro errors are considered as random biases. Then the conventional SDINS error model augmented with the sensor biases can be represented by

$$\dot{\underline{x}} = F\underline{x} \tag{1}$$

where the state variable $\underline{x}$ is given by

$$\begin{aligned} \underline{x} &= [\delta\underline{v}^{n^T} \vdots \underline{\psi}^{n^T} \vdots \underline{\nabla}^{b^T} \vdots \underline{\varepsilon}^{b^T}]^T \\ &= [\delta v_N, \delta v_E, \delta v_D \vdots \delta\psi_N, \delta\psi_E, \delta\psi_D \vdots \nabla_x, \nabla_y, \nabla_z \vdots \varepsilon_x, \varepsilon_y, \varepsilon_z]^T \end{aligned} \tag{2}$$

where $\delta\underline{v}$ is the velocity error; $\underline{\psi}$ is the attitude error; $\underline{\nabla}$ is the accelerometer bias error; $\underline{\varepsilon}$ is the gyro bias error; the superscripts n and b denote navigation and body frames respectively; the subscripts N, E and D denote the north, east and down components of the navigation frame respectively; the subscripts x, y and z denote the each components of the body frame. The system dynamics matrix F can be represented by

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0_{3\times3} \\ 0_{3\times3} & F_{22} & 0_{3\times3} & F_{24} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{3}$$

where $0_{3\times3}$ is zero matrix of indicated dimension and

$$F_{11} = \begin{bmatrix} 0 & \tilde{\omega}_D & -\tilde{\omega}_E \\ -\tilde{\omega}_D & 0 & \tilde{\omega}_N \\ \tilde{\omega}_E & -\tilde{\omega}_N & 0 \end{bmatrix} \tag{4}$$

where

$$\underline{\tilde{\omega}} = \begin{bmatrix} \tilde{\omega}_N \\ \tilde{\omega}_E \\ \tilde{\omega}_D \end{bmatrix} = \begin{bmatrix} (2\Omega + \dot{l})\cos L \\ -\dot{L} \\ -(2\Omega + \dot{l})\sin L \end{bmatrix} \tag{5}$$

in which Ω is the Earth rate, l is the geographic longitude, and L is the geographic latitude. F_{12} is the matrix

$$F_{12} = \begin{bmatrix} 0 & -f_D & f_E \\ f_D & 0 & -f_N \\ -f_E & f_N & 0 \end{bmatrix} \tag{6}$$

where f_N, f_E and f_D are the specific forces. And F_{13} and F_{24} represent the coordinate transformation matrix C_b^n relating the body frame to the navigation frame. Finally F_{22} is defined as follows.

$$F_{22} = \begin{bmatrix} 0 & \omega_D & -\omega_E \\ -\omega_D & 0 & \omega_N \\ \omega_E & -\omega_N & 0 \end{bmatrix} \tag{7}$$

where

$$\underline{\omega} = \begin{bmatrix} \omega_N \\ \omega_E \\ \omega_D \end{bmatrix} = \begin{bmatrix} (\Omega + \dot{l})\cos L \\ -\dot{L} \\ -(\Omega + \dot{l})\sin L \end{bmatrix}. \tag{8}$$

Consider a stationary self-alignment at the fixed position excluding the Earth pole. The stationary conditions are given as follows.

$$\begin{bmatrix} f_N \\ f_E \\ f_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g(0) \end{bmatrix}, \tag{9a}$$

$$\begin{bmatrix} \omega_N \\ \omega_E \\ \omega_D \end{bmatrix} = \begin{bmatrix} \Omega\cos L(0) \\ 0 \\ -\Omega\sin L(0) \end{bmatrix} = \begin{bmatrix} \Omega_N(0) \\ 0 \\ \Omega_D(0) \end{bmatrix} \tag{9b}$$

where index 0 means the value during the alignment; $g(0)$ is the gravitational acceleration at the alignment position. The sensor errors are considered as random biases and the disturbance is neglected. Then, at the end of gyrocompass alignment stage, when $\delta\underline{v}^n = \delta\dot{\underline{v}}^n = \dot{\underline{\psi}}^n = 0$ is satisfied in (1), the system is not completely observable and the steady-state alignment errors are affected by the sensor biases as follows[1,6-9].

