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# End-point Position Control of a Flexible Arm by PID Self-Tuning Fuzzy Controller

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Abstract

This paper presents an end-point position control of 1-link flexible robot arm by the PID self-tuning fuzzy algorithm. The governing equation is derived by the extended Hamilton's principle and based on the Bernoullie-Euler beam theory. The governing equation is solved by applying the Laplace transform and the numerical inversion method. The arm is mounted on the translational mechanism driven by a ballscrew whose rotation is controlled by dc servomotor. Tip position is controlled by the PID self-tuning fuzzy algorithm so that it follows a desired position. This paper shows the experimental and theoretical results of tip displacement, and also shows the good effects reducing the residual vibration of the end-point.

## 1. Introduction

Many papers are presented on flexible robot arm to meet the purpose of high speed and accuracy of robot system for the increase of both the performance of robot and the productability.[1-5]

In the case of flexible robot arm, it can transfer heavier weight compared to its own weight, and have many merits about the high speed mobility, low power and low energy consumption.

There are also studies about the position control of flexible robot arm using fuzzy theory proposed by Zadeh in 1965.[6] But this algorithm has a weak point to make the look-up table through inference using the quantitative values of input and output variables for real-time control.

Nevertheless, the FLC(Fuzzy Logic Controller) is popular to the control engineers. The reason is that this algorithm uses the expert's experiences and knowledges instead of the mathematical modelling of the control system.

The PID Self-Tuning Fuzzy Controller is used in this paper. The performance of this controller depends on the quantification set that fuzzy rule is quantified and changed to the look-up table. The velocity is used in coarse control, and the displacement is used to refine the output of controller to reduce the residual vibration effectively.[7-9]

This controller has good effects not only on the reduction of residual vibration but on the fastness and exactness of the end-point when the tip moves translationally from the initial position to the desired position. The controllability and adaptability was checked when comparing the result of the mathematical analysis system with that of the experimental system.

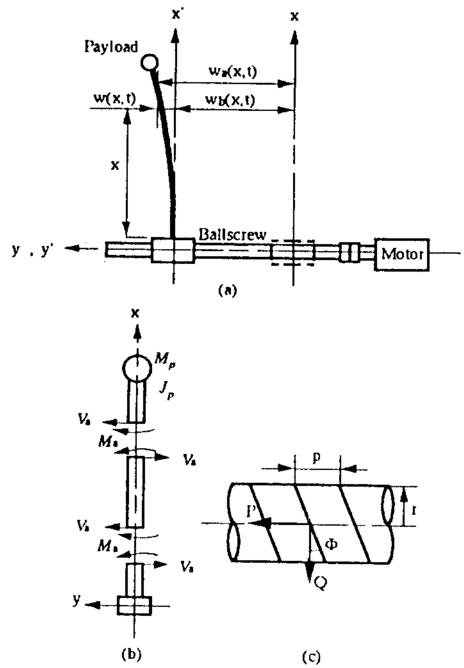


Fig.1 A sketch of the flexible arm and details of ballscrew with system variables.

## 2. The Governing Equation of the System

In this paper the system is composed of a dc servomotor, a ballscrew mechanism, flexible robot arm, and tip mass. To avoid the complexity on the theory analysis, the flexible arm is supposed to be Bernoulli-Euler beam neglecting the rotational interia and shearing deformation.

In this system the gravity force and the backlash are ignored, but the damping force is cosidered. And it is supposed that the flexible arm vibrates only on the x-y plane. $W_a(x, t)$ is the overall displacement of the arm from the origin of the absolute coordinate system, $W(x,t)$ the displacement caused by the deflection of the arm at the distant x from the base, and $W_b(t)$ the displacement of the base so that we obtain

$$W_a(x,t) = W(x,t) + W_b(x,t) \tag{1}$$

The Bernoulli-Euler beam equation, 3 boundary conditions, and force equilibrium of the base are obtained as given below by applying Extended Hamilton's principle to this system.