$$\psi_N(0) = \frac{\nabla_E(0)}{g(0)} \tag{10a}$$

$$\psi_E(0) = -\frac{\nabla_N(0)}{g(0)} \tag{10b}$$

$$\psi_D(0) = -\frac{\varepsilon_E(0)}{\Omega_N(0)} - \frac{\nabla_E(0)\tan L(0)}{g(0)} \tag{10c}$$

where index 0 means the value during the alignment; $\psi_i(0)(i = N, E, D)$ is the alignment error in the steady state. Provided that $|\frac{\varepsilon_E(0)}{\Omega_N(0)}| \gg |\frac{\nabla_E(0)\tan L(0)}{g(0)}|$, the equation (10c) can be reduced to

$$\psi_D(0) = -\frac{\varepsilon_E(0)}{\Omega_N(0)}. \tag{10d}$$

In (10), the sensor biases in navigation frame are related to those in body frame as follows.

$$\underline{\nabla}^n(0) = C_b^n(0)\underline{\nabla}^b \tag{11a}$$

$$\underline{\varepsilon}^n(0) = C_b^n(0)\underline{\varepsilon}^b \tag{11b}$$

where $C_b^n(0)$ is the transformation matrix during the alignment. When navigation starts, the steady-state alignment errors in (10) are assigned to initial attitude errors in navigation. Then in (1), if the initial attitude error and the sensor bias states are assumed to be jointly Gaussian random vectors of zero mean, they come to be correlated with each other.

$$E[\psi_N(0)\nabla_E] \neq 0 \tag{12a}$$

$$E[\psi_E(0)\nabla_N] \neq 0 \tag{12b}$$

$$E[\psi_D(0)\varepsilon_E] \neq 0 \tag{12c}$$

where $E[\cdot]$ means the expectation of $[\cdot]$.

Now let us investigate the navigation error propagation characteristics of such initial attitude errors. In order to simplify the analysis, several assumptions are used. That is, it is supposed that the coordinate transformation matrix from the body frame to the navigation frame has a roll-pitch-yaw convention and the body frame in the stationary alignment coincides with the navigation frame such as

$$C_b^n(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

Furthermore, the navigation at rest satisfies the stationary conditions as given in (9) and the initial conditions of velocity error states are given by

$$\delta\underline{v}^n(0) = \delta\dot{\underline{v}}^n(0) = \underline{0}. \tag{14}$$

Using these assumptions, first we analyze the relationship between initial east level tilt and the north component of accelerometer biases. Neglecting the Coriolis terms in (1), we take the north velocity error equation and consider the terms related to the north component of accelerometer biases. Then the resulting equation is written by

$$\delta\dot{v}_N = -f_D\psi_E + [C_b^n\underline{\nabla}^b]_N \tag{15}$$

where $[\cdot]_N$ means the north component of $[\cdot]$. Provided that an SDINS in stationary navigation maintains the alignment attitude, that is, $C_b^n = C_b^n(0)$, using (9a), (10b) and (13), (15) becomes

$$\begin{aligned} \delta\dot{v}_N &= g(0)\psi_E(0) + C_{11}\nabla_x \\ &= g(0)\cdot\{-\frac{C_{11}(0)\nabla_x}{g(0)}\} + C_{11}\nabla_x \\ &= 0 \end{aligned} \tag{16}$$

which shows that there is the cancellation between the initial east level tilt and the north component of accelerometer biases. Thus the north velocity error does not occur for $\delta v_N(0) = 0$. On the other hand, if the SDINS changes the heading by 180 degrees, that is,

$$C_b^n = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{17}$$

then (15) becomes

$$\delta\dot{v}_N = -2\nabla_x \tag{18a}$$

or equivalently

$$\delta\dot{v}_N = 2g(0)\psi_E(0). \tag{18b}$$

It is observed that the summation occurs between the initial level tilt and the north component of accelerometer biases. Note also that in this case, a pitch change of 180 degrees can also cause the summation.

Next we look into the relationship between initial heading error and the east component of gyro biases. We take the north velocity error equation and the east attitude error equation from (1), neglecting the Coriolis terms. Differentiate the north axis velocity error equation and insert the east axis attitude error equation into it. Selecting the terms related to the east component of gyro biases, we obtain

$$\begin{aligned} \delta\ddot{v}_N &= -f_D\dot{\psi}_E \\ &= -f_D\{\Omega_N\psi_D + [C_b^n\underline{\varepsilon}^b]_E\}. \end{aligned} \tag{19}$$

Then, if an SDINS maintains the alignment attitude, inserting (9a),(10d) and (13) into (19) yields

$$\begin{aligned} \delta\ddot{v}_N &= g(0)\{\Omega_N(0)\psi_D(0) + C_{22}\varepsilon_y\} \\ &= g(0)\{\Omega_N(0)\cdot\frac{-C_{22}(0)\varepsilon_y}{\Omega_N(0)} + C_{22}\varepsilon_y\} \\ &= 0 \end{aligned} \tag{20}$$

where since the east component of gyro biases is canceled by the initial heading error, the north velocity error does not appear for $\delta v_N(0) = \delta\dot{v}_N(0) = 0$. On the other hand, if the heading is rotated by 180 degrees, then inserting (17) into (19) results in

$$\delta\ddot{v}_N = -2g(0)\varepsilon_y \tag{21a}$$

or equivalently

$$\delta\ddot{v}_N = 2g(0)\Omega_N(0)\psi_D(0). \tag{21b}$$

We can again observe the summation between the initial heading error and the east component of gyro biases. It can be noted that in this case, a roll change of 180 degrees can also produce the summation.

III. CHARACTERISTICS OF GYROCOMPASS ALIGNMENT ERRORS IN MODIFIED SDINS ERROR MODEL

In this section, we derive a modified SDINS error model where the attitude error states are transformed into new states which are initially uncorrelated with the sensor biases. And we show through the mathematical analysis that the characteristics of gyrocompass alignment errors both in the modified error model and in the conventional error model are equivalent to each other.

A. Derivation of Modified SDINS Error Model

To make the initial attitude errors be uncorrelated with the sensor biases, we define a new state variable $\underline{\gamma}$, so-called a pseudo-state vector which is transformed from the attitude error state vector $\underline{\psi}$ of (1) by a linear transformation as follows.

$$\underline{z} = T\underline{x} \tag{22}$$

where the state variable $\underline{x}$ is given by (2) and the transformed state variable $\underline{z}$ is given by

$$\underline{z} = [\delta\underline{v}^{n^T} \vdots \underline{\gamma}^{n^T} \vdots \underline{\nabla}^{b^T} \vdots \underline{\varepsilon}^{b^T}]^T. \tag{23}$$

And the transformation matrix T is represented by

$$T = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & T_{23} & T_{24} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{24}$$

where $I_{3\times3}$ is the third-order identity matrix and

$$T_{23} = \begin{bmatrix} -\frac{C_{21}(0)}{g(0)} & -\frac{C_{22}(0)}{g(0)} & -\frac{C_{23}(0)}{g(0)} \\ \frac{C_{11}(0)}{g(0)} & \frac{C_{12}(0)}{g(0)} & \frac{C_{13}(0)}{g(0)} \\ 0 & 0 & 0 \end{bmatrix}, \tag{25a}$$

$$T_{24} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{C_{21}(0)}{\Omega_N(0)} & \frac{C_{22}(0)}{\Omega_N(0)} & \frac{C_{23}(0)}{\Omega_N(0)} \end{bmatrix}. \tag{25b}$$

In (22), the pseudo-state components can be rewritten in the following simple form.

$$\gamma_N = \psi_N - \frac{\nabla_E(0)}{g(0)} \tag{26a}$$

$$\gamma_E = \psi_E + \frac{\nabla_N(0)}{g(0)} \tag{26b}$$

$$\gamma_D = \psi_D + \frac{\varepsilon_E(0)}{\Omega_N(0)}. \tag{26c}$$

In (22)-(26), we see that since linear transformations of Gaussian random variables are also Gaussian random variables, the pseudo-states and the sensor biases come to be always initially uncorrelated and independent regardless of the alignment attitude.

$$E[\gamma_N(0)\nabla_E] = 0 \tag{27a}$$

$$E[\gamma_E(0)\nabla_N] = 0 \tag{27b}$$

$$E[\gamma_D(0)\varepsilon_E] = 0 \tag{27c}$$

which differ from (12) in the conventional error model because off-diagonal terms disappear. However, inserting (22) into (1) results in a modified SDINS error model as follows.