$$\rho A\frac{\partial^2}{\partial t^2}W(x,t) + EI(1 + c\frac{\partial}{\partial t})\frac{\partial^4}{\partial x^4}W(x,t) = 0 \quad (2)$$

$$W(0,t) = 0, \qquad \frac{\partial}{\partial x}W(0,t) = 0 \quad (3)$$

$$EI(1+c\frac{\partial}{\partial t})\frac{\partial^2}{\partial x^2}\cdot W(L,t) = -Jp\frac{\partial^2}{\partial t^2}\frac{\partial}{\partial x}\cdot W(L,t) \quad (4)$$

$$EI(1+c\frac{\partial}{\partial t})\frac{\partial^3}{\partial x^3}\cdot W(L,t) = Mp\frac{\partial^2}{\partial t^2}\cdot W(L,t) \quad (5)$$

$$M_b\frac{\partial^2}{\partial t^2}\cdot W(0,t) = C_b\cdot\frac{\partial}{\partial t}\cdot W(0,t) - EI(1+c\frac{\partial}{\partial t})\frac{\partial^3}{\partial x^3}\cdot W(0,t) + Q(t) \quad (6)$$

where E is the Young's modulus, $\rho$ the mass density, A cross sectional area, I the moment of inertia of the arm, $M_p$ the mass of the payroad, $J_p$ the polar moment of inertia of payroad, $M_b$ the mass of the base, $C_b$ the damping coefficient at the base, x the displacement, t the time, and c the damping coefficient at the arm.

As shown in Fig.1(a)-(c), the force equilibrium equation between the base and ballscrew, and the relationship between the rotation angle of the motor shaft and the base displacement are given by

$$P(t) = Q(t)\cdot\tan(\phi+\varphi) = Q(t)\cdot\frac{\mu\cdot\cos\phi + \sin\varphi}{\cos\phi - \mu\cdot\sin\varphi}$$

$$\theta(t) = \frac{2\pi}{p}W(0,t) \quad (7)$$

where Q(t) is the circumferential force acting on the screw thread, P(t) the axial force acting to the base trough the ballscrew, $\mu(=\tan\varphi)$ the friction coefficient between the base and ballscrew, $\phi$ the lead angle, $\varphi$ the friction angle, $\theta(t)$ the rotation angle of the motor, and p the pitch of the ballscrew.

The ballscrew connected to the motor by coupling is driven by motor torque, $T(t)=K_t\cdot i_a(t)$ and the equation of the moment equilibrium to the motor shaft is obtained as follows

$$(J_m + J_c + J_s)\frac{d^2\theta(t)}{dt^2} = -C_m\frac{d\theta(t)}{dt} - P(t)\cdot r + T(t) \quad (8)$$

where $J_m$, $J_c$, and $J_s$ are the inertia moments of the motor shaft, the flexible coupling, and the ballscrew respectively, $C_m$ the damping coefficient between the motor and ballscrew, r the pitch radius of the ballscrew.

In motor armature circuit, the circuit equation is defined to be

$$\frac{L_a}{R_a}\frac{dT(t)}{dt} + T(t) + \frac{K_tK_b}{R_a}\frac{2\pi}{p}\frac{dW_b(t)}{dt} = \frac{K_t}{R_a}\cdot e_a(t) \quad (9)$$

where $i_a(t)$ is the armature current, $L_a$ and $R_a$ are the inductance and the resistance of the motor armature, $K_b$ the back electromotive force constant, and $e_a(t)$ the amature voltage. After applying Laplace transform to the Eq.(8), (9) and introducing $\xi=\frac{L_a}{R_a}$, We get

$$M_{eq}s^2W_b + C_{eq}sW_b + EI(1+s)\frac{d^3W(0)}{dx^3} = K_{eq}E_a \quad (10)$$

where,

$$M_{eq} = \frac{2\pi}{p}\frac{1}{r\cdot\tan(\phi+\varphi)}(J_m + J_c + J_e) + M_b$$

$$C_{eq} = C_b + \frac{2\pi}{p}\frac{1}{r\cdot\tan(\phi+\varphi)}(C_m + \frac{K_tK_b}{R_a}\frac{1}{(1+\xi s)})$$

$$K_{eq} = \frac{K_tK_b}{R_a}\frac{K_t}{R_a}\frac{1}{(1+\xi s)}$$

Applying Laplace transform to the Eq.(2), and we have

$$\frac{d^4W(x)}{dx^4} - \zeta^4W(x) = \zeta^4W_b, \quad \zeta^4 = -\frac{\rho A\cdot s^2}{EI(1 + cs)} \quad (11)$$

Then, the general solution of the Eq.(11) is

$$W(x,s) = \alpha\cos\zeta x + \beta\sin\zeta x + \gamma\cosh\zeta x + \delta\sinh\zeta x \quad (12)$$

After calculating Laplace transform to the Eq.(3)-(5), we have 2-equations about $\alpha$, $\beta$, and $\gamma$ by substituting these equation into the Eq.(12). Applying Laplace transform to the equations (6)-(9) and using the boundary conditions, we can obtain the equation about $\alpha$, $\beta$, and $\gamma$ and the matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ W(L) \end{bmatrix} \quad (13)$$

where

$$a_{11} = -J_p\cdot s^2\cdot\zeta\sin\zeta L - EI(1+cs)\zeta^2\cos\zeta L$$
$$a_{12} = J_p\cdot s^2\cdot\zeta(\cos\zeta L - \cosh\zeta L) - EI(1+cs)\zeta^2(\sin\zeta L + \sinh\zeta L)$$
$$a_{13} = J_p\cdot s^2\cdot\zeta\sinh\zeta L + EI(1+cs)\zeta^2\cosh\zeta L$$
$$a_{21} = M_p\cdot s^2\cos\zeta L - EI(1+cs)\zeta^3\sin\zeta L$$
$$a_{22} = M_p\cdot s^2(\sin\zeta L - \sinh\zeta L) + EI(1+cs)\zeta^3(\cos\zeta L + \cosh\zeta L)$$
$$a_{23} = M_p\cdot s^2\cosh\zeta L - EI(1+cs)\zeta^3\sin\zeta L$$
$$a_{31} = \cos\zeta L$$
$$a_{32} = \sin\zeta L - \sinh\zeta L$$
$$a_{33} = \cosh\zeta L$$

Applying Cramer's rule to the Eq.(13) to get the coefficient $\alpha$, $\beta$, and $\gamma$ leads to Eq.(14)

$$\alpha = \frac{\Delta\alpha}{\Delta}, \quad \beta = \frac{\Delta\beta}{\Delta}, \quad \gamma = \frac{\Delta\gamma}{\Delta} \quad (14)$$

where $\Delta$, $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ are matrix equations defined by

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \quad \begin{aligned} \Delta\alpha &= W(L)(a_{12}a_{23} - a_{13}a_{22}) \\ \Delta\beta &= W(L)(a_{13}a_{21} - a_{11}a_{23}) \\ \Delta\gamma &= W(L)(a_{11}a_{22} - a_{12}a_{21}) \end{aligned}$$

By using the given boundary conditions, the armature voltage Ea(s) of the motor is computed as

$$E_a(s) = \frac{\left[(M_{eq}s^2 + C_{eq}s)\frac{\Delta\alpha + \Delta\gamma}{\Delta} - 2\zeta^3 EI(1+cs)\frac{\Delta\beta}{\Delta}\right]}{K_{eq}} \quad (15)$$

As a result, we obtain the following equation .

$$w(L) = \frac{\Delta\alpha(\cos\zeta L - 1) + \Delta\beta(\sin\zeta L - \sinh\zeta L) + \Delta\gamma(\cosh\zeta L - 1)}{\Delta}$$

$$+ \frac{K_{eq}}{M_{eq}s^2 + C_{eq}s} E_a(s) + 2\zeta^3 EI(1+cs)\frac{1}{M_{eq}s^2 + C_{eq}s}\frac{\Delta\beta}{\Delta} \quad (16)$$

In this paper, we obtain the theoretical results from the computer simulation, where the weeks' algorithm[10] is used to calculate inverse transform of the Eq(16).