$$\begin{aligned} \dot{\underline{z}} &= TFT^{-1}\underline{z} \\ &= \begin{bmatrix} \bar{F}_{11} & \bar{F}_{12} & \bar{F}_{13} & \bar{F}_{14} \\ 0_{3\times3} & \bar{F}_{22} & \bar{F}_{23} & \bar{F}_{24} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \begin{bmatrix} \delta\underline{v}^n \\ \underline{\gamma}^n \\ \underline{\nabla}^b \\ \underline{\varepsilon}^b \end{bmatrix} \end{aligned} \tag{28}$$

where

$$\bar{F}_{11} = F_{11} \tag{29a}$$

$$\bar{F}_{12} = F_{12} \tag{29b}$$

$$\bar{F}_{22} = F_{22} \tag{29c}$$

$$F_{13} = \begin{bmatrix} C_{11} + \frac{f_D C_{11}(0)}{g(0)} & C_{12} + \frac{f_D C_{12}(0)}{g(0)} & C_{13} + \frac{f_D C_{13}(0)}{g(0)} \\ C_{21} + \frac{f_D C_{21}(0)}{g(0)} & C_{22} + \frac{f_D C_{22}(0)}{g(0)} & C_{23} + \frac{f_D C_{23}(0)}{g(0)} \\ C_{31} - \frac{f_N C_{11}(0)}{g(0)} - \frac{f_E C_{21}(0)}{g(0)} & C_{32} - \frac{f_N C_{12}(0)}{g(0)} - \frac{f_E C_{22}(0)}{g(0)} & C_{33} - \frac{f_N C_{13}(0)}{g(0)} - \frac{f_E C_{23}(0)}{g(0)} \end{bmatrix} \tag{29d}$$

$$\bar{F}_{23} = \begin{bmatrix} -\frac{\omega_D C_{11}(0)}{g(0)} & -\frac{\omega_D C_{12}(0)}{g(0)} & -\frac{\omega_D C_{13}(0)}{g(0)} \\ -\frac{\omega_D C_{21}(0)}{g(0)} & -\frac{\omega_D C_{22}(0)}{g(0)} & -\frac{\omega_D C_{23}(0)}{g(0)} \\ \frac{\omega_N C_{11}(0)+\omega_E C_{21}(0)}{g(0)} & \frac{\omega_N C_{12}(0)+\omega_E C_{22}(0)}{g(0)} & \frac{\omega_N C_{13}(0)+\omega_E C_{23}(0)}{g(0)} \end{bmatrix} \tag{29e}$$

$$F_{14} = \begin{bmatrix} -\frac{f_E C_{21}(0)}{\Omega_N(0)} & -\frac{f_E C_{22}(0)}{\Omega_N(0)} & -\frac{f_E C_{23}(0)}{\Omega_N(0)} \\ \frac{f_N C_{21}(0)}{\Omega_N(0)} & \frac{f_N C_{22}(0)}{\Omega_N(0)} & -\frac{f_N C_{23}(0)}{\Omega_N(0)} \\ 0 & 0 & 0 \end{bmatrix} \tag{29f}$$

$$F_{24} = \begin{bmatrix} C_{11} + \frac{\omega_E C_{21}(0)}{\Omega_N(0)} & C_{12} + \frac{\omega_E C_{22}(0)}{\Omega_N(0)} & C_{13} + \frac{\omega_E C_{23}(0)}{\Omega_N(0)} \\ C_{21} - \frac{\omega_N C_{21}(0)}{\Omega_N(0)} & C_{22} - \frac{\omega_N C_{22}(0)}{\Omega_N(0)} & C_{23} - \frac{\omega_N C_{23}(0)}{\Omega_N(0)} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}. \tag{29g}$$

B. Characteristics of Gyrocompass Alignment Errors

We look into the characteristics of gyrocompass alignment errors in the modified SDINS error model and compare with those in the conventional SDINS error model. Considering the assumptions used in Section II, we first analyze the relationship between the initial east level tilt and the north component of accelerometer biases. From (28), in case that (9a) and $\gamma_E(0) = 0$ hold, we obtain the following equation related to the north component of accelerometer biases.

$$\begin{aligned} \delta\dot{v}_N &= [C_b^n\underline{\nabla}^b]_N - [C_b^n(0)\underline{\nabla}^b]_N \\ &= \{C_{11} - C_{11}(0)\}\nabla_x \end{aligned} \tag{30}$$

where it is seen that the cancellation occurs in case that an SDINS maintains the alignment attitude, and the summation takes place in case that the SDINS is rotated by 180 degrees with respect to heading or pitch axis. Now investigate the relationship between the initial heading error and the east component of gyro biases. Then from (28), we also obtain the following equation related to the east component of gyro biases.