## 3. Fuzzy Controller

To design the fuzzy controller, we have to define the input/output variables of the system, and make the membership function to these variables and the fuzzy control rules to construct the basic knowledge. Determining the fuzzification strategy calculating the output of the controller by using the measured input is also needed.

In this study, the control system is single input and single output system. The input variables of the controller are composed of error e(k), the sum of error soe(k), and the change of error de(k). The output variable is the change of control input du(k) as shown in Eq.(17) - (20).

$$e(k) = sp - t\_d(k) \quad (17)$$
$$soe(k) = \Sigma e(K) \quad (18)$$
$$de(k) = e(k) - e(k-1) \quad (19)$$
$$du(k) = u(k) \quad (20)$$

where sp is the desired tip position of the flexible arm, t_d(k) is the tip position of the flexible arm at the k-th sampling time and du(k) is the output of the controller at the k-th. This is named as the PID Fuzzy Control because the method to calculate du(k) from e(k), soe(k) and de(k) is same as the PID control. Therefore, in case of PID control, it is easy to obtain the following equation

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{d}{dt}e(t) \quad (21)$$

Let e(t) be e(k), soe(k) be $\int e(t)dt$, and de(k) be $\frac{d}{dt}e(t)$, then the following equation is derived .

$$du(k) = GP\cdot e(k) + soe(k-1) + GI\cdot e(k) + GD\cdot de(k) \quad (22)$$

The input/output relationship of the fuzzy controller is given from the definition of the input/output variables.

$$R : e(k) \times soe(k) \times de(k) \rightarrow du(k) \quad (23)$$

The general fuzzy control rules expressed by the "IF-THEN" types are

Ri = IF e(k) is Ei and soe(k) is SOEi,
IF e(k) is Ei and de(k) is DEi, (24)
THEN du(k) is DUi. (i=1,2,----,n)

where Ei, SOEi, DEi, and DUi are the input/output fuzzy variables, and these are defined by the set of the 7 linguistic variables as shown in below

{ PB PM PS ZO NS NM NB }

The membership functions of each linguistic variables use triangular type, and the fuzzy control rule is constructed by the above fuzzy variables and 3 meta rules proposed by Macvicar-Whelm[11].

The Max-Min composition as the one of the inference methods is used to obtain the output of the controller by the linguistic fuzzy control rules and membership functions.

$$m\cdot DU(du) = MAX\{ MIN[ m\cdot Ei(e(K)),\ m\cdot SOEi(soe(k)),\ m\cdot DEi(de(k))]\},\quad i=(1,2,\text{---},n) \quad (25)$$

where m is the value of membership function. Applying the inference results calculated by the above mentioned method to the defuzzification algorithm leads to the output of the controller. The center of gravity method is used in the defuzzification algorithm

$$DU(k) = \frac{\Sigma\mu_{DU}(du_i)\cdot du_i}{\Sigma\mu_{DU}(du_i)} \quad (26)$$

In this paper, the fuzzification method to the error and the change of error for real-time control is realized by mapping the real values to the membership function and using the on-line process in the inference.

## 4. Design of the Self-Tuning Fuzzy Controller

GP, GI, GD, and GU are the scaling factors of input/output variables. This scaling factors are the elements to define the entire set of each fuzzy variables in the membership function. These factors applied to the input / output of the fuzzy controller are tuned to obtain the desired response on the basis of the overall performance of the system. The OV(overshot) and RT(rising time) of the transient response as the criteria for the performance evaluation are used to tune the scaling factors about input variables, but in general the most effective parameter of the input/output is the output scaling factor determining the controller's gain.

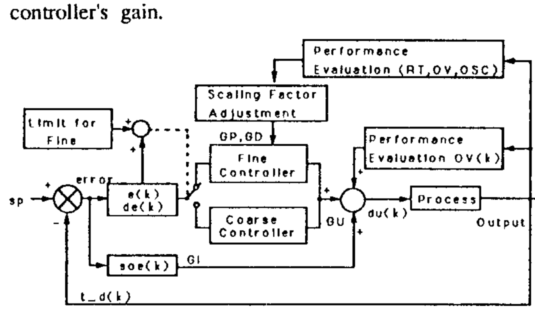


Fig. 2 The block diagram of the PID Self-Tuning Fuzzy Controller

Therefore, input scaling factors(GP, GI, GD) are set to constant and only output scaling factor(GU) is tuned at the process. That is, OV is the criterion for the performance evaluation. When the overshoot occurs, we have to decrease the output of the controller. Namely, GU must be decreased. In this paper, GU is tuned every sampling time without complex inference process and given by

$$\begin{aligned} GU(k) &= GU(k-1) + DGU(k) \\ DGU(k) &= \kappa \cdot OV(k) \end{aligned} \tag{27}$$

where $\kappa$ is the tunning coefficient, $OV(k)$ is the error when overshoot occurs.

It is an advantage that the velocity type controller has not so many control rules. But this is not suitable for the precise control moving the tip to the desired position. Because of this reason, this type is used in the coarse controller to decrease both the initial oscillation and rising time, and the position type is used in finer controller for the precision. The output is not continuous value but discrete one because the control input is made to the look-up table.

As a result, the precise control is not achieved near the desired position because of the residual vibration. By adding I-controller to the output of the PD type fuzzy controller, the residual vibration is decreased effectively. The self-tunning fuzzy controller used in this study is shown in Fig.2.

## 5. Experiment

Fig.3 shows the system block diagram for the experiment. The flexible arm is an aluminium beam with a rectangular cross section, the height 20mm, the width 2mm, and the length 850mm. The tip mass Mp is 0g, 50g, 75g, and 100g respectively.

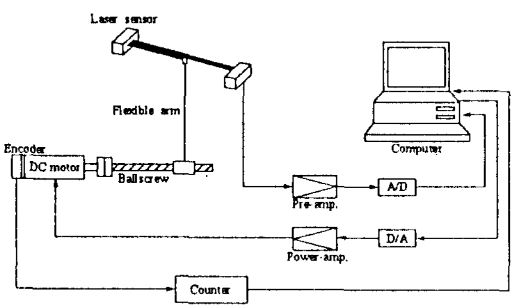


Fig.3 Experimental Setup

## 6. Theoretical and Experimental Results.

The desired tip displacement Wd* is set to 50mm. Fig.4 shows the theoretical and experimental results to the flexible arm of length 850mm with three different kinds of tip-mass by PID self tuning control algorithm. As shown in figure, a fluctuation appears in the simulation before the tip arriving at the desired position. But the residual oscillation of the tip decreases rapidly within about 3 seconds. Meanwhile, the experimental results show that the tip moves smoothly to the desired position without fluctuation. It also shows that heavier tip mass arm has lower natural frequency, and this means it has bigger flexibility. For example, in the case of arm length 850mm, we got the experimental data to have the natural frequencies of 2.25Hz, 1.53Hz, 1.24Hz, and 1.05Hz with repect to the tip mass of 0g, 50g, 75g, and 100g respectively. The settling times of these results are approximately in good agreement.