$$\begin{aligned} \delta\ddot{v}_N &= g(0)\{[C_b^n\underline{\varepsilon}^b]_E - [C_b^n(0)\underline{\varepsilon}^b]_E\} \\ &= g(0)\{C_{22} - C_{22}(0)\}\varepsilon_y \end{aligned} \tag{31}$$

which reveals the cancellation in case of no attitude change and the summation due to the rotation of 180 degrees with respect to heading or roll axis. Therefore, these results show that the characteristics of the initial level tilt and the initial heading error in the modified error model are equivalent to those in the conventional error model.

IV. COVARIANCE ANALYSIS

In the previous sections, we derived the modified SDINS error model and showed that the characteristics of gyrocompass alignment errors in the modified error model are equivalent to those in the conventional model. The main difference between the two error models is whether the initial attitude error states are correlated with the sensor biases or not. In this section, considering such a point of the two equivalent error models, we propose two approaches to covariance analysis on the characteristics of gyrocompass alignment errors. The first approach is to use the conventional SDINS error model. There, since the initial attitude error states are correlated with the sensor bias states, off-diagonal components are included in initial covariance matrix. The second approach is to use the modified SDINS error model, where since the initial attitude error states are uncorrelated with the sensor biases, the initial covariance matrix contains only diagonal components. In order to compare the two approaches, we introduce simplified covariance analysis examples for the initial east level tilt case and initial heading error case respectively. And for each case, we compare the analytic solutions of linear variance equations. The assumptions used in Section II also hold here.

A. Initial East Level Tilt Case

For this case, in order to perform covariance analysis using the first approach, from the conventional SDINS error model of (1), we take a simplified conventional error model including the relationship between the east level tilt and the north component of accelerometer biases as follows.

$$\begin{bmatrix} \delta\dot{v}_N \\ \dot{\psi}_E \\ \dot{\nabla}_x \end{bmatrix} = \begin{bmatrix} 0 & -f_D & C_{11} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v_N \\ \psi_E \\ \nabla_x \end{bmatrix} \qquad (32)$$

where all error states are considered as Gaussian random variables with zero mean. Using the system model of (32), let us perform covariance analysis on the stationary navigation error after gyrocompassing. Here, the discrete linear variance equation for covariance analysis is represented by

$$P(k) = \Phi(k,0)P(0)\Phi^T(k,0) \qquad (33)$$

where $\Phi(k,0) = e^{Ak\Delta t}$ is the state transition matrix and $P(k)$ is the covariance matrix whose initial covariance matrix includes the off-diagonal terms such as

$$P(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \psi_E^2(0) & \psi_E(0)\nabla_x \\ 0 & \psi_E(0)\nabla_x & \nabla_x^2 \end{bmatrix}. \qquad (34)$$

Then, through the analytic calculation using (9a),(10b) and (13), the standard deviation of the north velocity error after time k can be represented by

$$\sigma_{\delta v_N}(k) = \sqrt{P_{11}(k)} = | C_{11}(k) - C_{11}(0) |\ \nabla_x\, k\Delta t \qquad (35)$$

where P_{ij} is a ij-component of P and Δt is the time interval. In (35), if an SDINS in navigation maintains the alignment attitude, that is, $C_{11}(k) = C_{11}(0) = 1$, then $\sigma_{\delta v_N}(k)$ reveals to be zero so that the cancellation between them occurs. On the other hand, if the SDINS changes the heading by 180 degrees, rather there occurs the summation. Next to use the second approach, we modify the conventional error model of (32) using the pseudo-state γ_E which is defined in (26b). Then (32) is changed into

$$\begin{bmatrix} \delta\dot{v}_N \\ \dot{\gamma}_E \\ \dot{\nabla}_x \end{bmatrix} = \begin{bmatrix} 0 & -f_D & C_{11} + \frac{f_D C_{11}(0)}{g(0)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v_N \\ \gamma_E \\ \nabla_x \end{bmatrix}. \qquad (36)$$

If (36) is applied to covariance analysis where the initial covariance matrix is given by the diagonal matrix such as

$$P(0) = diag(0,0,\nabla_x^2), \qquad (37)$$

then, the standard deviation of the north velocity error after time k is represented by (35) as well. Thus we can also see the cancellation or summation according to heading change. Hence, this analysis shows that for the initial east level tilt case, the two approaches produce the same result.