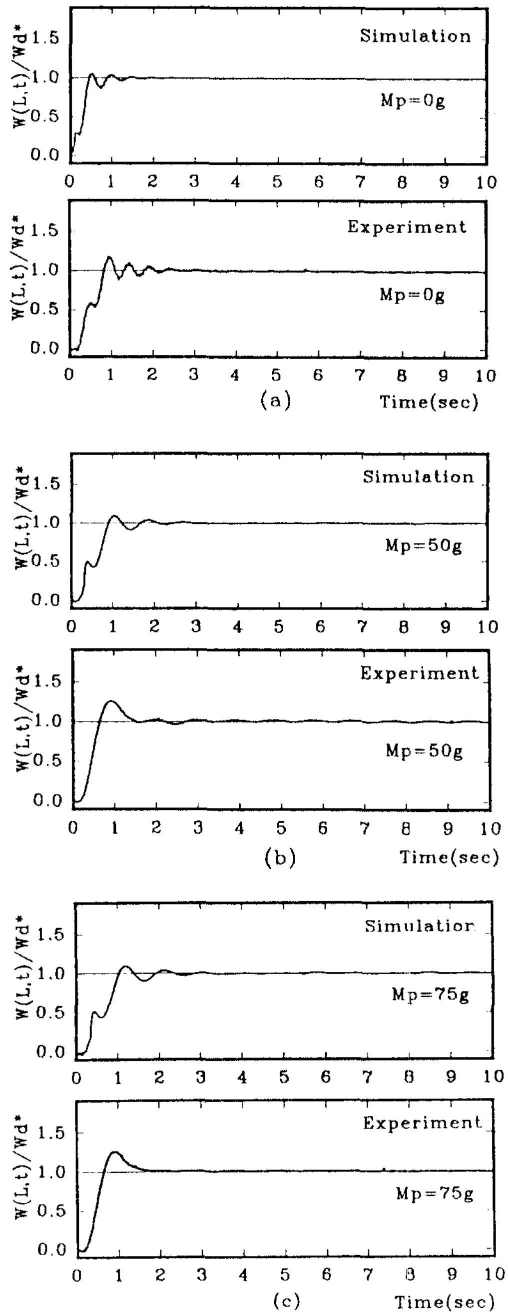


Fig.4 The theoretical and experimental results of the tip displacement with arm length 850mm in the case of three types of tip mass. (Mp= 0g, 50g, and 75g)

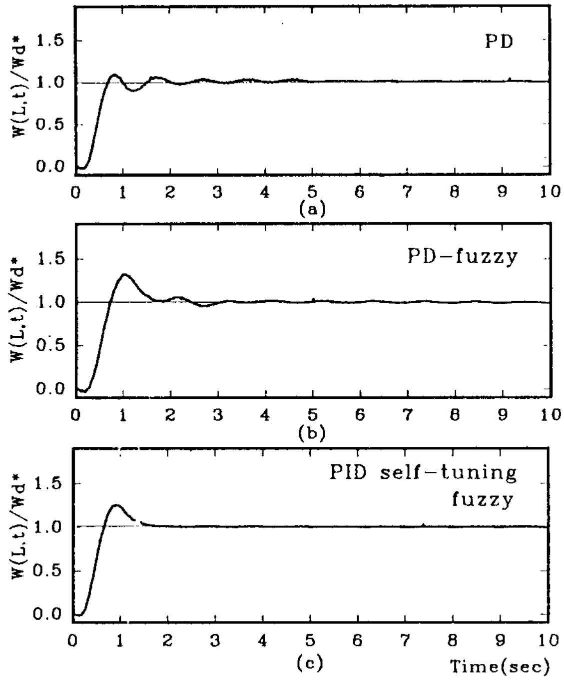


Fig.5 Experimental results of (a)PD-Control, (b)PD-Fuzzy control, and (c)PID Self Tuning Fuzzy control of flexible arm with length 850mm and tip mass 75g.

Fig.5 shows the experimental results of the PD, PD-type fuzzy, and PID self-tuning fuzzy control with respect to the flexible arm with length 850mm and tip mass 75g. As you can see, the PD-control has smaller overshoot than the other two control algorithms but residual oscillations near the desired position remain continuously for a long time.[12] The PD-type fuzzy controller has larger overshoot and longer settling time compared with the other two control algorithm.[8] But the PID self-tuning fuzzy controller studied in this paper shows good results compared with the PD-controller except for the overshoot and PD-type fuzzy controller. This controller gives the flexible arm to move smoothly and to settle rapidly to the desired position without residual oscilations after showing only one onershoot.

Fig.6 shows the relationship between the tip displacement and the base movement of the flexible arm, when the good experimental results appear in the above control algorithms.