B. Initial Heading Error Case

In order to apply the first approach, from the conventional SDINS error model of (1), we take a simplified conventional error model including the relationship between the heading error and the east component of gyro biases as follows.

$$\begin{bmatrix} \delta\dot{v}_N \\ \dot{\psi}_E \\ \dot{\psi}_D \\ \dot{\varepsilon}_y \end{bmatrix} = \begin{bmatrix} 0 & -f_D & 0 & 0 \\ 0 & 0 & \omega_N & C_{22} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v_N \\ \psi_E \\ \psi_D \\ \varepsilon_y \end{bmatrix}. \qquad (38)$$

If we try covariance analysis using (38) where initial covariance matrix is given by the non-diagonal matrix such as

$$P(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_D^2(0) & \psi_D(0)\varepsilon_y \\ 0 & 0 & \psi_D(0)\varepsilon_y & \varepsilon_y^2 \end{bmatrix}, \qquad (39)$$

then, using (9),(10d) and (13), the standard deviation of δv_N after time k is represented by

$$\sigma_{\delta v_N}(k) = \sqrt{P_{11}(k)} = | C_{22}(k) - C_{22}(0) |\ \varepsilon_y\, g(0)\frac{k^2\Delta t^2}{2}. \qquad (40)$$

In (40), provided that an SDINS maintains the alignment attitude, $\sigma_{\delta v_N}(k)$ becomes zero so that the cancellation occurs. On the other hand, if the SDINS undergoes a heading change of 180 degrees, there occurs the summation between them. Next in order to apply the second approach, we derive the modified error model using the pseudo-state variables γ_E and γ_D which are defined in (26b) and (26c). The only difference is that in (26b), γ_E is assumed to be equal to ψ_E, because the north component of accelerometer biases is not considered here. Then the resulting system model is represented by

$$\begin{bmatrix} \delta\dot{v}_N \\ \dot{\gamma}_E \\ \dot{\gamma}_D \\ \dot{\varepsilon}_y \end{bmatrix} = \begin{bmatrix} 0 & -f_D & 0 & 0 \\ 0 & 0 & \omega_N & C_{22} - \frac{\omega_N C_{22}(0)}{\Omega_N(0)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v_N \\ \gamma_E \\ \gamma_D \\ \varepsilon_y \end{bmatrix}. \qquad (41)$$

If we perform covariance analysis using (41) under the condition that the initial covariance matrix is given by the diagonal matrix such as

$$P(0) = diag(0,\ 0,\ 0,\ \varepsilon_y^2), \qquad (42)$$

then, the standard deviation of δv_N after time k is represented by (40) as well. Thus we can also see the cancellation or summation according to heading change. Therefore, this analysis shows that for the initial heading error case, the two approaches produce the same result.

As seen from the results of the case A and B, it is obvious that the two approaches which use the two equivalent SDINS error models respectively give the same result, when we utilize covariance analysis for the purpose of analyzing the characteristics of gyrocompass alignment errors. We note that although the special cases were chosen to demonstrate the identity between the two approaches, the general cases will involve more arguments, and the results will not be changed.

V. SUMMARY AND CONCLUSIONS

Two approaches to covariance analysis for SDINS have been presented to analyze the characteristics of gyrocompass alignment errors which are caused by the inertial sensor biases in stationary gyrocompassing. One is to introduce the correlation between the initial attitude errors and the inertial sensor biases into the conventional SDINS error model. The other is to use a modified SDINS error model where the attitude error states are transformed into new states initially uncorrelated with the sensor bias states. In the former approach, the initial covariance matrix includes off-diagonal components, while the latter approach does not.

Through the mathematical interpretation, it is shown that the characteristics of gyrocompass alignment errors in the modified SDINS error model are conceptually equivalent to those in the conventional SDINS error model. Furthermore, by comparing the analytic solutions of covariance analysis examples, it is shown that the two approaches produce the same result.

Due to the simplicity of covariance analysis, the two covariance analyses presented here can serve as more efficient and accurate means to evaluate a stationary performance of SDINS or to select a mission flight trajectory considering the gyrocompass characteristics.

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