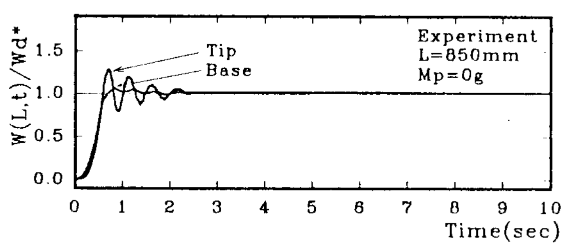


Fig.6 The experimental relationship between the tip displacement and base movement of the arm length 850mm without tip mass.

As we control the tip position of the flexible arm by using the data of the tip displacement, we often meet the resonance phenomena. To remove this resonance, experimently we let the direction of the base move against that of tip. As a result, in this paper the operating data must be sent to the system as late as the time corresponding to the half period of the natural frequency of the flexible arm. That is, the base follows the tip one half period later. Fig.6 explains one of this cases.

## 7. Conclusion

The PID Self-Tunning Fuzzy Algorithm has been used to increase the positioning accuracy and to reduce the residual vibration of 1-link flexible robot arm with tip mass moving translationally from the initial position to the commanded one. As a result, as the tip-mass of the flexible arm becomes heavier, the overshoot increases but the residual vibration decreases. The experimental results are approximately in good agreement with the theoretical results. Therfore, this algorithm is more effective on positioning the tip and reducing the residual vibration of 1-link flexible arm.

## References


[1]W.J.Book, et al., "Feedback Control of Beam Two Joint Systems with Distributed Flexibility," J. of Dynamic Systems, Meas. and Control, 97G(4), pp.424-431, 1975.

[2]R.H. Cannon and E. Schmitz, "Initial Experiments on the End-Point Control of a Flexible One-Link Robot," The Int. J. of Robotics Research, Vol.3, No.3, pp.62-75, 1984.

[3]E.Bayo, et al., "Inverse Dynamics of a Single-Link Flexible Robot. Analytical and Experimental Results," J. of Robotics and Automation, Vol.3, No.3, pp.150-157, 1988.

[4]S.C.Lee, S.Chonan and H.Inooka, "End-Point Positioning of One-Link Flexible Arm under Translational Motion," '90KACC, Vol.2, pp.890-895, 1990.

[5]S.C.Lee, D.Y.Pang, S.Chonan and H.Inooka, "Tip Position Control of 1-Link Flexible Arm by Inverse Dynamics," '91KACC, Vol.2, pp.1453-1458, 1991.

[6]J.W.Kim, Y.K.Ha, and Y.P.Park, "Vibration Control of a Flexible Arm Using Fuzzy Control Theory," Procedings of the 1st Int'l. Conference on Motion and Vibration Control, pp.517-522, Sept. 1992.

[7]Y.F.Li and C.C.Lau, "Development of Fuzzy Algorithms for Servo System," IEEE Control Systems Magazine, pp. 65-71, 1989.

[8]G.T.Yang, Y.C.Lee, and S.C.Lee, "Tip position Control of 1-Link Robot Arm by Fuzzy Algorithm," Procedings of the '92KSPE, pp.149-153, 1992.(In Korean)

[9]G.T.Yang, S.D.Ahn, and S.C.Lee, "Tip position Control of Flexible Robot Arm by Self-Tuning Fuzzy Algorithm," Procedings of the '93KSPE, pp.213-217, 1993.

[10]W.T.Weeks, "Numerical Inversion of Laplace Transforms Using Laguerre Funtions," J. of the Association for Computing Machinery, Vol.13, No.3, pp.419-426, 1966.

[11]P.J. Macvicar-Whelan, "Fuzzy Sets for Man-Machine Interaction," Int'l.J. of Man-Machine Studies, Vol.8, pp. 687-697, 1976

[12]S.C.Lee, et al., "Tip Vibration Control of a Single-Link Flexible Arm under Translational Motion," '92KACC, Vol.2, pp.607-612, 1992.